

Reconsidering “Does the Sum Rule Hold at the Big Bang?”, with Pre Planckian HUP, and Division Algebras

Andrew W. Beckwith*

Physics Department, Chongqing University, College of Physics, Chongqing University Huxi Campus, No. 44
Daxuechen Nanlu, Shapinba District, Chongqing 401331, People’s Republic of China

Rwill9955b@gmail.com; abeckwith@uh.edu

Abstract

In 2012, the author submitted an article to the Prespacetime journal based upon the premise of inquiry as to the alleged vanishing of disjoint open sets contributing to quantum vector measures no longer working. I.e. the solution in 2012 was that the author stated that quantum measures in 4 dimensions would not work, mandating, if measure theory were used, imbedding in higher dimensions was necessary for a singularity. The idea was to use the methodology of string theory as to come up with a way out of the impasse if higher dimensions do not exist. We revisit this question, taking into account a derived HUP, for metric tensors if we look at Pre-Planckian space-time introducing a pre-Quantum mechanical HUP which may be a way to ascertain a solution not mandating higher dimensions, as well as introducing cautions as to what will disrupt the offered solution. Note that first, measurable spaces allow disjoint sets. Also, that smooth relations alone do not define separability or admit sets Planck’s length, if it exists, is a natural way to get about the ‘bad effects’ of a cosmic singularity at the beginning of space-time evolution, but if a development is to be believed, namely by Stoica in the article, about removing the cosmic singularity as a break down point in relativity, there is nothing which forbids space-time from collapsing to a point. Without the use of a Pre Planckian HUP, for metric tensors, the quantum measures in four dimensions break down. We try to ascertain if a Pre Planckian HUP is sufficient to avoid this pathology and also look at if division algebras which can link Octonionic geometry and E8, to Quark spinors, in the standard model and add sufficient definition to the standard model are necessary and sufficient conditions for a metric tensor HUP which may remove this break down of the sum rule in the onset of the ‘Big Bang’.

Keywords: quantum measures, spatial diffeomorphism, cylinder sets, Caratheodary-Hahn –Huvaneck theorem, Big Bang singularity, causal sets, Modified Pre Planckian HUP, Division Algebras

I. Introduction

As stated, in 2012 [1] the author came to the conclusion as to the existence of a situation where a Stoica induced “non pathological” singularity would disrupt the existence of a functional quantum measure in 4 dimensions. The relationship of this construction to the Sum rule is explained, Afterwards, we introduce constructions as due to a new HUP, as given by [2,3,4] which among other things demonstrates the method which can give a start to cosmological expansion of the universe, and we assert that in certain cases, the answer in terms of an energy flux may indeed falsify the conclusion given in [1]. To start off we consider in this paper whether in the absence of an HUP, as given in [2,3,4] if singularity behavior in space-time can be affected by co-ordinate choices¹. Afterwards, we will examine the role of the HUP given in [2,3,4] and also consider the formulation of the problem needs more than four dimensions, or if it does not. This delineation of the number of dimensions needed to solve the initial problem as given in [1]

* Correspondence: Dr. Andrew W. Beckwith, Dept. of Physics, Chongqing Univ, Chongqing, PRC, 401331 .
E-mail: abeckwith@uh.edu; rwill9955b@gmail.com

¹ As noted by one of the Reviewers: “one can indeed look at the induced metric at light-cone boundary[. It] is just $r_M^2 d\Omega^2$. There is nothing pathological in its topology nor in the topology of future future lightcone when seen as parts of Minkowski space,[p]erfectly well defined but metrically 2-D since radial direction gives no contribution as a light-like direction. This example suggests that one should be extremely cautious in considerations related to singularity: so much depends on a choice of proper coordinates. In fact, holography suggests that one should consider light-cone boundary or initial singularity as sub-manifold of space-time.”

will be central to determine if the HUP as given in [2,3,4] has any applicability to space-time initial conditions.

II. What if we neglect the modification of the HUP as offered in [2,3,4] This segment is from [1]

In this situation, without considering a modified HUP in Pre Planckian space, as given by [2,3,4] we digress back to the older situation as given by [1]. I.e. we re forced then to consider having to have more than four dimensions.

As stated before in [1] we then unequivocally state that the following will hold. From [1], we have the following quote

Quote:

A proper choice of coordinates is going to involve more than four dimensions and that what is chosen in four-dimensional space-time usually in the Robertson Walker metric will lead to a singularity problem. We claim that this affects the quantum measure problem in four dimensions. The main point of the article is below where we outline how to fix the glaring problems in four-dimensional measure theory which we state as unphysical.

