Conjecture on the infinity of primes obtained concatenating a prime \( p \) with \( p + 30k \)

**Abstract.** In this paper I make the following conjecture: for any \( p \) prime, \( p > 5 \), there exist an infinity of \( k \) positive integers such that the number \( q \) obtained concatenating to the right \( p \) with \( p + 30k \) is prime (examples: for \( p = 13 \), the least \( k \) for which \( q \) is prime is 2 because 1373 is prime; for \( p = 104729 \), the least \( k \) for which \( q \) is prime is 3 because 104729104819 is prime). It is notable the small values of \( k \) for which primes \( q \) are obtained, even in the case of primes \( p \) having 20 digits, so this formula could be a way to easily find, starting from a prime \( p \), a prime \( q \) having twice as many digits!

**Conjecture:**

For any \( p \) prime, \( p > 5 \), there exist an infinity of \( k \) positive integers such that the number \( q \) obtained concatenating to the right \( p \) with \( p + 30k \) is prime (examples: for \( p = 13 \), the least \( k \) for which \( q \) is prime is 2 because 1373 is prime; for \( p = 104729 \), the least \( k \) for which \( q \) is prime is 3 because 104729104819 is prime).

The sequence of the least \( k \) for which \( q \) is prime:
(for \( p \geq 7 \))

\[
\begin{align*}
: & \text{ for } p = 7, \quad q = 797 \text{ is prime}, \quad \text{so } k = 3; \\
: & \text{ for } p = 11, \quad q = 1171 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 13, \quad q = 1373 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 17, \quad q = 1747 \text{ is prime}, \quad \text{so } k = 1; \\
: & \text{ for } p = 23, \quad q = 2383 \text{ is prime}, \quad \text{so } k = 3; \\
: & \text{ for } p = 29, \quad q = 29179 \text{ is prime}, \quad \text{so } k = 5; \\
: & \text{ for } p = 31, \quad q = 3191 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 37, \quad q = 3767 \text{ is prime}, \quad \text{so } k = 1; \\
: & \text{ for } p = 41, \quad q = 41131 \text{ is prime}, \quad \text{so } k = 3; \\
: & \text{ for } p = 43, \quad q = 4373 \text{ is prime}, \quad \text{so } k = 1; \\
: & \text{ for } p = 47, \quad q = 47137 \text{ is prime}, \quad \text{so } k = 3; \\
: & \text{ for } p = 53, \quad q = 53113 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 59, \quad q = 59119 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 61, \quad q = 61121 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 67, \quad q = 67157 \text{ is prime}, \quad \text{so } k = 3; \\
: & \text{ for } p = 71, \quad q = 71161 \text{ is prime}, \quad \text{so } k = 3; \\
: & \text{ for } p = 73, \quad q = 73133 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 79, \quad q = 79139 \text{ is prime}, \quad \text{so } k = 2; \\
: & \text{ for } p = 83, \quad q = 83203 \text{ is prime}, \quad \text{so } k = 4; \\
: & \text{ for } p = 89, \quad q = 89119 \text{ is prime}, \quad \text{so } k = 1; \\
: & \text{ for } p = 97, \quad q = 97127 \text{ is prime}, \quad \text{so } k = 1;
\end{align*}
\]
for p = 101, q = 101161 is prime, so k = 2;
for p = 103, q = 103133 is prime, so k = 1;
for p = 107, q = 107137 is prime, so k = 1;
for p = 109, q = 109139 is prime, so k = 1;
for p = 113, q = 113143 is prime, so k = 1;
for p = 127, q = 127157 is prime, so k = 1;

[note the chain of 5 primes q (103133, 107137, 109139, 113143, 127157) obtained for k = 1 from 5 consecutive primes p]

(...)
for p = 104651, q = 104651104771 is prime, so k = 4;
for p = 104659, q = 104659104749 is prime, so k = 3;
for p = 104677, q = 104677104737 is prime, so k = 2;
for p = 104681, q = 104681104831 is prime, so k = 5;
for p = 104683, q = 104683104833 is prime, so k = 5;
for p = 104693, q = 104693104723 is prime, so k = 1;
for p = 104701, q = 104701104821 is prime, so k = 4;
for p = 104707, q = 104707104797 is prime, so k = 3;
for p = 104711, q = 104711104921 is prime, so k = 7;
for p = 104717, q = 104717104837 is prime, so k = 4;
for p = 104723, q = 104723104753 is prime, so k = 1;
for p = 104729, q = 104729104819 is prime, so k = 3;
(...)
for p = 982451501, q = 982451501982451561 is prime, so k = 2;
for p = 982451549, q = 982451549982451609 is prime, so k = 2;
for p = 982451567, q = 982451567982451597 is prime, so k = 1;
(...)

The value of the least k for 5 random 20 digit primes p:

for p = 48112959837082048697, q = 4811295983708204869748112959837082049237, prime, so k = 18;
for p = 54673257461630679457, q = 5467325746163067945754673257461630680777, prime, so k = 44;
for p = 29497513910652490397, q = 2949751391065249039729497513910652490847, prime, so k = 15;
for p = 12764787846358441471, q = 1276478784635844147112764787846358441741, prime, so k = 9;
for p = 71755440315342536873, q = 7175544031534253687371755440315342537023, prime, so k = 5.
Note the small value of \( k \) for which first prime \( q \) is obtained, even in the case of primes \( p \) having 20 digits! This formula could be a way to easily find, starting from a prime \( p \), a prime \( q \) having twice as many digits!