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Causality of the Coulomb field of relativistic electron bunches

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Abstract Recent experiments with relativistic electron bunches indicate that Coulomb field is rigidly attached to the charge's instantaneous position. We claim that despite a widespread opinion, this fact does not violate causality in moving reference frames. To see that, one should properly take into account the interaction dependence of the Lorentz boost generator.

1 Experiment at Frascati

In 2012 the team of Prof. Pizzella at the Frascati National Laboratory performed a remarkable experiment [1] that observed rigid connection between electric charge and its Coulomb field. In order to see how surprising this result is from the point of view of the traditional Maxwell–Liénard–Wiechert electrodynamics, let us turn to Fig. 1.

In three panels (a)→(b)→(c) we show the sequence of events expected in the traditional theory. Before the electron bunch leaves the accelerator's pipe, the bunch's field is shielded by the pipe's metal, so there is no electric field in the surrounding space.[1] Once the bunch leaves the pipe, the emerged force field has two components. First, there is an expanding spherical electromagnetic wave (radiation field) centered at the pipe's exit. Second, there is a non-radiating bound Coulomb field,[2] which is squeezed (or Lorentz-contracted [2]) in the direction of the bunch's motion, thus forming a narrow "disk."

Electric field lines of the Coulomb disk are shown in Figs. 1(b)-(c). In special relativity, these field lines cannot "leak" outside the "light cone" bounded by the expanding sphere of the electromagnetic wave. Therefore, the transverse dimension d of the disk grows with time, according to formula

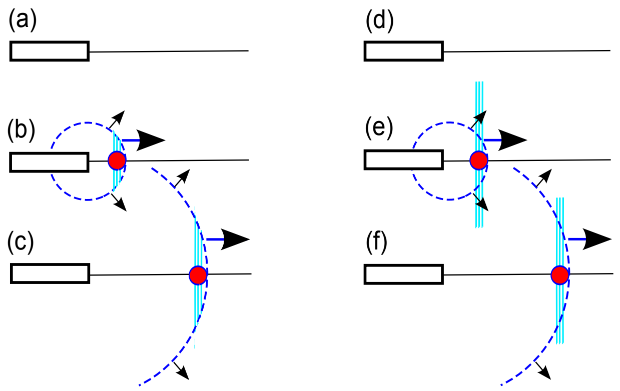

Fig. 1 Time evolution of the electric field of an electron bunch (small circle) leaving the accelerator pipe (rectangle): (a)→(b)→(c) Maxwell–Liénard–Wiechert theory; (d)→(e)→(f) Frascati experiment. Broken line circle fragments show the expanding spherical electromagnetic wave. Parallel lines perpendicular to the beam's axis represent the Coulomb field disk viewed from its side

$$d \approx 2ct/\gamma \approx 2L/\gamma \tag{1}$$

where L is the distance traveled by the bunch from the pipe's exit and $\gamma \equiv (1-v^2/c^2)^{-1/2}$ is the usual relativistic factor. In the Pizzella's experiment, the energies of electrons in the beam were 500 MeV, which corresponded to the Lorentz contraction factor of $\gamma \approx 1000$. Within the experimental room length of 5 m the expected size of the Coulomb disk was not greater than 1 cm — too small to have any effect on the field sensors placed around the beam.

In the parallel sequence of time frames (d)→(e)→(f) we present results of the Pizzella's experiment. In contrast to theoretical expectations, the size of the Coulomb field's disk did not grow in a linear fashion (1). Instead, measurements

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[1]See Fig. 1(a).

[2]There is also an associated bound magnetic field, but we will not discuss it in this Letter.

