Four conjectures on the Smarandache prime partial digital sequence

Abstract. In this paper I make the following four conjectures on the Smarandache prime-partial-digital sequence defined as the sequence of prime numbers which admit a deconcatenation into a set of primes: (I) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n = m*h - h + 1, where h positive integer; (II) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k - 1, such that n = m*h + h - 1, where h positive integer; (III) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n + m - 1 is prime or power of prime; (IV) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k - 1, such that n - m + 1 is prime or power of prime. Note that almost all from the first 65 primes obtained concatenating two primes of the form 6k + 1 (exceptions: 3779, 4373, 6173, 6719, 6779), and all the first 65 primes obtained concatenating two primes of the form 6k - 1, belong to one of the four sequences considered by the conjectures above.

The Smarandache prime-partial-digital sequence (see A019549 in OEIS):

Conjecture 1:

There exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n = m*h - h + 1, where h positive integer.

Note that all primes n larger than 7 of the form 6*k + 1 can be written as 7*h - h + 1, where h positive integer, so all the primes obtained concatenating a prime of the form 6*k + 1 with 7 is term of this sequence.

The sequence of primes p:
: 137, 197, 317, 617, 677, 719, 743, 761, 773, 797, 977, 1097, 1277, 1361 (61 = 13*5 - 5 + 1), 1373 (73
Example of larger $p$: 

\[ p = 499943 \text{ where } 4999 = 43 \times 119 - 119 + 1. \]

**Conjecture 2:**

There exist an infinity of primes $p$ obtained concatenating two primes $m$ and $n$, both of the form $6k - 1$, such that $n = m \times h + h - 1$, where $h$ positive integer.

Note that all primes $n$ larger than 5 of the form $6k - 1$ can be written as $5h + h - 1$, where $h$ positive integer, so all the primes obtained concatenating a prime of the form $6k - 1$ with 5 is term of this sequence.

The sequence of primes $p$: 

\[ 541, 547, 571, 1123, 2311, 4723, 5101, 5107, 5113, 5167, 5179, 5197, 5227, 5233, 5237, 5239, 5243, 5431, 5443, 5449, 5479, 5503, 5521, 5557, 5641, 5647, 5653, 5659, 5683, 5701, 5743, 5821, 5827, 5839, 5857, 5881, 5953 (\ldots) \]

**Conjecture 3:**

There exist an infinity of primes $p$ obtained concatenating two primes $m$ and $n$, both of the form $6k + 1$, such that $n + m - 1$ is prime or power of prime.

The sequence of primes $p$: 

\[ 137 (13 + 7 + 1 = 19), 197 (19 + 7 - 1 = 25 = 5^2), 317 (31 + 7 - 1 = 37), 617 (61 + 7 - 1 = 67), 677 (67 + 7 - 1 = 73), 719 (71 + 9 - 1 = 79), 743 (7 + 43 - 1 = 49 = 7^2), 761 (7 + 67 - 1 = 73), 773 (7 + 73 - 1 = 79), 797 (7 + 97 - 1 = 103), 977 (97 + 7 - 1 = 103), 1319 (13 + 9 - 1 = 31), 1361 (13 + 61 - 1 = 73), 1367 (13 + 67 - 1 = 79), 1637 (163 + 7 - 1 = 169 = 13^2), 1913 (19 + 13 - 1 = 31), 1931 (19 + 31 - 1 = 49 = 7^2), 1979 (19 + 79 - 1 = 97), 2237 (223 + 7 - 1 = 229), 2777 (277 + 7 - 1 = 283), 2837 (283 \ldots) \]
\[7 - 1 = 289 = 17^2, \ 3119 \ (31 + 19 - 1 = 49 = 7^2), \ 3167 \ (31 + 67 - 1 = 97), \ 3677 \ (367 + 7 - 1 = 373), \ 3761 \ (37 + 61 - 1 = 97), \ 3767 \ (37 + 67 - 1 = 103), \ 4397 \ (43 + 97 - 1 = 139), \ 5237 \ (523 + 7 - 1 = 529 = 23^2), \ 5717 \ (571 + 7 - 1 = 577), \ 6113 \ (61 + 13 - 1 = 73), \ 6143 \ (61 + 43 - 1 = 103), \ 6197 \ (61 + 97 - 1 = 157), \ 7283 \ (7 + 283 - 1 = 289 = 17^2), \ 7331 \ (73 + 31 - 1 = 103) \text{ or } 7 + 331 - 1 = 337), \ 7349 \ (349 - 7 + 1 = 343 = 7^3) (...) \\

