

# Theoretical determination of fundamental physical constants

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The article presents a theoretical method of determination of fundamental physical constants. The results of analytical calculations, including: the fine structure constant, the wave length of Compton, the electron mass, elementary charge, Planck constant, Planck mass, length and time, Planck, Newton's gravitational constant, the lifetime of the neutron.

Keywords: the fine-structure constant, the wave length of Compton, the electron mass, elementary charge, Planck constant, Planck mass length and time, Newton's gravitational constant, the lifetime of the neutron, the baryon asymmetry of the Universe

Notes:

If the parameter has a subscript « $\pi$ » means that this theoretical parameter has a numeric value that can be used instead of the true parameter value.

Used measurement units: length  $u_{\pi l} = 1,0[\text{cm}]$ , mass  $u_{\pi m} = 1,0[\text{r}]$ , time  $u_{\pi t} = 1,0[\text{c}]$  and the surface density of mass  $u_{\pi \rho S} = \frac{u_{\pi m}}{u_{\pi l}^2}$  of the Unitary of system of units.

## 1. The basic relationships

Let us write the expression

$$k_{\pi 0}^{n-1} \cdot \lambda_{\pi} \cdot (1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^n = (\sqrt{2} \cdot \pi)^n \cdot \lambda_{\pi 0}^n, \quad (1.1)$$

Where

$$k_{\pi 0} = \lambda_{\pi} \cdot \alpha_{\pi} \cdot \beta_{\pi}; \quad \lambda_{\pi 0}^n = \pi^{n-1} \cdot k_{\pi 0}^n; \quad (1.2)$$

$\alpha_{\pi}$ ,  $\beta_{\pi}$ ,  $\Delta y_{\pi}$  – numeric parameters;  $\lambda_{\pi}$ ,  $k_{\pi 0}$ ,  $\lambda_{\pi 0}$  – the parameters with dimension of length;  $n = 1, 2, 3, \dots$  – integer from natural number.

It is known [1, p. 37] algebraic equation with an unknown  $x$  degree  $n$  type:

$$f(x) \equiv a_0 \cdot x^n + a_1 \cdot x^{n-1} + \dots + a_{n-1} \cdot x + a_n = 0 \quad (a_0 \neq 0). \quad (1.3)$$

Here  $n$  – non-negative integer,  $a_0, a_1, \dots, a_n$  – is a real numbers,  $f(x)$  – the polynomial [2, p. 7] extent  $n$  on one variable  $x$ :

$$f(x) = (1+x)^n = 1 + n \cdot x + \frac{n \cdot (n-1)}{2!} \cdot x^2 + \frac{n \cdot (n-1) \cdot (n-2)}{3!} \cdot x^3 + \dots + \frac{n!}{(n-r)! \cdot r!} \cdot x^r + \dots \quad (1.4)$$

$$f(x) = (1-x)^n = 1 - n \cdot x + \frac{n \cdot (n-1)}{2!} \cdot x^2 - \frac{n \cdot (n-1) \cdot (n-2)}{3!} \cdot x^3 + \dots + (-1)^r \cdot \frac{n!}{(n-r)! \cdot r!} \cdot x^r + \dots \quad (1.5)$$

If  $n$  – positive integer, the expressions (1.4) and (1.5) consists of a *finite number* of members.

The algebraic equation of the form (1.3) is called valid if all its coefficients  $a_i$  – are real numbers. It is known [1, p. 39] that the corresponding equation (1.3) is a valid polynomial  $f(x)$  of the form (1.4) and (1.5) for all valid values  $x$  can take values. In the article are only valid algebraic equation of the form (1.3).

