Two conjectures on Smarandache's proper divisor products sequence

Abstract. In this paper I make the following two conjectures on the *Smarandache's proper divisor products sequence* where a term P(n) of the sequence is defined as the product of the proper divisors of n: (1) there exist an infinity of numbers n divisible by 3 such that the number obtained concatenating the value of P(n) to the right with 1 is prime; (2) there exist an infinity of numbers n divisible by 3 such that the number obtained concatenating the value of P(n) to the right with 1 is semiprime p*q with the property that q – p + 1 is prime.

The *Smarandache's proper divisor products sequence* (see A007956 in OEIS):
: 1, 1, 1, 2, 1, 6, 1, 8, 3, 10, 1, 144, 1, 14, 15, 64, 1, 324, 1, 400, 21, 22, 1, 13824, 5, 26, 27, 784, 1, 27000, 1, 1024, 33, 34, 35, 279936, 1, 38, 39, 64000, 1, 74088, 1, 1936, 2025, 46, 1, 5308416, 7, 2500, 51, 2704, 1, 157464, 55, 175616, 57, 58, 1, 777600000, 1, 62, 3969, 32768, 65 (...)

Conjecture 1:

There exist an infinity of numbers n divisible by 3 such that the number m obtained concatenating the value of P(n) to the right with 1 is prime.

The sequence of primes m:
: m = 11, prime, for (n, P(n)) = (3, 1);
: m = 61, prime, for (n, P(n)) = (6, 6);
: m = 31, prime, for (n, P(n)) = (9, 3);
: m = 151, prime, for (n, P(n)) = (15, 15);
: m = 211, prime, for (n, P(n)) = (21, 21);
: m = 138241, prime, for (n, P(n)) = (24, 13824);
: m = 271, prime, for (n, P(n)) = (27, 27);
: m = 270001, prime, for (n, P(n)) = (30, 27000);
: m = 331, prime, for (n, P(n)) = (33, 33);
: m = 2799361, prime, for (n, P(n)) = (36, 279936);
: m = 571, prime, for (n, P(n)) = (57, 57);
: m = 418121194241, prime, for (n, P(n)) = (84, 41812119424);
: m = 98011, prime, for (n, P(n)) = (99, 9801);
: m = 14815441, prime, for (n, P(n)) = (114, 1481544);
: m = 1231, prime, for (n, P(n)) = (123, 123);
: m = 1291, prime, for (n, P(n)) = (129, 129);
: m = 216091, prime, for (n, P(n)) = (147, 21609);
: m = 44921251, prime, for (n, P(n)) = (165, 4492125);

: m = 52680241, prime, for (n, P(n)) = (174, 5268024);
: m = 1831, prime, for (n, P(n)) = (183, 183);
: m = 2011, prime, for (n, P(n)) = (201, 201);
: m = 2131, prime, for (n, P(n)) = (213, 213);
: m = 109410481, prime, for (n, P(n)) = (222, 10941048);
: m = 2371, prime, for (n, P(n)) = (237, 237);
: m = 148869361, prime, for (n, P(n)) = (246, 14886936);
: m = 171735121, prime, for (n, P(n)) = (258, 17173512);
: m = 2671, prime, for (n, P(n)) = (267, 267);
: m = 203464171, prime, for (n, P(n)) = (273, 20346417);
: m = 321574321, prime, for (n, P(n)) = (318, 32157432);
: m = 3271, prime, for (n, P(n)) = (327, 327);
: m = 3391, prime, for (n, P(n)) = (339, 339);
: m = 1317691, prime, for (n, P(n)) = (363, 131769);
: m = 11026624842058533121, prime, for (n, P(n)) = (378, 1102662484205853312);
: m = 13723100667900000001, prime, for (n, P(n)) = (390, 1372310066790000000);
: m = 3931, prime, for (n, P(n)) = (393, 393);
: m = 635211991, prime, for (n, P(n)) = (399, 63521199);
: m = 269042006251, prime, for (n, P(n)) = (405, 26904200625);
: m = 4111, prime, for (n, P(n)) = (411, 411);
: m = 773087761, prime, for (n, P(n)) = (426, 77308776);
: m = 840276721, prime, for (n, P(n)) = (438, 84027672);
: m = 967025791, prime, for (n, P(n)) = (459, 96702579);
(...)

Examples of larger m:
: m = 1006131972417915371063869440000000000000001, prime, for (n, P(n)) = (720, 100613197241791537106386944000000000000000);
: m = 8953382542587164451099000000000001, prime, for (n, P(n)) = (990, 895338254258716445109900000000000).

Conjecture 2:

There exist an infinity of numbers n divisible by 3 such that the number m obtained concatenating the value of P(n) to the right with 1 is semiprime m = p*q with the property that q – p + 1 is prime.

The sequence of semiprimes m:

: m = 3241 = 7*463 (where 463 – 7 + 1 = 457, prime), for (n, P(n)) = (18, 324);
: m = 391 = 17*23 (where 23 – 17 + 1 = 7, prime), for (n, P(n)) = (39, 39);
: m = 20251 = 7*2893 (where 2893 – 7 + 1 = 2887, prime), for (n, P(n)) = (45, 2025);
: m = 511 = 7*73 (where 73 – 7 + 1 = 67, prime), for (n, P(n)) = (51, 51);
: m = 59049000001 = 215161*274441 (where 274441 – 215161 + 1 = 59281, prime), for (n, P(n)) = (90, 5904900000);
: m = 10612081 = 37*286813 (where 286813 – 37 + 1 = 286777, prime), for (n, P(n)) = (102, 1061208);
: m = 136891 = 367*373 (where 373 – 367 + 1 = 7, prime), for (n, P(n)) = (117, 13689);
: m = 1411 = 17*83 (where 83 – 17 + 1 = 67, prime), for (n, P(n)) = (141, 141);
: m = 1591 = 37*43 (where 43 – 37 + 1 = 7, prime), for (n, P(n)) = (159, 159);
: m = 2191 = 7*313 (where 313 – 7 + 1 = 307, prime), for (n, P(n)) = (219, 219);
: m = 2491 = 47*53 (where 53 – 47 + 1 = 7, prime), for (n, P(n)) = (249, 249);
: m = 1046035320300000001 = 37*28271224872972973 (where 28271224872972973 – 37 + 1 = 28271224872972937, prime), for (n, P(n)) = (270, 104603532030000000);
: m = 291 = 41*71 (where 71 – 41 + 1 = 31, prime), for (n, P(n)) = (291, 291);
: m = 261980731 = 3037*86263 (where 86263 – 3037 + 1 = 83227, prime), for (n, P(n)) = (297, 26198073);
: m = 3091 = 11*281 (where 281 – 11 + 1 = 271, prime), for (n, P(n)) = (309, 309);
: m = 454992931 = 331*1374601 (where 1374601 – 331 + 1 = 1374271, prime), for (n, P(n)) = (357, 45499293);
: m = 3811 = 37*103 (where 103 – 37 + 1 = 67, prime), for (n, P(n)) = (381, 381);
: m = 649648081 = 17*38214593 (where 38214593 – 17 + 1 = 38214577, prime), for (n, P(n)) = (402, 64964808);
: m = 2275291 = 139*16369 (where 16369 – 139 + 1 = 16231, prime), for (n, P(n)) = (477, 227529);
: m = 1126785871 = 421*2676451 (where 2676451 – 421 + 1 = 2676031, prime), for (n, P(n)) = (477, 112678587);
: m = 4891 = 67*73 (where 73 – 67 + 1 = 7, prime), for (n, P(n)) = (489, 489);

(...)