Example of larger p:

: \[p = 499979 \text{ where } 4999 + 79 - 1 = 5077, \text{ prime.}\]

Conjecture 4:

There exist an infinity of primes \(p\) obtained concatenating two primes \(m\) and \(n\), both of the form \(6k - 1\), such that \(n - m + 1\) is prime or power of prime.

The sequence of primes \(p\):

: \[541 \ (41 - 5 + 1 = 37), \ 547 \ (47 - 5 + 1 = 43), \ 571 \ (71 - 5 + 1 = 67), \ 1117 \ (17 - 11 + 1 = 7), \ 1123 \ (23 - 11 + 1 = 13), \ 1129 \ (29 - 11 + 1 = 19), \ 1153 \ (53 - 11 + 1 = 43), \ 1171 \ (71 - 11 + 1 = 61), \ 1741 \ (41 - 17 + 1 = 25 = 5^2), \ 1747 \ (47 - 17 + 1 = 31), \ 1753 \ (53 - 17 + 1 = 37), \ 1759 \ (59 - 17 + 1 = 43), \ 1783 \ (83 - 17 + 1 = 67), \ 1789 \ (89 - 17 + 1 = 73), \ 2311 \ (23 - 11 + 1 = 13), \ 2341 \ (41 - 23 + 1 = 19), \ 2347 \ (47 - 23 + 1 = 25 = 5^2), \ 2371 \ (71 - 23 + 1 = 49 = 7^2), \ 2383 \ (83 - 23 + 1 = 61), \ 2389 \ (89 - 23 + 1 = 67), \ 2971 \ (71 - 29 + 1 = 43), \ 4111 \ (41 - 11 + 1 = 31), \ 4129 \ (41 - 29 + 1 = 13), \ 4153 \ (53 - 41 + 1 = 13), \ 4159 \ (59 - 41 + 1 = 19), \ 4723 \ (47 - 23 + 1 = 25 = 5^2), \ 4729 \ (47 - 29 + 1 = 19), \ 4759 \ (59 - 47 + 1 = 13), \ 4783 \ (83 - 47 + 1 = 37), \ 4789 \ (89 - 47 + 1 = 43), \ 5101 \ (101 - 5 + 1 = 97), \ 5107 \ (107 - 5 + 1 = 103), \ 5113 \ (113 - 5 + 1 = 109), \ 5167 \ (167 - 5 + 1 = 163), \ 5179 \ (179 - 5 + 1 = 173), \ 5197 \ (197 - 5 + 1 = 193), \ 5227 \ (227 - 5 + 1 = 223), \ 5233 \ (233 - 5 + 1 = 227), \ 5323 \ (53 - 23 + 1 = 31), \ 5347 \ (53 - 47 + 1 = 7), \ 5647 \ (647 - 5 + 1 = 643), \ 5743 \ (743 - 5 + 1 = 739), \ 5827 \ (827 - 5 + 1 = 823), \ 5857 \ (857 - 5 + 1 = 853), \ 5881 \ (881 - 5 + 1 = 877), \ 5923 \ (59 - 23 + 1 = 37), \ 7129 \ (71 - 29 + 1 = 43), \ 7159 \ (71 - 59 + 1 = 13) (...) \\

Example of larger p:

: \[p = 499711 \text{ where } 4997 - 11 + 1 = 4987, \text{ prime.}\]
Note:
Almost all from the first 65 primes obtained from $m = 6x + 1$, prime, concatenated with $n = 6y + 1$, prime (exceptions: 3779, 4373, 6173, 6719, 6779), and all the first 65 primes obtained from $m = 6x - 1$, prime, concatenated with $n = 6y - 1$, prime, belong to one of the 4 sequences considered by the conjectures above.

Note:
Up to the number 7349 there are 65 primes obtained concatenated two primes of the form $6k + 1$ and 65 primes obtained concatenated two primes of the form $6k - 1$!