We note here that higher dimensions will, as in String theory, remove this problem. We wish to reconcile the four and higher dimensional examples of coordinate behavior and reflect upon what the four dimensional representation does to quantum measures, especially if there is a removal of the standard four dimensional representation of a mathematical singularity at the start of inflation. To do this, we will give an argument which will point in the direction of vanishing of disjoint sets in four dimensions leading to a break up of the quantum measure in four dimensions.

Our initial goal is to show that disjoint sets, are due to separability in a topological sense, and that at a point in space – time, that the very notion of separability breaks down completely [5].

Separability in a topological sense can be constructed as follows. A topological space X is said to be separable if X has a countable dense subset. In other words, there is a countable subset D of X such that $\text{closure}(D) = X$.

Equivalently, each nonempty open set in X intersects D . The fact is, that if there is a space – time point, that the countable subset D of X is such that the $\text{closure}(D) = X$. breaks down completely.

Afterwards, we should note that disjoint sets in a topological space, X , are due to working with X being a Hausdorff space. We then note the properties of Hausdorff spaces can be written follows:

1. If K is a compact subset of X and $y \in X$ is a point outside of K then y and K have disjoint neighborhoods, i.e. there exist an open neighborhood W_y of y and an open set $V_y \supset K$ for which $W_y \cap V_y = \emptyset$
2. Every compact subset of X is closed.
3. Any two disjoint compact subsets of X have disjoint open neighborhoods, i.e. if C and D are compact disjoint subsets of X , then there exist open sets $U \supset C$ and $V \supset D$ for which $U \cap V = \emptyset$

Note that when one has a point in space time, there is not a comparable construction to closure $(D) = X$.
or $U \cap V = \phi$.

This lack of having at a point in space-time a topological set X with open subsets with these constructions dooms having these properties. I.e if one does not have a Hausdorff space, one is going to find it impossible to form disjoint sets in a separable X if X is itself a point

When one does not have separable sub sets, at a single point, then the construction used for quantum measures breaks down. We review in Appendix A what happens due to Stoica's treatment [6] of the Friedman and acceleration equations and show it implies a smoothness condition which eliminates disjoint sets at a point, entirely. i.e. no pressure, density and scale factor.

While the existence of the pathological singularity can be treated by use of Planck's length, which can be used to construct disjoint sets, if Stoica is believable, this Planck's length is no longer essential, which brings up interesting questions so far avoided by main stream cosmologists. This paper merely brings up that issue, and asks what can be done to correct for it, at the point of the big bang. To do this, we later revisit what happened in Surya's paper [7] in the DICE 2010 conference, and make a few suggestions of our own afterwards. Appendix A summarizes how Surya built up her quantum measures and is mandatory reading for those wishing to understand how quantum measures are built up outside the point regime so specified by Surya which is claimed to break down in usual singularity regimes at the origin of the big bang².

Our contribution is to examine quantum measures assuming a non-string theory treatment of cosmology. And to argue that the break down of a quantum measure in four dimensions necessitates use of higher dimensional embedding of the start of cosmological inflation.

End of quote of [1].

This concludes our introduction as to what is done if the HUP of [2,3,4, 8, 9] is not used to generate a vacuum energy which could change the conclusion as to the necessity of higher dimensions in order to remove the break down of the vector measure of the so called quantum vector measure.

What we will do next is to elaborate the HUP which is cited in [2,3,4, 8, 9] which given certain conditions will possibly conserve the quantum vector measure.

III. What if we put in the construction of [2,3,4] and from first principles, include in a HUP for metric tensors based on [8,9, 10,11] ?

In order to do this we will be looking at the following construction. From [2,3,4, 10, 11] we can cite the following

IIIa. Examining what happens in Pre Planckian Space time $\dot{\phi}^2 \gg V_{SUSY}$ due to $\phi \sim \xi^+ \ll M_{Planck}$?

