Proton-Neutron Mass Difference by Electroweak Interactions

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Abstract – The weak interaction due to different compositions of up and down quarks leads to the neutron-proton mass difference. The radius of the nucleon is fixed by the strong interaction. In a first calculation, the weak coupling is introduced by the hand. In a second one, both the mass difference and the weak coupling are determined.

1 – Introduction

Proton and Neutron both have isospin I = 1 ⁄ 2, but with different projections, $m_I = \pm ½$ along a preferential axis of the isospin space [1,2]. Strong interaction is only sensible to the I quantum number, and therefore does not distinguish proton from neutrons (nucleons). Feynman and Speisman [3] have attributed all deviations of the isotropic spin symmetry solely to electromagnetic effects. One of the consequences of this viewpoint [3] is that proton-neutron mass difference would come from electrodynamics. The S-matrix method has been used by Dashen [4]as a means to calculate the proton-neutron mass difference. As was pointed out by Dashen [4], neutrons and protons are treated as bound-states poles in the π-N scattering amplitude and the mass difference is obtained by finding the electromagnetic corrections to their binding energies. Kwei-Chou Yang et al. [5] used the method of the QCD sum rules to investigate the neutron-proton mass difference. The electromagnetic contribution to the mass of nucleons was also investigated by Pagels [6]. Recently, Sz. Borzanyi [7] performed lattice QCD and QED calculations and computed the neutron-proton mass-splitting.

In this paper we will take in account the electroweak interactions as a means to obtain the proton-neutron mass difference.

2 – Mass difference given by the weak interactions

To compute the proton-neutron mass splitting, the next very simplified model is considered:

. For the neutron picture, we consider two down quarks performing circular orbit of radius R, having the up quark localized in the center. The two down quarks keep diametrically opposed positions, during their motion.

. For the proton picture, we consider two up quarks performing circular orbit of radius R around the down quark. The up quarks also keep the diametrically opposed sites, while executing their circular motion.

. The radius R of the orbit is fixed by the strong interaction.

. Let us take g_w as the unit of weak charge and attribute the charges: $+⅔\ g_w$ to the up quark and $-⅓\ g_w$ to the down quark. We also assume that weak charges of equal signal repel and of different signals attract. This seems to be consistent with the fact that at some high energy, we have the unification between weak and electric forces.

Now let us compute these weak interactions. We have

$$U_n = 2\ U_{u\text{-}d} + U_{d\text{-}d}, \tag{1}$$

$$U_p = 2\ U_{u\text{-}d} + U_{u\text{-}u}. \tag{2}$$

$$U_{u\text{-}d} = -\ [1/(4\pi)]\ (2/9)\ g_w^2/R, \tag{3}$$

$$U_{d\text{-}d} = +\ [1/(4\pi)]\ (1/9)\ g_w^2/(2R), \tag{4}$$

$$U_{u\text{-}u} = +\ [1/(4\pi)]\ (4/9)\ g_w^2/(2R). \tag{5}$$

$$U_p - U_n = U_{u\text{-}u} - U_{d\text{-}d} = [1/(4\pi)]\, g_w^{\,2}/(6R). \qquad (6)$$

Defining: $$[1/(4\pi)]\, g_w^{\,2} = \alpha_w, \qquad (7)$$

we have

$$m_n - m_p = U_p - U_n = \alpha_w/(6R). \qquad (8)$$

In (6) we have done $\hbar = c = 1$.

