Two conjectures on Smarandache’s divisor products sequence

Abstract. In this paper I make the following two conjectures on the Smarandache’s divisor products sequence where a term $P(n)$ of the sequence is defined as the product of the positive divisors of $n$: (1) there exist an infinity of $n$ composites such that the number $m = P(n) + n - 1$ is prime; (2) there exist an infinity of $n$ composites such that the number $m = P(n) - n + 1$ is prime.

The Smarandache’s divisor products sequence (see A007955 in OEIS):
: 1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19, 8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 81000, 31, 32768, 1089, 1156, 1225, 10077696, 37, 1444, 1521, 256000, 41, 3111696, 43, 85184, 91125, 2116, 47, 254803968 (...)

Conjecture 1:

Let $P(n)$ be the Smarandache’s divisor products sequence where a term $P(n)$ of the sequence is defined as the product of the positive divisors of $n$: there exist an infinity of $n$ composites such that the number $m = P(n) + n - 1$ is prime.

Note that for $n$ primes, because $P(n) = n$, $P(n) + n - 1 = 2*n - 1$ and is already conjectured that there exist an infinity of primes of the form $2*q - 1$, where $q$ prime.

The sequence of primes $m$:
: $m = 3$, prime, for $(n, P(n)) = (2, 2)$;
: $m = 11$, prime, for $(n, P(n)) = (4, 8)$;
: $m = 41$, prime, for $(n, P(n)) = (6, 36)$;
: $m = 71$, prime, for $(n, P(n)) = (8, 64)$;
: $m = 109$, prime, for $(n, P(n)) = (10, 100)$;
: $m = 1739$, prime, for $(n, P(n)) = (12, 1728)$;
: $m = 239$, prime, for $(n, P(n)) = (15, 225)$;
: $m = 1039$, prime, for $(n, P(n)) = (16, 1024)$;
: $m = 5849$, prime, for $(n, P(n)) = (18, 5832)$;
: $m = 461$, prime, for $(n, P(n)) = (21, 441)$;
: $m = 149$, prime, for $(n, P(n)) = (25, 125)$;
: $m = 701$, prime, for $(n, P(n)) = (26, 676)$;
: $m = 1259$, prime, for $(n, P(n)) = (35, 1225)$;
: $m = 1481$, prime, for $(n, P(n)) = (38, 1444)$;
: $m = 2560039$, prime, for $(n, P(n)) = (40, 2560000)$;
\[ m = 2161, \text{ prime, for } (n, P(n)) = (46, 2116); \]

Examples of larger m:

\[ m = 46656000059, \text{ prime, for } (n, P(n)) = (60, 46656000000); \]
\[ m = 782757789791, \text{ prime, for } (n, P(n)) = (96, 782757789696); \]
\[ m = 1586874323051, \text{ prime, for } (n, P(n)) = (108, 158687422944); \]
\[ m = 63456228123711976, \text{ prime, for } (n, P(n)) = (168, 63456228123711976). \]

Note that m is prime for \( n = 12, 60, 96, 108, 168 \). I conjecture that m is prime for an infinity of \( n \) of the form \( 12k \).

**Conjecture 2:**

Let \( P(n) \) be the Smarandache’s divisor products sequence where a term \( P(n) \) of the sequence is defined as the product of the positive divisors of \( n \): there exist an infinity of \( n \) composites such that the number \( m = P(n) - n + 1 \) is prime.

Note that for \( n \) primes, because \( P(n) = n \), \( P(n) - n + 1 = 1 \).

The sequence of primes \( m \):

\[ m = 5, \text{ prime, for } (n, P(n)) = (4, 8); \]
\[ m = 31, \text{ prime, for } (n, P(n)) = (6, 36); \]
\[ m = 19, \text{ prime, for } (n, P(n)) = (9, 27); \]
\[ m = 211, \text{ prime, for } (n, P(n)) = (15, 225); \]
\[ m = 1009, \text{ prime, for } (n, P(n)) = (16, 1024); \]
\[ m = 421, \text{ prime, for } (n, P(n)) = (21, 441); \]
\[ m = 463, \text{ prime, for } (n, P(n)) = (22, 484); \]
\[ m = 331753, \text{ prime, for } (n, P(n)) = (24, 331776); \]
\[ m = 149, \text{ prime, for } (n, P(n)) = (25, 125); \]
\[ m = 1123, \text{ prime, for } (n, P(n)) = (34, 1156); \]
\[ m = 254803921, \text{ prime, for } (n, P(n)) = (48, 254803968); \]

Examples of larger m:

\[ m = 531440999911, \text{ prime, for } (n, P(n)) = (90, 531441000000); \]
\[ m = 389328928561, \text{ prime, for } (n, P(n)) = (208, 389328928768). \]

Note that m is prime for \( n = 24, 48 \). I conjecture that m is prime for an infinity of \( n \) of the form \( 12k \).