We will be looking at the value of Eq. (1) if $\phi \sim \xi^+ \ll M_{Planck}$. In short, we have then that

² The Reviewer noted that "[i]n Einstein's equations for RW cosmology of course ρ , p , and scale factor are scalar invariants which become infinite at the initial moment. This is a real physical singularity. In cosmic string dominated primordial cosmology however the mass per comoving volume vanishes at the singularity like a so that in this sense everything is non-singular. As mentioned Stoica shows that the equations can be redefined to get well-defined equations also at the singularity."

$$\begin{aligned}
 (\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\
 (\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A
 \end{aligned}
 \tag{1}$$

If we use the following, from the Roberson-Walker metric [2,3,4,6] .

$$\begin{aligned}
 g_{tt} &= 1 \\
 g_{rr} &= \frac{-a^2(t)}{1-k \cdot r^2} \\
 g_{\theta\theta} &= -a^2(t) \cdot r^2 \\
 g_{\phi\phi} &= -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2
 \end{aligned}
 \tag{2}$$

Following Unruth [8,9] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters}
 \tag{3}$$

Then, if $\Delta T_{tt} \sim \Delta \rho$ [2,3,4,6]

$$\begin{aligned}
 V^{(4)} &= \delta t \cdot \Delta A \cdot r \\
 \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\
 \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}}
 \end{aligned}
 \tag{4}$$

This Eq. (14) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [2,3,4,6]

$$T_{ii} = \text{diag}(\rho, -p, -p, -p)
 \tag{5}$$

Then [2,3,4,6, 8, 9, 10,11]

$$\Delta T_{tt} \sim \Delta \rho \sim \frac{\Delta E}{V^{(3)}}
 \tag{6}$$

$$\text{Then, } \delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2}
 \tag{7}$$

$$\text{Unless } \delta g_{tt} \sim O(1)$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen.

We next will then go into a description of what Eq (7) will do to the issue of if not not the quantum measure breaks down To to that we will work from the stand point of what a traditional quantum measure will do and to put in what the situation is, if we use Eq. (7) and if we do NOT assume Eq (7), in terms of a local HUP.

IV. Aftermath of Spatial Diffeomorphism Leading to Quantum Measures if not using HUP

From [1] we will make the following quote:

The main point of the formalism for Appendix B is of bi-additivity of D leading to the finite additivity of μ_V . The author asks readers to go to Appendix B to see the construction leading to the following equation, which in its creation uses disjoint sets, in an interval [12]

$$\mu_V \left(\bigcup_{i=1}^n \alpha_i \right) = \sum_{i=1}^n \mu_V (\alpha_i) \quad (8)$$

The use of finite additivity of μ_V is essential to the quantum measure prospect and in Appendix B inherently involves use of disjoint sets. The reason for stating this shows up in the next section, C. We leave the issue of if a Planck's length is mandatory for initial cosmology to the conclusion with our own point of view. Should the existence of Planck's length be mandatory due to space-time evolution, then there is no question that (8) (1) holds.

IVa. Arguments against Eq. (8) (1) in the Vicinity/ Origin of the Big Bang Singularity

The main problem, as the author sees it, is insuring the existence of disjoint sets at a point of space-time. If one views a finite, infinitely small region of space-time, as given by Planck's interval as 1.616×10^{-35} meters as contravening a space-time singularity, in relativity, then even in this incredibly small length, there can be disjoint sets, and then the math construction of Surya [7] goes through verbatim. Classical relativity theory though does not have a Planck interval, i.e. the singularity of space-time, so in effect in General relativity in its classical form will not have the construction so alluded to in Eq.(8) above. [6] written by Cristi Stoica gives a view of a beginning of space-time starting that does away completely with the space-time singularity, so mathematically, in a cosmos as constructed, if there is no singularity problem, there is then no restriction as to the collapse of space-time to an infinitely small point. In which then there would be no reason to appeal to a Planck's length graininess of space-time to enforce some rationality in the behavior of (quantum?) cosmology.

The precondition for a quantum measure μ_V for a quantum measurement is given by Eq. (8) [13] for n disjoint sets $\alpha_i \in A$. This Eq. (8) is a math precondition for μ_V being a vector measure over A . Eq (1) right at the point of the big bang cannot insure the existence of n disjoint sets $\alpha_i \in A$. Therefore at the loci of the big bang one would instead get, due to non-definable disjoint sets $\alpha_i \in A$, a situation definable as, at best.

$$\mu_V \left(\bigcup_{i=1}^n \alpha_i \right) \neq \sum_{i=1}^n \mu_V (\alpha_i) \quad (9)$$

