Observation on the lenght of the period of the rational number which is the sum of 1÷(pi-1) where pi are the 2-Poulet numbers

Abstract. In this paper I make the following observation: let p1, p2,..., pi be the ordered set of the 2-Poulet numbers; then the lenght of the period of the rational number which is the sum 1/(p1 – 1) + 1/(p2 – 1) +...+ 1/(ni - 1) seems to be always (for any i > 2) divisible by 240. This is not the fact when the numbers p1, p2,..., pi are not the ordered set of 2-Poulet numbers but few randomly taken (even consecutive) 2-Poulet numbers. For a related topic see my previous paper "A pattern that relates Carmichael numbers to the number 66" where I noticed that the lenght of the period of the rational number which is the sum 1/(c1 – 1) + 1/(c2 – 1) +...+ 1/(ci - 1), where c1, c2, ..., ci is the ordered set of Carmichael numbers, seems to be always divisible by 66.

Observation:

Let p1, p2,..., pi be the ordered set of the 2-Poulet numbers; then the period of the rational number which is the sum 1/(p1 – 1) + 1/(p2 – 1) +...+ 1/(ni - 1) seems to be always (for any i > 2) divisible by 240. This is not the fact when the numbers p1, p2,..., pi are not the ordered set of 2-Poulet numbers but few randomly taken (even consecutive) 2-Poulet numbers.

Note:

For the sequence of 2-Poulet numbers see A214305 on OEIS. The sequence is: 341, 1387, 2047, 2701, 3277, 4033, 4369, 4681, 5461, 7957, 8321, 10261, 13747 (...).

Verifying the observation:
(true up to i = 13)

: for p1 = 341, p2 = 1387 and p3 = 2047 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 equal to 240;
: for p4 = 2701 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 equal to 240;
: for p5 = 3277 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 equal to 240;
: for p6 = 4033 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 equal to 240;

: for p7 = 4369 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 equal to 240;

: for p8 = 4681 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 + 1/4680 equal to 240;

: for p9 = 5461 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 + 1/4680 + 1/5460 equal to 240;

: for p10 = 7957 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 + 1/4680 + 1/5460 + 1/7956 equal to 240;

: for p11 = 8321 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 + 1/4680 + 1/5460 + 1/7956 + 1/8320 equal to 240;

: for p12 = 10261 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 + 1/4680 + 1/5460 + 1/7956 + 1/8320 + 1/10260 equal to 720 = 3*240;

: for p13 = 13747 we have the lenght of the period of the number 1/340 + 1/1386 + 1/2046 + 1/2700 + 1/3276 + 1/4032 + 1/4368 + 1/4680 + 1/5460 + 1/7956 + 1/8320 + 1/10260 + 1/13746 equal to 65520 = 273*240.

Note:

As I mentioned in Abstract, the observation doesn’t apply when the numbers p1, p2,..., pi are not the ordered set of 2-Poulet numbers but few randomly taken (even consecutive) 2-Poulet numbers. Examples:

: for [p1, p2, p3] = [1387, 2047, 2701] we have the lenght of the period of the number 1/(p1 – 1) + 1/(p2 – 1) + 1/(p3 – 1) equal to 30 which is not divisible by 240;

: for [p1, p2, p3, p4] = [1387, 2047, 2701, 3277] we have the lenght of the period of the number 1/(p1 – 1) + 1/(p2 – 1) + 1/(p3 – 1) + 1/(p4 – 1) equal to 30 which is not divisible by 240;

: for [p1, p2, p3] = [2701, 3277, 4033] we have the lenght of the period of the number 1/(p1 – 1) + 1/(p2 – 1) + 1/(p3 – 1) equal to 6 which is not divisible by 240.

Note:

For a related topic see my previous paper “A pattern that relates Carmichael numbers to the number 66” where I noticed that the lenght of the period of the rational number which is the sum 1/(c1 – 1) + 1/(c2 – 1) +...+ 1/(ci - 1), where c1, c2, ..., ci is the ordered set of Carmichael numbers, seems to be always divisible by 66.