We write (1.1) in the form

$$\frac{(1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^n}{(\sqrt{2} \cdot \pi)^n} \cdot k_{\pi 0}^{n-1} \cdot \lambda_{\pi} = \lambda_{\pi 0}^n. \quad (1.6)$$

We denote the left part of equation (1.6) as

$$\lambda_{\pi S}^{n-1} = \frac{(1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^n}{(\sqrt{2} \cdot \pi)^n} \cdot k_{\pi 0}^{n-1}, \quad (1.7)$$

then (1.6) can be written:

$$\lambda_{\pi S}^{n-1} \cdot \lambda_{\pi} = \lambda_{\pi 0}^n. \quad (1.8)$$

Taking into account (1.2), the expression (1.7) can be written as

$$\lambda_{\pi S}^{n-1} = \frac{(1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^n \cdot (\alpha_{\pi} \cdot \beta_{\pi})^{n-1}}{(\sqrt{2} \cdot \pi)^n} \cdot \lambda_{\pi}^{n-1}. \quad (1.9)$$

At the same time, taking into account (1.2) and (1.8)  $\lambda_{\pi S}^{n-1}$  can be written in the form

$$\lambda_{\pi S}^{n-1} = \pi^{n-1} \cdot (\alpha_{\pi} \cdot \beta_{\pi})^n \cdot \lambda_{\pi}^{n-1}. \quad (1.10)$$

Equating (1.9) and (1.10), we obtain:

$$(\sqrt{2} \cdot \pi)^n \cdot \pi^{n-1} \cdot \alpha_{\pi} \cdot \beta_{\pi} = (1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^n. \quad (1.11)$$

It is known [1, p. 38] that a general formula expressing the roots of algebraic equations through the coefficients and containing only a *finite number* of operations, additions and subtractions, multiplications, divisions and root extractions only exist for equations of degree  $n \leq 4$ . With this in mind, we write the equation (1.1) for the case  $n = 3$  in the form of:

$$k_{\pi 0}^2 \cdot \lambda_{\pi} \cdot (1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^3 = (\sqrt{2} \cdot \pi)^3 \cdot \lambda_{\pi 0}^3, \quad (1.12)$$

then  $\lambda_{\pi 0}^n$  from (1.2) can be written as

$$\lambda_{\pi 0}^3 = \pi^2 \cdot k_{\pi 0}^3, \quad (1.13)$$

and (1.6) as

$$\frac{(1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^3}{(\sqrt{2} \cdot \pi)^3} \cdot k_{\pi 0}^2 \cdot \lambda_{\pi} = \lambda_{\pi 0}^3. \quad (1.14)$$

Designating the area  $s_{\pi}$  as

$$s_{\pi} = \frac{(1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^3}{(\sqrt{2} \cdot \pi)^3} \cdot k_{\pi 0}^2, \quad (1.15)$$

we write (1.14), taking into account (1.15), as

$$s_{\pi} \cdot \lambda_{\pi} = \lambda_{\pi 0}^3. \quad (1.16)$$

Taking into account (1.2), (1.15) can be written as

$$s_{\pi} = \frac{(1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^3 \cdot (\alpha_{\pi} \cdot \beta_{\pi})^2}{(\sqrt{2} \cdot \pi)^3} \cdot \lambda_{\pi}^2. \quad (1.17)$$

At the same time, taking into account (1.2), (1.13) and (1.16), the area  $s_{\pi}$  can be written as

$$s_{\pi} = \pi^2 \cdot (\alpha_{\pi} \cdot \beta_{\pi})^3 \cdot \lambda_{\pi}^2. \quad (1.18)$$

Let us denote in (1.18) elementary scalar radius  $r_{\pi S}$  as

$$r_{\pi S} = \alpha_{\pi} \cdot \beta_{\pi}. \quad (1.19)$$

scalar the length of a circle  $l_{\pi S}$ , given (1.19), equal

$$l_{\pi S} = 2 \cdot \pi \cdot \alpha_{\pi} \cdot \beta_{\pi} \quad (1.20)$$

and the scalar square  $s_{\pi S}$  is equal to

$$s_{\pi S} = 4 \cdot \pi^2 \cdot r_{\pi S}^2, \quad (1.21)$$

scalar volume  $v_{\pi S}$ :

$$v_{\pi S} = \pi^2 \cdot r_{\pi S}^3. \quad (1.22)$$

Equating (1.17) and (1.18), we obtain the equation:

$$(\sqrt{2} \cdot \pi)^3 \cdot \pi^2 \cdot \alpha_{\pi} \cdot \beta_{\pi} = (1 \pm \Delta y_{\pi} \cdot \alpha_{\pi})^3. \quad (1.23)$$