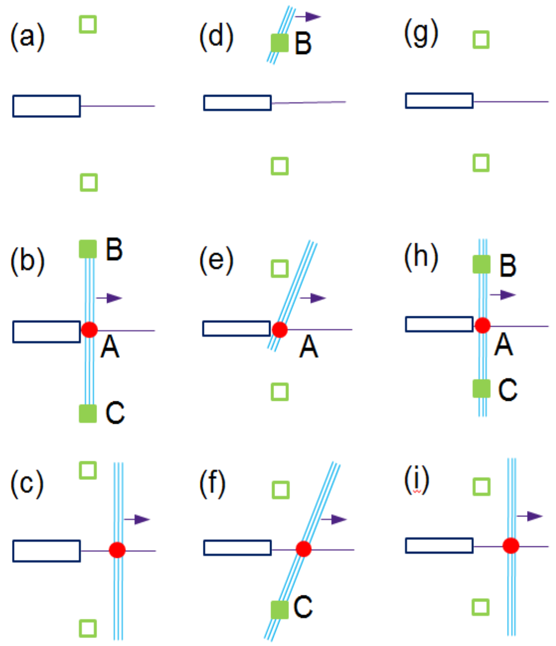

Fig. 2 Time evolution of the bunch's instantaneous Coulomb field: (a)→(b)→(c) in the rest frame; (d)→(e)→(f) in the moving frame, as predicted by Lorentz transformations (2) – (5); (g)→(h)→(i) in the moving frame, as predicted by the Wigner–Dirac theory. Squares mark positions of electric field sensors. Filled squares indicate "clicking" sensors

suggested that the field's disk emerged from the accelerator fully formed in the entire space (even beyond the light cone) and did not change with time, apart from the uniform movement together with the bunch.

This fact can be interpreted as an indication of an instantaneous force between charges in the bunch and in the field sensors. The ideas about electromagnetic interactions being composed of both instantaneous (bound) and retarded (radiation) parts are not new. They were repeatedly expressed theoretically [3–5], and electromagnetic superluminal effects were seen in experiments as well [6–8]. However, these ideas and experiments are usually met with scepticism, because they violate the special relativistic ban on faster-than-light propagation of signals.

2 Ban of superluminal signals

To understand why special relativity rejects such superluminal effects, consider Fig. 2. It focuses on the exact moment when the electron bunch leaves the accelerator's pipe. This time, for clarity, we did not show the spherical electromagnetic waves, because experiment [1] has demonstrated that their signals are weaker by an order of magnitude. We also placed two electric field sensors next to the pipe's exit.

In three consecutive time frames (a)→(b)→(c) we show the time evolution of the instantaneous Coulomb field[3] in the laboratory reference frame O. The time frame (a) is taken just before the bunch left the pipe, and the snapshot (c) is taken a moment after this event. The field's disk forms instantaneously (panel (b)), so the three events A (the bunch's exit from the pipe), B and C (clicks of the sensors) occur simultaneously, despite the fact that B and C are caused by the event A and separated from it by a considerable distance.

In the three time frames (d)→(e)→(f) we show how this experimental situation looks from the point of view of an inertial observer O' moving with high velocity in the vertical direction from the bottom of the page to its top.[4] To draw these frames we employed Lorentz transformations of special relativity

$$t' = t\cosh\theta - (y/c)\sinh\theta \quad (2)$$
$$x' = x \quad (3)$$
$$y' = y\cosh\theta - ct\sinh\theta \quad (4)$$
$$z' = z \quad (5)$$

where parameter θ was related to the observer's velocity by formula $v = c\tanh\theta$.

The absurdity of this situation is clear already from the panel (d), where a portion of the Coulomb field's disk emerges even before the bunch has left the accelerator's pipe. This means that the "effect" B occurred earlier than the "cause" A, in a clear violation of the principle of causality. Two other panels show the further sequence of events: the bunch leaves the accelerator (e), the lower sensor clicks (f).

To avoid such absurd violations of causality, special relativity forbids faster-than light formation of the field's disk. However, in a clear disrespect of this ban, Pizzella's experiment did show an instantaneous creation of the field's disk in the laboratory. This extraordinary observation demands an explanation.

3 Relativistic interactions between particles

The usual attitude is that instantaneous potentials of the type seen in Pizzella's experiment are impossible because (i) they cannot be relativistically invariant and (ii) they violate causality. We will debunk the latter statement in section 4. Here we would like to mention that the former statement is not correct as well.