Not being able to have a guarantee of having n disjoint sets $\alpha_i \in A$ because of singular conditions at the big bang will bring into question whether equation (8) can hold and the overall research endeavor of analyzing the existence of quantum measures μ_V . I.e., the triple (Ω, A, μ_V) for quantum measures μ_V cannot be guaranteed to exist. Especially if there is no bar to a singularity existing as given by [14,15] And we look at whether there is sufficiently convergent behavior for μ_V , so that uniqueness of convergent sequences is guaranteed by the Caratheodary-Hahn –Huvaneck theorem. If so, the following supremum expression for all FINITE partitions will lead to the equality expression for vector measures. This is what becomes very problematic if [14,15] is true about non pathological consequences of a BB singularity.

$$|\mu_V(\alpha)| = \sup_{\pi(\alpha)} \sum_{\rho} \|\mu_V(\alpha_{\rho})\| \quad (10)$$

The singularity will not allow us to analyze disjoint partitions. What happens if instead of Eq. (10) a situation for which there is longer finite partitions, ordered sets, but the replacement for Eq. (10) (3) is now an inequality written as:

$$|\mu_V(\alpha)| \neq \sup_{\pi(\alpha)} \sum_{\rho} \|\mu_V(\alpha_{\rho})\| \quad (11)$$

Or worse, a situation where there is no finite partially ordered set, i.e., no *causal* set? The inequality of Eq.(11) can occur if there is no finite disjoint sets to make a supremum over.

Eq. (9) depends upon having [14,15] an "*unconditional convergence of the vector measure over all partitions.*" Replace partitions with causal set structure, and one still has the same requirement of an *unconditional convergence of the vector set over all "causal set structure"* within a finite geometric regime of space-time. One does not get about the necessity of convergence of sequences and sub sequences in a causal set structure. The convergence of sequences and sub sequences has the same rules as when causal set structure is replaced by partitions. Surya's construction of taking a least upper bound (supremum) over finite partitions does not work if there are no finite partitions at a singularity.

V. Pointing out how Eq. (11) is influenced by the HUP of Eq.(7)

In order to reformulate the conclusions of Eq. (11) we will be examining if the existence of Eq. (7) stops physical disruptions of a disjoint partition. i.e. what we will be examining if we have an effective way to examining disjoint partitions, as showing up for why in the case of not using Eq. (7) we had effectively removed the singularity at the beginning of space-time

Va. What would be necessary to remove a singularity as given by Stoica? And Hawkings?

Usually as given by Penrose–Hawking singularity theorems [16] as well as the Penrose theorem [17] we have that there is, initially, a closed trapped surface that was introduced. A closed trapped surface exits, as given by [18] if

Quote
The strategy of the sketched proof presented was to assume that null geodesics were complete, proving that then the boundary of the future of the closed trapped surface is compact.

End of quote.

Strategy here, is to remove the caveat of compactness. Compactness, according to [18,19] is the situation combining closed, and bounded, and in this situation, it is part and parcel of the classification theorem for compact surfaces which can be accessed in [19]

We now should go to a new version of the modified HUP, and it will be stated as approximately as that unless an inflaton field exists in the Pre Planckian space-time so that [20]

$$\delta g_{tt} \sim a^2(t) \cdot \phi \xrightarrow{\phi \sim \text{Very Large}} 1 \quad (12)$$

Then by [2,3,4, 10]

$$\Delta E \approx \frac{\hbar}{\delta g_n \delta t} \neq \frac{\hbar}{2 \cdot \delta t} \tag{13}$$

Unless $\delta g_n \sim O(1)$

Preforce, the enormity of the change in energy, will the remove the possibility of a closed surface, in the Pre Planckian space-time. i.e. in doing so, the change in energy disrupts conditions as given in [21]

We shall next go to the division algebra results and gravitons, to give more structure to applying Eq. (13) above.

Vb. What leads to the Division Algebra results? I.e. what happens if there is a break down of initial singularities ? Referencing [22, 23]

In [22] , Dixon references [23] where in [22] there is a statement which resonates, i.e. if there is no bounded initial space, and there is a break down of compact bounded surfaces, due to what we have with Eq. (13), then one has to seriously consider the following quote

Quote:

“ Each of the four Division Algebras R, C, H, and O can also be viewed as a spinor space “, and later “ The mathematics linking these pairs is an SU(2) group”

i.e. so what permits the existence of a spinor space in a non Compact domain ? To whit the existence of gravitons, in a non compact space-time, and we will state then that there are theoretical arguments that a *massless* spin-2 particle has to be a graviton. The basic idea is that massless particles have to couple to conserved currents, and the only available one is the stress-energy tensor, which is the source for gravity. If a graviton is massless, a given as given by [24] is that there exist 2 polarization states $h_+(+)$ and $h_-(x)$, as are given , and that if there are massive gravitons, we will have by [25] that there are additional states. We assert in the case that the mass of a graviton, is about 10^{-62} grams, as given in [26] that initially it would be appropriate to look at the two helicity states, in the massless cases as an approximation , i.e. $h_+(+)$ and $h_-(x)$, and to compare directly with generic SU(2) states, via the following identification. Normally, a spin S object will have $2S+1$ polarization states. But for massless particles the transverse modes can't exist due to Lorentz invariance. Only the positive and negative helicity states remain. So, the graviton will have only 2 helicity states. The SU(2) **up and down states**, as in the case of the Pauli matrices, can be seen to be simply

$$\begin{aligned} |Up\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ |Down\rangle &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned} \tag{14}$$

We can of course, make a simple identification of $h_+(+)$ and $h_-(x)$ with the Up and Down states of Eq. (14), due to the fact that in the massless case, there are only 2 helicity states. We could as an example, make a simple relations

$$\begin{aligned} |Up\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \leftrightarrow h_+(+) \\ |Down\rangle &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \leftrightarrow h_-(x) \end{aligned} \tag{15}$$

As stated by [22, 23] whe have that the division algebras, in the quaternion setting may be linked by SU(2) via the methodology for complexification of the Quaternions in [22] , via

$$P = C \otimes H \tag{16}$$

According to [22], R is real, C is complex number, H , is Quaternion, and the O is octonionic

In the case of Eq. (16), [22] on page 47 claims that it is a

Quote:

“complexification of Quaternions’ and is equivalent to a pair of Paul Spinors, and if we form a Column matrix of two such elements we get a pair of Dirac spinors.”

End of quote

Furthermore from page 47-48 of [22] we have that

Quote:

“It is very interesting that the set of all unit quaternions is a copy of $SU(2)$ (Since H is 4 dimensional, the set of unit quaternions is topologically the same as the set of all points in 4 space a distance 1 from the origion, which is the 3-sphere, one of our Parallelizable spheres)” .

End of quote

This is the language of what [22] calls 1,3 spinor space-time.

In addition,

$$T = C \otimes H \otimes O \tag{17}$$

Is called in [22] are called a 1,9 spinor space-time. , where [22] states on page 48, that

Quote:

“ P and T spinors are $SU(2)$ doublets , so that leaves us with the reduced group, $SO(1,3) \times U(1) \times SU(2) \times SU(3)$ ”

The summary of what we are looking at is [27], i.e. here on page 284,

$$O = H \oplus H \tag{18}$$

Each element of Eq.(18) is given by

$$\begin{aligned} R &\simeq S^1 \\ C &\simeq S^2 \\ H &\simeq S^4 \\ O &\simeq S^8 \end{aligned} \tag{19}$$

This refers to 1,2, 4, and 8 ‘dimensional’ spheres, and 1, 2, 4, 8 are the Cayley numbers. This construction should be seen as a way of quantifying, as an example, Eq. (18) as a direct construction of 4 $SU(2)$ ‘spinors, and 2 Quaternion spinors. The moral being that we can build up a systematic algebra this way, which can use the set of spin $\frac{1}{2}$ wave function eigenvalue entries to build up through Eq.(17), Eq. (18) and Eq. (19) a linkage of 1,3 spinors as given by a proper interpretation of Eq. (16) as with comparisons with 1,9 spheres given in Eq. (17), Eq. (18) and Eq.(19). This construction though, and a linkage to massless versions of the Graviton, works well, if we wish to tie in the usual construction which may be appropriate for the interpretation of the 2 graviton polarization states as having a tine in, via Eq.(14) and Eq. (15) with the UP and Down basis spinors of $SU(2)$ and by extension the build up of the spinors of the Octonian as alluded to in Eq(18).

The relevance, in terms of space-time, in the case of massless gravitons is as follows, namely that if we can make the identification of Eq. (18) and link that to the idea of Eq. (15). Then the following situation occurs, namely

The change in geometry is occurring when we have first a pre quantum space time state, in which, in commutation relations [27] (Crowell, 2005) in the pre Octonionic space time regime no approach to QM commutations is possible as seen by.

$$[x_j, p_i] \neq -\beta \cdot (l_{Planck} / l) \cdot \hbar T_{ijk} x_k \text{ and does not } \rightarrow i\hbar \delta_{i,j} \quad (20)$$

