Pi Formulas

Part 6: Some general formulas for the constant Pi

Edgar Valdebenito

abstract

In this note we give some formulas for the constant Pi

Algunas Fórmulas Para La Constante Pi: π

Edgar Valdebenito V

Abril 2008

Resumen. Se muestran algunas fórmulas para la constante Pi:

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right)$$

1. Fórmulas

$$\pi = \frac{\sqrt[3]{\frac{\sqrt{3}}{8} + \frac{3}{4}\sqrt[3]{\frac{\sqrt{3}}{8} + \frac{3}{4}\sqrt[3]{\frac{\sqrt{3}}{8} + \dots\dots}}}}{\frac{4}{9}\left(1 - \left(\frac{4}{9}\right)^2\right)\left(1 - \left(\frac{4}{18}\right)^2\right)\left(1 - \left(\frac{4}{27}\right)^2\right)\dots} \tag{1}$$

$$\pi = \frac{\frac{\sqrt{3}}{6} + \frac{4}{3}\left(\frac{\sqrt{3}}{6} + \frac{4}{3}\left(\frac{\sqrt{3}}{6} + \frac{4}{3}\left(\frac{\sqrt{3}}{6} + \dots\right)^3\right)^3\right)^3}{\frac{1}{9}\left(1 - \left(\frac{1}{9}\right)^2\right)\left(1 - \left(\frac{1}{18}\right)^2\right)\left(1 - \left(\frac{1}{27}\right)^2\right)\dots} \tag{2}$$

$$\pi = 4\sum_{n=1}^{\infty}\left(\frac{1}{4n-3} - \sum_{k=1}^{m-1}\frac{1}{2^{k+1}(2n-1)-1} - \frac{1}{2^{m+1}n-1}\right) \tag{3}$$

$$m \in \mathbb{N} = \{1,2,3,\dots\}$$

$$\pi = 4\sum_{n=1}^{\infty}\left(-\frac{1}{4n-1} + \sum_{k=2}^{m-1}\frac{1}{2^{k+1}(2n-1)-3} + -\frac{1}{2^{m+1}n-3}\right) \tag{4}$$

$$m \in \mathbb{N} = \{1,2,3,\dots\}$$

$$\pi\sqrt{3} = 18\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{\frac{n(n-1)}{2}}\sum_{k=1}^{n}\frac{(-1)^{k-1}}{\left(2k+n(n-1)-1\right)3^{k}} \tag{5}$$

$$\frac{1}{\pi} = 256\sum_{n=1}^{\infty}\sum_{k=1}^{n}\binom{2k+n(n-1)-2}{k+\frac{n(n-1)}{2}-1}^{3}\frac{42k+21n(n-1)-37}{2^{12k+6n(n-1)}} \tag{6}$$

$$\pi\sqrt{3} = 6\sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\frac{(-1)^{k+m(n-1)}}{\left(2\left(k+m(n-1)\right)+1\right)3^{k+m(n-1)}} \tag{7}$$

$$m \in \mathbb{N} = \{1,2,3,...\}$$

$$\pi = 4\sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{(-1)^{\frac{n(n-1)}{2}+k-1}}{2k+n(n-1)-1} \tag{8}$$

$$\pi = 4\sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\frac{(-1)^{k+m(n-1)}}{2\left(k+m(n-1)\right)+1} \tag{9}$$

$$m \in \mathbb{N} = \{1,2,3,...\}$$

$$\frac{1}{\pi} = \sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\binom{2\left(k+m(n-1)\right)}{k+m(m-1)}^{3}\frac{42\left(k+m(n-1)\right)+5}{2^{12\left(k+m(n-1)\right)+4}} \tag{10}$$

$$m \in \mathbb{N} = \{1,2,3,...\}$$

$$\frac{\pi}{\sqrt{3}} = 3\sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\frac{1}{\left(k+m(n-1)+1\right)\binom{2\left(k+m(n-1)\right)+2}{k+m(n-1)+1}} \tag{11}$$

$$m \in \mathbb{N} = \{1,2,3,...\}$$

$$\pi = 4\sum_{n=0}^{\infty}\sum_{k=1}^{2^{m}}\frac{(-1)^{2^{m}n+k-1}}{2^{m+1}n+2k-1} \tag{12}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\pi\sqrt{3} = 6\sum_{n=0}^{\infty}\sum_{k=1}^{2^m}\frac{(-1)^{2^m n+k-1}}{\left(2^{m+1}n+2k-1\right)3^{2^m n+k-1}} \tag{13}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\pi = 4\sum_{n=0}^{\infty}\sum_{k=1}^{2^m}\left(\frac{1}{2}\right)^{2^m n+k}\sum_{m=0}^{2^m n+k-1}\frac{(-1)^m\binom{2^m n+k-1}{m}}{2m+1} \tag{14}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\frac{\pi}{3\sqrt{3}} = \sum_{n=0}^{\infty}\sum_{k=1}^{2^m}\frac{1}{\left(2^m n+k\right)\binom{2^{m+1}n+2k}{2^m n+k}} \tag{15}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\pi = 4\sum_{n=0}^{\infty}\sum_{k=1}^{2^m}\frac{25\cdot 2^m n+25k-28}{\binom{3\cdot 2^m n+3k-3}{2^m n+k-1}2^{2^m n+k}} \tag{16}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\frac{1}{\pi} = 256\sum_{n=0}^{\infty}\sum_{k=1}^{2^m}\binom{2^{m+1}n+2k-2}{2^m n+k-1}^3\frac{21\cdot 2^{m+1}n+42k-37}{2^{3\cdot 2^{m+2}n+12k}} \tag{17}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\pi = 16\sum_{n=0}^{\infty}\sum_{k=1}^{2^m}\left(\frac{1}{2}\right)^{2^{m+2}n+4k}\left(\frac{4}{2^{m+3}n+8k-7}-\frac{2}{2^{m+3}n+8k-4}-\frac{1}{2^{m+3}n+8k-3}-\frac{1}{2^{m+3}n+8k-2}\right) \tag{18}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\pi = 16 \sum_{n=0}^{\infty} \sum_{k=1}^{2^m} \frac{\binom{2^{m+1}n + 2k - 2}{2^m n + k - 1}}{2^{2^{m+1}n+2k} \left(2^{m+1}n + 2k + 1\right)} \tag{19}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$ln\left(\frac{\pi}{2}\right) = \sum_{n=0}^{\infty} \sum_{k=1}^{2^m} (-1)^{2^m n+k-1} ln\left(\frac{2^m n + k + 1}{2^m n + k}\right) \tag{20}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

$$\pi = 4 \sum_{n=0}^{\infty} \sum_{k=1}^{2^m} \frac{(-1)^{2^m n+k-1}}{2^{m+1}n + 2k - 1} \left(2^{-\left(2^{m+1}n+2k-1\right)} + 3^{-\left(2^{m+1}n+2k-1\right)}\right) \tag{21}$$

$$m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$$

Referencias

[1] Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover , 1965.

[2] I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey) , Academic Press, New York, London, and Toronto, 1980.

[3] M. R. Spiegel, Mathematical Handbook, McGraw-Hill Book Company, New York, 1968.

[4] E. Valdebenito, Pi Handbook, manuscript, unpublished, 1989, (20000 fórmulas).