

The Leibniz theorem in the Bohr model and the parity oscillation

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Abstract

We unify the Bohr energy formula with the Leibniz continuity theorem in order to get the aufbau of photon. During the electron transition in this model the photon is created by the continual way. The oscillation of parity of K-meson is discussed.

Key words: Bohr energy formula, Leibniz theorem, continual process, K-meson, parity.

The classical mechanics and the Larmor formula, predict that the electron will release electromagnetic radiation while orbiting a nucleus. Electron losing energy, rapidly spirals inwards, along the possible specific spirals (hyperbolic spiral, the Fermat spiral, the lituus, the Archimedean spiral with polar equation $r = a + \varphi^{1/c}$, etc.), collapsing into the nucleus. This atom model predicts that all atoms are unstable. To overcome this difficulty, Niels Bohr proposed, in 1913 model of the atom with the following physical ingredients (Bohr, 1913a; 1913b; 1913c; 1914; 1921):

1. Electrons in atoms orbit the nucleus.
2. The electrons are moving without radiating in certain orbits (called by Bohr the stationary orbits at a certain discrete set of distances from the nucleus. These orbits are associated with definite energies.
3. Electrons can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting electromagnetic radiation of frequency

$$E_i - E_f = \hbar\omega, \quad (1)$$

where E_i, E_f , are the initial and the final energy of electron in case of the emitting process and $E_f < E_i$, \hbar being the Planck constant. The angular momentum L is restricted to be an integer multiple of a fixed unit: $L = n\hbar$, or,

$$mvr = n\hbar, \quad (2)$$

where $n = 1, 2, 3, \dots$ is called the principal quantum number, and $\hbar = h/2\pi$. The lowest value of n is 1; this gives a smallest possible orbital radius of 0.0529 nm known as the Bohr radius. Starting from the angular momentum quantum rule, Bohr was able to calculate the energies of the allowed orbits of the hydrogen atom and other hydrogen-like atoms and ions.

Bohr's formula assumes that during a quantum jump a discrete amount of energy is radiated in the form nowadays known as photons. However, Bohr did not believe in the existence of photons.

In 1924 de Broglie introduced standing wave condition that the electron is a wave and a whole number of wavelengths must fit along the circumference of the electron orbit:

$$n\lambda = 2\pi r. \quad (3)$$

Substituting de Broglie's wavelength of $\lambda = h/p$ reproduces Bohr's rule. In 1925 the new theory was proposed by Heisenberg as the matrix mechanics and the physically equivalent theory, wave mechanics, was proposed by Schrödinger, who employed de Broglie matter waves in the three-dimension equation. The Schrödinger wave equation was able to overcome many of the shortcomings of the Bohr model.

It is natural statement that the motion of an electron is along some spiral trajectory from the energy levels E_i to E_f and viceversa. Such trajectory is logically necessary because of the principle of continuity in nature which is expressed by the Leibniz theorem - **Natura non facit saltus**. The Leibniz theorem is of the universal validity for all time of the existence of universe and it is the hidden integral part of all physical theories. It can be reformulated by the sentence - **every jump in nature is short continual process**. It also follows from the Leibniz theorem that the quantization of space, time and space-time is physically meaningless.

The radius of an electron circular orbit with number n in the hydrogen atom is:

$$r_n = \frac{n^2 \hbar^2}{e^2 m} \quad (4)$$

and the velocity of an electron on the n -th orbit in the hydrogen atom is

$$v_n = \sqrt{\frac{e^2}{m r_n}}. \quad (5)$$

The segment of the spiral between two concentric circles is the trajectory which connects continuously the i -th circle and f -th circle in such a way that the starting point and the end points are the tangent points of the hypothetical spiral.

In case of the emission of photon the tangent point is the point where electron has the velocity v_i on the initial orbit and the end point is the point where the velocity is v_f on the final orbit. During the motion of electron along the segment of the spiral, the photon with frequency $\omega = (E_i - E_f)/\hbar$ is created.

We postulate that the length of the spiral segment Δl is the wave length of photon. This is so called formation length and the formation time is

$$\Delta t = \Delta l / v, \quad (6)$$

where $v \approx \frac{1}{2}(v_i - v_f)$ with good approximation for dimensions in H-atom.

Let us remark that the formation length and formation time of the creation of photon in the hydrogen atom is not involved in the Bohr theory, or in the Bohr-Sommerfeld theory, or, nor

in the Heisenberg, nor in the Schrödinger quantum mechanics. In such a way we here define the missing problems of quantum mechanics. The experimental verification of the existence of the formation length and formation time of photon created in the hydrogen atom is the crucial problem of the experimental physics.

Let us notice, that the Bohr formula

$$\hbar\omega = E_i - E_f \quad (7)$$

can be generalized by the elementary way which has the deep physical meaning.

$$E_i - E_f = \int_{E_i}^{E_f} dE = \int_{r_i}^{r_f} \frac{dE}{dr} dr = \int_{r_i}^{r_f} \varrho(r) dr, \quad (8)$$

where $dE/dr = \varrho(r)$ is some function expressing the energy loss of electron during its motion on the spiral and forming photon with energy $\hbar\omega$.

The last formula is the logical necessity for the Bohr model which without this formula is incomplete and at the same time quantum mechanics is without this formula incomplete too.