Eq. (20) is such that even if one is in flat Euclidian space, and $i=j$, then

$$[x_j, p_j] \neq i \cdot \hbar \quad (21)$$

In the situation when we approach quantum “ octonion gravity applicable” geometry, Eq.(20) becomes

$$[x_j, p_i] = -\beta \cdot (l_{Planck} / l) \cdot \hbar T_{ijk} x_k \xrightarrow{\text{Approaching-flat-space}} i\hbar \delta_{i,j} \quad (22)$$

Eq.(22) is such that even if one is in flat Euclidian space, and $i=j$, then

$$[x_j, p_j] = i \cdot \hbar \quad (22a)$$

.Also the phase change in gravitational wave data due to a change in the physics and geometry between regions where Eq. (21)and Eq. (22) hold will be given by a change in phase of GW, which may be measured inside a GW detector.

Vc . Discussion of the geometry alteration due to the evolution from pre Planckian to Planckian regimes of space time, if Eq. (15) and Eq. (18) hold

The simplest way to consider what may be involved in alterations of geometry is seen in the fact that in pre **octonionic** space time regime (which is Pre Planckian), one would have [27] (Crowell, 2005)

This Pre Octonionic space-time behavior should be seen to be separate from the flatness condition as referred to in [27]. But retuning to [27] we have that, in Pre Planckian space- time, that

$$[x_j, x_i] \neq 0 \text{ under ANY circumstances, with low to high temperatures, or flat or curved space.} \quad (23)$$

Whereas in the **octonion** gravity space time regime where one would have Eq. (22) hold that for enormous temperature increases Eq. (22) , then by [27] (Crowell, 2005)

$$[x_j, x_i] = i \cdot [\Theta_{ji}] \xrightarrow{\text{Temp} \rightarrow \infty} 0 \quad (24)$$

Here,

$$\Theta_{ji} \sim \Lambda_{NC}^{-2} \sim [\Lambda_{4-Dim}]^{-2} \propto 1/[T^{2\beta}] \xrightarrow{T \rightarrow \infty} 0 \quad (25)$$

We argue that Eq. (24) holds, and that the Stoica non pathological singularity is removed, if there is a sufficiently large energy flux given by change in energy, in Eq. (13), but this requires that it not be infinite

We shall next go to the conclusions and to first review the conclusions made if we do not have the modifications due to an ultra large change in initial energy due to Eq. (13)

VI. Conclusions, part a, if we do not use Eq. (13). This is the case where the Stoica non Pathological singularity is kept

First of all, the question we need to ask is, is the existence of a Planck length, as a minimum length mandatory as to space-time? If it is, the problem of the existence of disjoint intervals is solved. I.e. we need not worry, even if it is 10^{-35} meters in length. If this minimum length exists, Eq. (8) holds everywhere.

If a mandatory minimum non-zero space-time interval is necessary then there is nothing which forbids the existence of (8) above. If such an interval does not exist, then (1) breaks down. Furthermore, the space of all infinitely differentiable functions is also separable, and a fundamental sequence is the sequence of all powers of x . This is shown by Taylor series and Weierstrass's theorem [28]. But having either Weierstrass theorem or Taylor series at a single point of space-time is a non starter, and also the dodge of using the simplification of a finite dimensional normed space breaks down. No longer at a point can, many of the computations be simplified by the existence of a finite basis, where every vector in the space is a linear combination of some subset of vectors in the basis. **One does not have a finite basis in a point of space time** [28].

It should be noted that Connes [29] outlines conditions for non commutative geometry in space-time for the development of exotic basis which in higher dimensions could restore separable space, i.e. even Hausdorff behavior, as would be necessary for disjoint sets to exist. But such a development would be involving encasing the four dimensional singularity as embedded in a hierarchy of higher dimensional geometric spaces. With 3 dimensional space and time at a singular point, one does not have a Hausdorff metric space X , separability and without having either of the above, then the construction for a quantum measure, as outlined and developed in the given Appendix A will not work out.

In essence, for making a consistent cosmology, our results argue in favor of a string theory style embedding of the start of inflation and what we have argued so far is indicating how typical four dimensional cosmologies have serious mathematical measure theoretic problems. These quantum measure theoretic problem are unphysical especially in light of the Stoica findings. [7]

VI. Conclusions, part b, if we do use Eq. (13). This is the case where the Stoica non Pathological singularity is removed

We argue in this second case, that then the problems are consistent with regards to the sort of measure theory as advocated by Tao [30], and that the use of Eq. (13) removes the problem cited in [1].