## 2. Protoparameters

### 2.1. Scalar parameter of structure of space-time

We write the equation (1.23) in the form

$$(\sqrt{2} \cdot \pi)^3 \cdot \pi^2 \cdot \alpha_{\pi 0} \cdot \bar{\beta}_{\pi} = (1 + \Delta y_{\pi} \cdot \alpha_{\pi})^3, \quad (2.1.1)$$

where:

$$\bar{\beta}_{\pi} = 1 + \bar{\beta}_{\pi 0}; \quad (2.1.2)$$

$$\Delta y_{\pi 0} = \sqrt[4]{2 \cdot \pi}; \quad (2.1.3)$$

$$\varphi_{\pi 0} = \frac{\alpha_{\pi 0}}{\beta_{\pi 0}}, \quad (\varphi_{\pi 0} = \sqrt{2} \cdot \pi). \quad (2.1.4)$$

The right part of (2.1.1) is a polynomial (1.4) for the case of  $n = 3$  [2, p. 8]:

$$(1 + x)^3 = 1 + 3 \cdot x + 3 \cdot x^2 + x^3. \quad (2.1.5)$$

Denoting  $x = \Delta y_{\pi 0} \cdot \alpha_{\pi 0}$ , we write (2.1.5) as

$$(1 + \Delta y_{\pi 0} \cdot \alpha_{\pi 0})^3 = 1 + 3 \cdot \Delta y_{\pi 0} \cdot \alpha_{\pi 0} + 3 \cdot \Delta y_{\pi 0}^2 \cdot \alpha_{\pi 0}^2 + \Delta y_{\pi 0}^3 \cdot \alpha_{\pi 0}^3. \quad (2.1.6)$$

Equation (2.1.5) is written in general form as [3, p. 304]:

$$a \cdot x^3 + b \cdot x^2 + c \cdot x + d = 0 \quad (a \neq 0). \quad (2.1.7)$$

Using any of the known methods for solving cubic equations (for example, the solution of Cardano [1, p. 43] or search procedure – for example, a method of half division [3, p. 472]) yields the parameter  $\alpha_{\pi 0}$  – is the real root of the equation (2.1.1).

We write the equation (1.23) in the form:

$$(\sqrt{2} \cdot \pi)^3 \cdot \pi^2 \cdot \alpha_{\pi e} \cdot \beta_{\pi e} = (1 - \Delta y_{\pi e} \cdot \alpha_{\pi e})^3, \quad (2.1.8)$$

in which the parameter  $\beta_{\pi e}$

$$\beta_{\pi e} = 1 + \frac{\bar{\beta}_{\pi 0}}{\beta_{\pi}^3}. \quad (2.1.9)$$

Right hand side of (2.1.8) is a polynomial (1.5) for the case of  $n = 3$  [2, p. 8]:

$$(1 - x)^3 = 1 - 3 \cdot x - 3 \cdot x^2 - x^3. \quad (2.1.10)$$

Denoting  $x = \Delta y_{\pi e} \cdot \alpha_{\pi e}$ , we write (2.1.10) in the form

$$(1 - \Delta y_{\pi e} \cdot \alpha_{\pi e})^3 = 1 - 3 \cdot \Delta y_{\pi e} \cdot \alpha_{\pi e} - 3 \cdot \Delta y_{\pi e}^2 \cdot \alpha_{\pi e}^2 - \Delta y_{\pi e}^3 \cdot \alpha_{\pi e}^3. \quad (2.1.11)$$

For finding the coefficient  $\Delta y_{\pi e}$  in (2.1.11) we write the quadratic equation

$$\frac{1}{\varphi_{\pi 0}} \cdot \alpha_{\pi x}^2 + \alpha_{\pi x} - \bar{\beta}_{\pi} = 0. \quad (2.1.12)$$

As is known [1, p. 43] an algebraic equation of the 2nd degree is written in the form:

$$a \cdot x^2 + b \cdot x + c = 0 \quad (a \neq 0). \quad (2.1.13)$$

The roots of the equation are determined by the formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}. \quad (2.1.14)$$