There is a classical and quantum energy for the hydrogen atom:

$$E = -\frac{e^2}{2r}; E_n = -\frac{e^4 m}{2n^2 \hbar^2} \quad (9)$$

leading to

$$\frac{dE}{dr} = \varrho(r) = \frac{e^2}{2r^2} \quad (10)$$

and

$$E_i - E_f = \int_{r_i}^{r_f} \varrho(r) dr = \int_{r_i}^{r_f} \left(\frac{e^2}{2r^2} \right) dr = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad (11)$$

where $R = e^4 m / 2\hbar^2$ is so called the Rydberg constant.

Now, let us unify the Leibniz theorem with the problem of parity oscillation of K-meson, which follows the ideas on the parity violation considered by Lee and Yang (Lee and Yang, 1957). We know that Bruno Pontecorvo was the first physicist who introduced the electron-neutrino, muon-neutrino and tau-neutrino mutual oscillation (Pontecorvo, 1968). We used here his idea to define the so called parity oscillation of K_+, K_- mesons with the parity $P = 1, -1$ and to show that it is in harmony with the Leibniz theorem.

It is well known that the parity operator P acts on the wave function $\psi(x)$ as follows:

$$P\psi(x) = \lambda\psi(-x), P^2\psi(x) = \lambda^2\psi(x) = \psi(x), \quad (12)$$

from which follows that the eigen value of parity λ is ± 1 . The wave function $\psi(x)$ corresponds to the state with positive parity and $-\psi(x)$ corresponds to the state with negative parity. Because $|\psi|^2$ does not depend on the parity number P , the parity as such has no immediate physical meaning. The same statement is not valid for the Muirhead imaginary parity (Muirhead, 1968).

Now, let us be specific and let us consider the $\theta - \tau$ puzzle. If the decay is according to form $\theta \rightarrow \pi + \pi$, then parity of θ is $(-1)(-1) = 1$, and if $\tau \rightarrow \pi + \pi + \pi$, then parity of τ is $(-1)(-1)(-1) = -1$, because William Chinowsky and Jack Steinberger demonstrated that the

pion has negative parity (Chinowsky and Steinberger, 1954). However, it was experimentally confirmed that θ and τ are the same particle called K-meson. So we have two decays:

$$K \rightarrow \pi + \pi, \quad (13)$$

$$K \rightarrow \pi + \pi + \pi. \quad (14)$$

The first process can be evidently described by Hamiltonian $H_1 = AB_+$, and the second process can be described by Hamiltonian $H_2 = AB_-$, where operator A is of the positive parity. The adequate Hamiltonian was later postulated in the form

$$H = A + (C_1B_+ + C_2B_-), \quad (15)$$

where C_1, C_2 are some constants.

After application of the parity operator on the Hamiltonian (15), we get

$$PH = A + (C_1B_+ - C_2B_-), \quad (16)$$

It is possible to show, (Wu et al., 1957), that the angular distribution $N(\varphi)$ of electrons in beta decay of Cobalt C_{60} is described by function, which follows from the Hamiltonian, which is combination of positive parity term and negative parity term in (15). The resulting distribution of electrons in beta-decay is

$$N(\varphi) = c_1(1 + c_2 \cos \varphi). \quad (17)$$

It means that the correct description of the process of the violation of parity is described by the Hamiltonian with the positive and negative parity term.

The main idea of this section is in the following principle: Theorem: If system K is described by Hamiltonian $H_1 = AB_+$, where A, B_+ are the positive parity operators, and at the same time it can be described by the Hamiltonian $H_2 = AB_-$, where B_- is the negative parity operator, then it can be described by the Hamiltonian $H = A(C_1B_+ + C_2B_-)$ (following from the so called superposition principle of operators).

The immediately statement following from the last principle is, that K system oscillates between positive and negative parity states to form the positive parity states, or the negative parity states. If the system is K -meson, then this meson oscillates between positive and negative parity mesons, forming θ particle and τ particle.

The oscillation of parity is the crucial problem of the future particle physics, which after general experimental confirmation will establish the Leibniz famous statement. To support logically the Leibniz statement in modern particle physics it is enough to consider the natural logical limits:

$$\lim_{t \rightarrow 0_+} (\pi + \pi) \rightarrow K_+ \quad (18)$$

$$\lim_{t \rightarrow 0_+} (\pi + \pi + \pi) \rightarrow K_- \quad (19)$$

The application of a laser as a source of intense electromagnetic radiation enables to study the new class of physical processes which are running in the intense field of the electromagnetic

wave. The probability of some processes are increased, or, decreased in comparison with the processes in vacuum. The polarization of the electromagnetic wave plays important role too. The situation with the two electromagnetic waves is the next step of the laser physics of elementary particles. So, with regard to this situation in particle physics it is reasonable to consider the parity oscillation in laser field, or two laser fields and so on. At the same time the influence of the laser field on the parity violation (or restoration) in case of the K-mesons was, to our knowledge never considered. So, we expect new experiments in this area of physics.

Quantum theory which cannot give information on the motion of electron between two stationary states is evidently incomplete. At present time there is no theory which gives the description of the electron motion between two stationary energy states. At the same time there is no proposed experiment which would be feasible for the determination of the state of electron between energy levels. While the stationary states ψ_i, ψ_f in quantum mechanics have concrete and specified physical meaning, the state between energies E_i, E_f , which is the transition between E_i and E_f and being denoted as $\psi_{i \rightarrow f}$ was never defined in modern quantum mechanics. So, quantum mechanics is incomplete, because it does not involve the Leibniz theorem on the continual processes.

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