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Appendix A

This is straight from reference [1] and is applied only if we do not apply Eq. (13). Upon application of Eq. (13), this application is no longer used, and [6] is not pertinent. And [31] as an elaboration is not considered.

If Eq (13) is not used, which removes, this construction, we state that Stoica [6] does a re-scaling of the pressure and density along the following lines, namely the initial Friedman equation is changed i.e. it starts with

$$\rho = \frac{3}{\kappa} \cdot \frac{\dot{a}^2 + k}{a^2} \quad (1a)$$

Furthermore we also have the acceleration equation given by

$$\rho + 3p = -\frac{6}{\kappa} \cdot \frac{\ddot{a}}{a^2} \quad (2a)$$

Using the re-scaling of [6] using Σ as part of a ‘typical space’

$$\det g = -a^6 \det_3 g\Sigma \Leftrightarrow \sqrt{-g} = a^3 \sqrt{g\Sigma} \quad (3a)$$

We then re-scale the density and also the pressure as follows:

$$\begin{aligned} \tilde{\rho} &= \rho a^3 \sqrt{g\Sigma} \\ \tilde{p} &= p a^3 \sqrt{g\Sigma} \end{aligned} \quad (4a)$$

This will lead to

$$\begin{aligned} \tilde{\rho} &= \frac{a}{\kappa} (\dot{a}^2 + k) \sqrt{g\Sigma} \\ \tilde{\rho} + 3\tilde{p} &= -\frac{6}{\kappa} \cdot a^2 \ddot{a} \sqrt{g\Sigma} \end{aligned} \quad (5a)$$

The upshot is, as explained in [2] that then;

$$\begin{aligned}
 a(0) = 0 &\Leftrightarrow \tilde{\rho}(0) = \rho a(0)^3 \sqrt{g\Sigma} = 0 \\
 a(0) = 0 &\Leftrightarrow \tilde{p}(0) = p a(0)^3 \sqrt{g\Sigma} = 0
 \end{aligned}
 \tag{6a}$$

So then the acceleration equation and Friedman equation vanish at $a(0) = 0$

Appendix B

This is straight from reference [1] and is applied only if we do not apply Eq. (13). Upon application of Eq. (13), this application is no longer used, and [6] is not pertinent. Also, we then would, if we apply Eq. (13) NOT use the additional details in [31]. With that, let us commence the review.

We introduce the formalism by appealing to the concept of spatial diffeomorphism [12] as a necessary condition for linking the physics of what happens at a singularity to outside of the singularity of inflation generated space time geometry. Trivially, a diffeomorphism involves an infinitely differentiable, one-to-one mapping of the model to itself. In contrast, there is a breakdown of differentiability at the start of the big bang, based on non-loop-quantum-gravity theories.

We submit that the difficulties in terms of consistency of Eq. (8) of this document. In terms of initial causal structural breakdown -- which we claim leads to Eq. (8) being re written as an inequality -- one has to come up with a different way to embed quantum measures within a superstructure, as noted in the conclusions of this paper. Spatial diffeomorphisms as stated in [12] do not work unless there is a lattice structure, effectively doing away with a singularity. If the lattice structure is not used, differentiability breaks down and one does not have one-to-one mapping of the physics of the big bang singularity onto the rest of the inflationary process. We submit that this breakdown would then make Eq. (b1) not definable. As to the measure set structure, the readers are referred to [12] to get the foundations of the measure theory structure understood. The rest of this text is an adoption of what was done in [6], with the author's re interpretation of what the significance is of quantum measures as stated in [6], in the vicinity of a singularity.

The author's main point is that there is a break down of measurable structure, starting with definitions given in [6] and [12] where the concept of disjoint sets becomes meaningless in a point of space. In the causal set approach, the probabilities are held to be Markovian [6], label-independent and adhere to Bell's inequality. The author of [6] refers to a sequential growth called a classical transition percolation model. Then reference [6] extends the classical transition percolation model to complex models involving quantum measures in the definition of a (quantum) complex percolation model. Reference [6] defines the amplitude of transition as follows. For a quantum measure space defined as triple as given by (Ω, A, μ_ν) , with μ_ν a yet to be defined vector measure, A is an event algebra or set of propositions about the system, and Ω is the sample space of histories or space-time configurations.