First explicit construction of a relativistically invariant multiparticle model with instantaneous forces was undertaken by Bakamjian and Thomas [9]. They used the theory

[3] same as in Fig. 1 (d)→(e)→(f)

[4] To be consistent with [1], we call this y-direction. For added realism, the whole picture has been Lorentz-contracted in the y-direction.

of unitary representations of the Poincaré group developed earlier by Wigner [10] and Dirac [11]. According to this theory, interactions must modify generators of the Poincaré group representation, so that the commutators of generators remain intact. The full Hamiltonian (= the generator of time translations) is obtained by adding the "potential energy" operator V to the free Hamiltonian: $\mathrm{H} = \mathrm{H}_0 + \mathrm{V}$. In the Dirac's instant form of relativistic dynamics, the invariance of this addition is achieved by also adding an interacting "potential boost" $\mathbf{Z}$ to the noninteracting boost generator $\mathbf{K}_0$

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{Z} \tag{6}$$

Generators of space translations and rotations remain interaction-free: $\mathbf{P} = \mathbf{P}_0$, $\mathbf{J} = \mathbf{J}_0$.

If this construction is performed in a two-particle system,[5] then the Hamiltonian H is a function of positions and momenta of the two particles. Time dependencies of these observables are readily obtained

$$\mathbf{r}_j(t) = e^{\frac{i}{\hbar}\mathrm{Ht}}\mathbf{r}_j e^{-\frac{i}{\hbar}\mathrm{Ht}} \tag{7}$$

$$\mathbf{p}_j(t) = e^{\frac{i}{\hbar}\mathrm{Ht}}\mathbf{p}_j e^{-\frac{i}{\hbar}\mathrm{Ht}} \tag{8}$$

where $j = 1,2$ is particle label. These expressions can be viewed either as quantum mechanical time-dependent operators in the Heisenberg picture or as particle trajectories in the $\hbar \to 0$ classical limit. In the latter case one should replace quantum commutators with classical Poisson brackets.

The force acting on the particle 2

$$\begin{aligned}\mathbf{f}_2(t) &= \frac{d}{dt}\mathbf{p}_2(t) = -\frac{i}{\hbar}[\mathbf{p}_2(t), \mathrm{H}] \\ &\equiv \mathbf{f}_2(\mathbf{r}_1(t), \mathbf{p}_1(t); \mathbf{r}_2(t), \mathbf{p}_2(t))\end{aligned} \tag{9}$$

depends on positions and momenta of both particles at the same time instant t. This demonstrates that interaction propagates instantaneously in the reference frame O at rest.

4 Causality

Now consider the above two-particle system from the point of view of the moving reference frame O'. Particle trajectories in this frame are[6]

$$\mathbf{r}_j(\theta, t') = e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta} e^{\frac{i}{\hbar}\mathrm{Ht'}}\mathbf{r}_j e^{-\frac{i}{\hbar}\mathrm{Ht'}} e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \tag{10}$$

$$\mathbf{p}_j(\theta, t') = e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta} e^{\frac{i}{\hbar}\mathrm{Ht'}}\mathbf{p}_j e^{-\frac{i}{\hbar}\mathrm{Ht'}} e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \tag{11}$$

[5] For example, we can assume that charge 1 represents an electron in the bunch and charge 2 is a part of the electric field sensor.

[6] Here t' is time measured by the clock of the observer O'.

The Hamiltonian in the reference frame O' is

$$\mathrm{H}(\theta) = e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta}\mathrm{H}e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \tag{12}$$