In order to estimate R , we make use of the MIT bag model [8],as giving by Xiangdong Ji [9]. We write

$$V(r) = 3/r + B\,(4/3)\,\pi\, r^3. \qquad (9)$$

In (9) the first term gives the contribution of the free quarks inside the bag and the volume term accounts for the vacuum pressure over the bag. Taking the minimum of V at r = R and equaling V(R) to the nucleon mass M, we get

$$R = 4/M. \qquad (\hbar = c = 1) \qquad (10)$$

Inserting the result of (10) into (8) we obtain

$$\Delta M = m_n - m_p = \alpha_w\, M/24. \qquad (11)$$

Taking $\alpha_w = 1/30$ (in the appendix , based in reference [10], we give a justification for this choice) and putting in the numbers, we find

$$\Delta M = m_n - m_p = (940/720)\ \text{MeV} \approx 1.31\ \text{MeV}. \tag{12}$$

The above value is close to the measured neutron-proton mass difference of 1.29 MeV, as can be inferred from ref.[11].

3 –Another estimate of the weak coupling

In order to make another estimate of the weak coupling, besides the reasoning to be treated in the appendix, we turn now to an independent evaluation of the mass difference ΔM.

First we consider the neutron-proton transmutation, without looking inside at the quarks, leading to an excess of electrostatic energy ΔU_e , given by

$$\Delta U_e = \tfrac{1}{2}(\hbar/R)\,\alpha c = \tfrac{1}{2}\,\alpha/R. \tag{13}$$

In the last step we took $\hbar = c = 1$.

Inserting (10) into (13),, we get

$$\Delta U_e = \alpha M/8. \tag{14}$$

Now let us take in account the neutron β-decay:

$$n \rightarrow p + e^- + \underline{\upsilon}_e \tag{15}$$

In an event where we could approximately neglect the energy carried out by neutrinos, we can write (being m_e the electron mass)

$$\Delta M = \Delta U_e + m_e \, . \tag{16}$$

Next we make the requirement of the equality between ΔM as given by (11) or (16) and we write

$$\alpha_w \, M/24 = \alpha \, M/8 + m_e, \tag{17}$$

which leads to

$$\alpha_w = 24 \, (m_e/M) + 3\alpha. \tag{18}$$

Naturally α_w in (18) can be evaluated, by considering the experimental values for the nucleon-electron mass ratio (m_e / M), as well for the fine-structure constant α. However let us take a step further. Geometric reasonings were employed before [12,13] as a means to evaluate the proton-electron mass ratio. The obtained result was

$$m_p \, / m_e = M / m_e = 6 \, \pi^5. \tag{19}$$

Inserting (19) into (18) gives

$$\alpha_w = 4/\pi^5 + 3 \, \alpha. \tag{20}$$

Putting in numbers, we find

$$\alpha_w = 1/28.6 \ . \qquad (21)$$

as we can see, this value for the weak coupling $\alpha_w \approx 1/29$, is close to that of $1/30$ we have used to determine ΔM (please see (11) and (12)).

4 – Concluding remarks

It seems that relation (20) nicely fits to the Griffiths observation [10] which states that: “Weak interactions are feeble not because the intrinsic coupling (it isn’t) but because the mediators are so massive, etc…”.

Appendix

The use of $\alpha_w = 1/30$, on determining the proton-neutron mass difference (ΔM) will be now justified. In a discussion of the muon decay, Griffiths [10] notice that the weak charge g_w and the mass of the boson which intermediates the weak interaction M_w, do not appear separately in the relation for the muon lifetime and both could be absorbed in the Fermi constant G_F. In the Fermi original theory of beta decay there was no W, as was pointed out by Griffiths [10].Therefore α_w can be obtained from the experimental determination of G_F, once M_w is given. However, the weak coupling is also a running coupling constant. Griffiths found that

$$\alpha_w(m_\mu) = 1/29 \ . \qquad (22)$$

Taking the constituent mass of quarks as $m_q = 313$ MeV, and using an expression for the running coupling constant as deduced in reference [14], we can write

$$\alpha_w(m_q) = \alpha_w(m_\mu)/[1 + \tfrac{1}{2}\,\alpha_w(m_\mu)\,\ln(m_q/m_\mu)]. \qquad (23)$$

Putting in numbers, we find (m_μ = 116 MeV)

$$\alpha_w(m_q) \approx 1/30. \qquad (24)$$

This result has been used in (12).

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