Let $p \in C$ be amplitude of transition, instead of a probability; and set $\psi(C^n)$ as the amplitude for a transition from an empty set to n element of a causal set C^n , and with $Cyl(C^n)$ cylinder set as a subset of Ω containing labeled past finite causal sets whose first n elements form the causal subset C^n . Note that the cylinder sets form event algebra A with measure given by form the sub-causal set C^n . Here, ψ is a complex measure on A , so then ψ is a vector measure [3]. This is the primary point of breakdown that occurs in the case of a space time singularity. Away from the singularity we will be working with the physics of

$$D(Cyl(C^n), Cyl(C^m)) = \psi^*(C^n)\psi(C^m) \quad (b1)$$

This is done for a cylinder set, where γ is a given path, and γ' as a truncated path, with $cyl(\gamma')$ a subset of Ω and $\mu(cyl(\gamma')) = P(\gamma')$, with $P(\gamma')$ the probability of a truncated path, with a given initial (x_i, t_i) to final (x_f, t_f) spatial and times. Note that the μ measure would be for $\mu: A \rightarrow R^+$ obeying the weaker Quantum sum rule [31]

$$\mu(\alpha \cup \beta \cup \gamma) = \mu(\alpha \cup \beta) + \mu(\alpha \cup \gamma) + \mu(\beta \cup \gamma) - \mu(\alpha) - \mu(\beta) - \mu(\gamma) \quad (b2)$$

This probability would be a quantum probability which would *not* obey the classical rule of Kolmogorov [6]

$$P(\gamma_1 \cup \gamma_2) = P(\gamma_1) + P(\gamma_2) \quad (b3)$$

The actual probability used would have to take into account quantum interference. That is due to Eq. (1b) and Kolmogorov probability no longer applying, leading to

$$cyl(\gamma') \equiv \left\{ \gamma \in \Omega \mid \gamma(t') = \gamma'(t') \text{ for all } 0 \leq t' \leq t \right\} \quad (b4)$$

Here, $D: A \times A \rightarrow C$ is a decoherence functional, which is (i) Hermitian, (ii) finitely biadditive, and (iii) strongly additive, i.e., the eigenvalues of D constructed as a matrix over the histories $\{\alpha_i\}$ are non-negative. A quantum measurement is then defined via

$$\mu(\alpha) = D(\alpha, \alpha) \geq 0 \quad (b5)$$

A quantum vector measurement is defined via

$$\mu_v(\alpha) := [\chi_\alpha] \in H \quad (b6)$$

Where

$$\chi_\alpha(\beta) = \begin{cases} 1 \\ 0 \end{cases}, \quad \chi_\alpha(\beta) = 1 \text{ if } \beta = \alpha, \chi_\alpha(\beta) = 0 \text{ if } \beta \neq \alpha \quad (\text{b7})$$

Also V is the vector space over A with an inner product given by

$$\langle u, v \rangle_V \equiv \sum_{\alpha \in A} \sum_{\beta \in A} u^*(\alpha) v(\beta) \cdot D(\alpha, \beta) \quad (\text{b8})$$

with a Hilbert space H constructed by taking a sequence of Cauchy sequences $\{u_i\}$ sharing an equivalence relationship

$$\{u_i\} \sim \{v_i\} \text{ if } \lim_{i \rightarrow \infty} \|u_i - v_i\|_V = 0 \quad (\text{b9})$$

So then the following happens,

$$[\{u_i\}] + [\{v_i\}] \equiv [\{u_i + v_i\}] \quad (\text{b10})$$

$$[\{\lambda u_i\}] \equiv \lambda [\{u_i\}] \quad (\text{b11})$$

$$\langle [\{u_i\}], [\{v_i\}] \rangle \equiv \lim_{i \rightarrow \infty} \langle u_i, v_i \rangle_V \quad (\text{b12})$$

This is for all $[\{u_i\}], [\{v_i\}] \in H$ and $\lambda \in C$ so then the quantum measure is defined for $\mu_V : A \rightarrow H$ so the inner product on H is

$$\langle \mu_V(\alpha), \mu_V(\beta) \rangle = D(\alpha, \beta) \quad (\text{b13})$$

b

The claim associated with Eq. (b1) above is that since ψ is a complex measure of A , Eq. (b1) corresponds to an unconditional convergence of the vector measure over all partitions. Secondly according to the Caratheodary-Hahn theorem there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

The main point of the formalism for Eq. (b13) is of bi-additivity of D leading to the finite additivity of μ_V

$$\mu_V \left(\bigcup_{i=1}^n \alpha_i \right) = \sum_{i=1}^n \mu_V(\alpha_i) \quad (\text{b14})$$