Note that

$$x_1 + x_2 = -\frac{b}{a} \quad \text{и} \quad x_1 \cdot x_2 = \frac{c}{a}. \quad (2.1.15)$$

The ratio of the roots of equation (2.1.12) we write as:

$$\Delta_{\pi x} = \frac{\alpha_{\pi x 2}}{\alpha_{\pi x 1}}. \quad (2.1.16)$$

Find  $\Delta y_{\pi e}$  out how

$$\Delta y_{\pi e} = \frac{\Delta_{\pi x}}{\Delta y_{\pi 0}^3}. \quad (2.1.17)$$

Using any of the known methods of solving cubic equations, or a search procedure, we find the parameter

$\alpha_{\pi e}$  – is the real root of the equation (2.1.8).

For determine the scalar parameter of structure of space-time  $f_{\pi s}$  we write the ratio:

$$\frac{\alpha_{\pi 0} \cdot \bar{\beta}_{\pi}}{\alpha_{\pi e} \cdot \beta_{\pi e}} = \frac{(\alpha_{\pi e} \cdot \beta_{\pi e})^3}{(\alpha_{\pi} \cdot \beta_{\pi})^3}. \quad (2.1.18)$$

Write (2.1.18) in the form

$$(\alpha_{\pi} \cdot \beta_{\pi})^3 = \frac{\alpha_{\pi e}^4 \cdot \beta_{\pi e}^4}{\alpha_{\pi 0} \cdot \bar{\beta}_{\pi}}. \quad (2.1.19)$$

From (2.1.19):

$$\alpha_{\pi} \cdot \beta_{\pi} = \sqrt[3]{\frac{(\alpha_{\pi e} \cdot \beta_{\pi e})^4}{\alpha_{\pi 0} \cdot \bar{\beta}_{\pi}}}. \quad (2.1.20)$$

Scalar parameter of structure of space-time  $f_{\pi s}$ :

$$f_{\pi s} = \alpha_{\pi} \cdot \beta_{\pi}. \quad (2.1.21)$$

## 2.2. Electromagnetic constant

For determine the electromagnetic constant  $\alpha_{\pi}$  let us write the expression:

$$\frac{[\alpha \cdot \beta]_{\pi}}{\alpha_{\pi 0} \cdot \bar{\beta}_{\pi}} = \frac{(\alpha_{\pi e} \cdot \beta_{\pi e})^3}{[\alpha \cdot \beta]_{\pi}^3} \quad (2.2.1)$$

Let us write the expression (2.2.1) in the form

$$[\alpha \cdot \beta]_{\pi}^4 = (\alpha_{\pi e} \cdot \beta_{\pi e})^3 \cdot \alpha_{\pi 0} \cdot \bar{\beta}_{\pi}. \quad (2.2.2)$$

From (2.2.2):

$$[\alpha \cdot \beta]_{\pi} = \sqrt[4]{(\alpha_{\pi e} \cdot \beta_{\pi e})^3 \cdot \alpha_{\pi 0} \cdot \bar{\beta}_{\pi}}. \quad (2.2.3)$$

We denote the relation (2.2.3) to (2.1.20):

$$k_{\pi}^4 = \frac{[\alpha \cdot \beta]_{\pi}}{\alpha_{\pi} \cdot \beta_{\pi}}. \quad (2.2.4)$$

The ratio  $k_{\pi}$  of (2.2.4):

$$k_{\pi} = \sqrt[4]{\frac{[\alpha \cdot \beta]_{\pi}}{\alpha_{\pi} \cdot \beta_{\pi}}}. \quad (2.2.5)$$