Therefore, the force acting on the particle 2

$$\begin{aligned}&\mathbf{f}_2(\theta, t') \\ &= \frac{d}{dt'}\mathbf{p}_2(\theta, t') = -\frac{i}{\hbar}[\mathbf{p}_2(\theta, t'), \mathrm{H}(\theta)] \\ &= -\frac{i}{\hbar}\left[e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta}e^{\frac{i}{\hbar}\mathrm{Ht'}}\mathbf{p}_2 e^{-\frac{i}{\hbar}\mathrm{Ht'}}e^{\frac{ic}{\hbar}\mathrm{K_y}\theta}, e^{-\frac{ic}{\hbar}\mathrm{K_y c}\theta}\mathrm{H}e^{\frac{ic}{\hbar}\mathrm{K_y}\theta}\right] \\ &= -\frac{i}{\hbar}e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta}\left[e^{\frac{i}{\hbar}\mathrm{Ht'}}\mathbf{p}_2 e^{-\frac{i}{\hbar}\mathrm{Ht'}}, \mathrm{H}\right]e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \\ &= -\frac{i}{\hbar}e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta}[\mathbf{p}_2(0, t'), \mathrm{H}]\,e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \\ &= e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta}\mathbf{f}_2(0, t')e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \\ &= e^{-\frac{ic}{\hbar}\mathrm{K_y}\theta}\mathbf{f}_2\left(\mathbf{r}_1(0, t'), \mathbf{p}_1(0, t'); \mathbf{r}_2(0, t'), \mathbf{p}_2(0, t')\right)e^{\frac{ic}{\hbar}\mathrm{K_y}\theta} \\ &= \mathbf{f}_2(\mathbf{r}_1(\theta, t'), \mathbf{p}_1(\theta, t'); \mathbf{r}_2(\theta, t'), \mathbf{p}_2(\theta, t'))\end{aligned} \tag{13}$$

is a function of positions and momenta of both particles at the same time instant t'. Thus, from the point of view of the moving observer O', the interaction propagates instantaneously, exactly as for the observer at rest O. Moreover, in agreement with the principle of relativity, the force function $\mathbf{f}_2$ in (13) has the same form as in the rest frame (9).

In regard to the Pizzella's experiment, this means that clicks of the sensors (events B and C) in the frame O' must occur exactly at the time of the bunch's exit from the pipe (event A). This prediction of the Wigner – Dirac theory is shown in the sequence of snapshots 2 (g)→(h)→(i). In other words, in this theory the three events A, B, C remain simultaneous in all frames, and instantaneous potentials do not contradict causality.

5 Discussion

The above discussion implies that in the presence of interaction ($V \neq 0$, $\mathbf{Z} \neq 0$), particle trajectories in the moving frame (10) cannot be obtained from rest-frame trajectories (7) by applying Lorentz formulas (2) - (5). One can say that interacting Wigner – Dirac theory does not have "invariant worldlines" [12, 13]. We should note that this idea is rather controversial, and the majority of theoreticians believe that the "invariant worldline" condition must be respected in all dynamical theories. Based on this belief, a multitude of approaches appeared [14–23], which deviated from the pure Hamiltonian-based Wigner – Dirac theory. So far, the predictive power of these approaches remains rather limited.

Another idea is that particle equations of motion (including boost transformations) should be extracted from our most successful theory – quantum electrodynamics (QED).

At the first sight it seems that Maxwell's equations as well as Liénard–Wiechert retarded potentials must follow from the QED Lagrangian in the classical limit. However, this conclusion is not so obvious, because time evolution in QED is an ill-defined concept as this theory is formulated in terms of fictitious bare particles, and the Hamiltonian contains formally divergent renormalization counterterms. One can also find authoritative opinions that observables (positions and momenta) of individual interacting particles have no meaning in QED [24] or even that *"there are no particles, there are only fields"* [25]. For more works on the same theme see [26–30].

A somewhat different and more pragmatic attitude is taken in research programs that try to replace the field-based language of quantum field theory with ideas of physical particles interacting through effective relativistic potentials. One way to obtain such potentials is to fit them to the known renormalized QED S-matrix. This can be done not only in the lowest second perturbation order, resulting in the classic Coulomb–Darwin–Breit potential [24, 31], but taking into account radiative corrections as well [32–35]. Another way forward is to apply a unitary dressing transformation [36–38] to the QED Hamiltonian. This method has the added advantage that it explicitly preserves the relativistic invariance of the theory [39].

It appears that the effective electromagnetic interactions of physical particles separate into bound and radiation types. As expected, the radiation force field propagates with the speed of light [40, 41]. However, this is not true for the bound (Coulomb and magnetic) potentials. Even in high perturbation orders they are expressed by functions that depend on instantaneous positions and momenta of the charges. These potentials are well suited to describe the field dynamics of relativistic electron bunches observed in Frascati.

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