The electromagnetic constant  $\alpha_{\pi}$  is defined as

$$\alpha_{\pi} = \frac{\alpha_{\pi e}}{k_{\pi}}. \quad (2.2.6)$$

## 3. Fundamental physical constants

From the relation

$$\frac{\lambda_{\pi}^3}{2 \cdot \pi^2 \cdot f_{\pi s}^3 \cdot \lambda_{\pi}^2} = 2 \cdot \pi \cdot f_{\pi s} \cdot u_{\pi l} \quad (3.1)$$

find the wavelength of  $\lambda_{\pi}$

$$\lambda_{\pi} = 4 \cdot \pi^3 \cdot f_{\pi s}^4 \cdot u_{\pi l}. \quad (3.2)$$

Taking into account (3.2) and the substitution of the fine-structure constant  $\alpha$  in the form

$$\alpha_{\pi th} = 2 \cdot \pi \cdot \alpha_{\pi} \quad (3.3)$$

in the formula for Rydberg constant  $R_{\infty}$  (represented on the website National Institute of Standards and Technology (NIST) at the address <http://physics.nist.gov/cuu/Constants/index.html>):

$$\lambda_C / 2\pi = \frac{\alpha^2}{4\pi \cdot R_\infty}, \quad (3.4)$$

we define from (3.4) Rydberg constant  $R_{\pi\infty}$  in the form

$$R_{\pi\infty} = \frac{\alpha_{\pi th}^2}{8 \cdot \pi^3 \cdot f_{\pi s}^4} \cdot u_{\pi l}^{-1}. \quad (3.5)$$

The matching coefficient  $k_{\pi R}$  defined as:

$$k_{\pi R} = \frac{R_{\pi\infty}}{R_\infty}. \quad (3.6)$$

The wavelength of Compton  $\lambda_{\pi C}$  we find, given (3.6) in the form

$$\lambda_{\pi C} = k_{\pi R} \cdot \lambda_\pi. \quad (3.7)$$

The mass of an electron  $m_{\pi e}$  find, taking into account (1.18) in the form

$$m_{\pi e} = \pi^2 \cdot f_{\pi s}^3 \cdot \lambda_{\pi C}^2 \cdot u_{\pi \rho S}. \quad (3.8)$$

The elementary charge  $e_\pi$  can be found in the form

$$e_\pi = \pm(\sqrt{\alpha_\pi}) \cdot (m_{\pi e} \cdot \lambda_{\pi C})^{1/2} \cdot c. \quad (3.9)$$

The Planck mass  $m_{\pi P}$ , given (1.21), we find in the form

$$m_{\pi P} = k_{\pi R}^{1/3} \cdot 4 \cdot \pi^2 \cdot f_{\pi s}^2 \cdot u_{\pi l}^2 \cdot u_{\pi \rho S}. \quad (3.10)$$

Find Planck's constant  $h_\pi$  in the form

$$h_\pi = m_{\pi e} \cdot \lambda_{\pi C} \cdot c. \quad (3.11)$$

Based on the known formula for the Planck mass

$$m_p = \sqrt{\frac{h \cdot c}{G}} \quad (3.12)$$

and from the equality (3.10) and (3.12):

$$k_{\pi R}^{1/3} \cdot 4 \cdot \pi^2 \cdot f_{\pi s}^2 \cdot u_{\pi l}^2 \cdot u_{\pi \rho S} = \sqrt{\frac{h_\pi \cdot c}{G_\pi}}, \quad (3.13)$$

find from (3.13) gravitational constant Newton  $G_\pi$  in the form

$$G_\pi = \frac{h_\pi \cdot c}{k_{\pi R}^{2/3} \cdot 16 \cdot \pi^4 \cdot f_{\pi s}^4 \cdot u_{\pi l}^4 \cdot u_{\pi \rho S}^2}. \quad (3.14)$$

From the relation

$$m_{\pi P} \cdot l_{\pi P} = m_{\pi e} \cdot \lambda_{\pi C} \quad (3.15)$$

can find the length of the Planck  $l_{\pi P}$  in the form of

$$l_{\pi P} = \frac{m_{\pi e}}{m_{\pi P}} \cdot \lambda_{\pi C} \quad (3.16)$$

and the Planck time as

$$t_{\pi P} = \frac{l_{\pi P}}{c}. \quad (3.17)$$

To determine the lifetime of the neutron we write the equation (1.23) in the form

$$(\sqrt{2} \cdot \pi)^3 \cdot \pi^2 \cdot f_{\pi s} = (1 + \Delta y_\pi \cdot \alpha_\pi)^3 \quad (\Delta y_\pi - \text{parametric offset machining}). \quad (3.18)$$

Parameter  $f_{\pi s}$  from (3.18):

$$f_{\pi s} = \frac{(1 + \Delta y_\pi \cdot \alpha_\pi)^3}{(\sqrt{2} \cdot \pi)^3 \cdot \pi^2}. \quad (3.19)$$

The lifetime  $\tau_{\pi nS}$  of short-lived neutrons  $n_{\pi S}$  determine, taking into account (3.19), as

$$\tau_{\pi nS} = \frac{k_{\pi R}^{1/3}}{f_{\pi s}} \cdot u_{\pi t} = \frac{(\sqrt{2} \cdot \pi)^3 \cdot \pi^2 \cdot k_{\pi R}^{1/3}}{(1 + \Delta y_{\pi} \cdot \alpha_{\pi})^3} \cdot u_{\pi t}, \quad (3.20)$$

The lifetime  $\tau_{\pi nL}$  of long-lived neutrons determine from (3.20), provided  $\Delta y_{\pi} \cdot \alpha_{\pi} = 0$ , in the form of

$$\tau_{\pi nL} = (\sqrt{2} \cdot \pi)^3 \cdot \pi^2 \cdot k_{\pi R}^{1/3} \cdot u_{\pi t}. \quad (3.21)$$

Note that, when presence of the neutron has two life times, the baryon asymmetry of the Universe finds its natural explanation.

The **Table** presents the results of theoretical calculations of fundamental constants (*italics* protoconstants).

**Table**

The name of the parameter	Symbol	The numerical value (SGS)
<i>scalar parameter of structure of space-time</i>	$f_{\pi s}$	1.161 712 977 019 596 928 9703 · 10 <sup>-3</sup>
<i>electromagnetic constant</i>	$\alpha_{\pi}$	1.161 409 733 400 893 939 4882 · 10 <sup>-3</sup>
fine-structure constant	$\alpha_{\pi th}$	7.297 352 572 519 857 423 5458 · 10 <sup>-3</sup>
Wavelength	$\lambda_{\pi}$	2.258 941 438 338 421 412 6297 · 10 <sup>-10</sup> sm
Rydberg constant	$R_{\pi \infty}$	1.178 679 395 222 205 270 7871 · 10 <sup>5</sup> sm <sup>-1</sup>
the matching coefficient*	$k_{\pi R}$	1.074 091 696 0293
Compton wavelength	$\lambda_{\pi C}$	2.426 310 240 7358 · 10 <sup>-10</sup> sm
electron mass	$m_{\pi e}$	9.109 382 325 3407 · 10 <sup>-28</sup> g
elementary charge	$e_{\pi}$	4.803 204 354 1651 · 10 <sup>-10</sup> g <sup>-1/2</sup> sm <sup>3/2</sup> s <sup>-1</sup>
Planck constant	$h_{\pi}$	6.626 069 154 6019 · 10 <sup>-27</sup> g sm <sup>2</sup> s <sup>-1</sup>
Planck mass	$m_{\pi P}$	5.456 379 113 3014 · 10 <sup>-5</sup> g
Planck length	$l_{\pi P}$	4.050 706 001 8742 · 10 <sup>-33</sup> sm
Planck time	$t_{\pi P}$	1.351 170 082 4289 · 10 <sup>-43</sup> s
Newtonian constant of gravitation	$G_{\pi}$	6.672 177 502 9339 g <sup>-1</sup> sm <sup>3</sup> s <sup>-2</sup>
lifetime neutron $n_{\pi S}$	$\tau_{\pi nS}$	881.552 698 044 s
lifetime neutron $n_{\pi L}$	$\tau_{\pi nL}$	886.423 939 853 s

\* – value (CODATA 2010) from the NIST website:  $R_{\infty} = 1.097\,373\,156\,8539(55)\text{ sm}^{-1}$  (SGS), the speed of light in vacuum  $c = 2.997\,924\,58 \cdot 10^{10}\text{ sm} \cdot \text{s}^{-1}$  (SGS).

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