Each unidimensional system is Hamiltonian, so that each unidimensional system is quantizable

**Abstract**

I prove that the field of classical trajectories can be a field Hamiltonian projection of higher dimension.

I hypothesize that the same is valid for any dimension: each system is Hamiltonian, and each system is quantizable using the correspondence principle.

**Unidimensional Hamiltonian System**

Each unidimensional trajectory can be described by a differential equation of high order, and high degree (each derivable function can be approximate by a sum of Taylor, Fourier and Laplace series, that is a solution of a linear differential equation, but an improved approximation is a nonlinear differential equation):

\[ 0 = F(y, \dot{y}, \ddot{y}, \cdots) = a_{10} + a_{01} \dot{y} + a_{001} \ddot{y} + \cdots + a_{010} y + \cdots \quad (1) \]

\[ 0 = F(y, \dot{y}, \ddot{y}, \cdots) = \sum_{i_0, \ldots, i_n} a_{i_0, \ldots, i_n} \frac{d^{i_0} y}{dt^{i_0}} \cdots \frac{d^n y}{dt^n} \quad (2) \]

the derive of the differential equation is linear in the higher derivative:

\[ 0 = \frac{dF(y, \dot{y}, \ddot{y}, \cdots)}{dt} = \frac{d}{dt} \sum_{i_0, \ldots, i_n} a_{i_0, \ldots, i_n} \prod_{s=1}^{n} \left( \frac{d^i y}{dt^i} \right)^{i_s} = \sum_{i_0, \ldots, i_n} a_{i_0, \ldots, i_n} \prod_{s=1}^{n} \left( \frac{d^i y}{dt^i} \right)^{i_s-\delta_{sk}} \frac{d^{k+1} y}{dt^{k+1}} \quad (3) \]

\[ \frac{d^N y}{dt^N} = \mathcal{G} \left( y, \frac{dy}{dt}, \ldots, \frac{d^{N-1} y}{dt^{N-1}} \right) \quad (4) \]

so that each polynomial differential equation can be write linearly in the maximum derivative; so that:

\[
\begin{align*}
  y &= y_0 \\
  \dot{y}_0 &= y_1 \\
  \dot{y}_1 &= y_2 \\
  &\vdots \\
  \dot{y}_{N-2} &= y_{N-1} \\
  \dot{y}_{N-1} &= \mathcal{G}(y_0, \cdots, y_{N-1})
\end{align*}
\]
this system can be the half of an Hamiltonian system \( H = \sum_i p_i f_i \), that have \( N \) new momenta:

\[
H = \sum_{i=0}^{N} p_i f_i = \sum_{j=0}^{N} p_i \{ y_{i+1} + \delta_{i,N} [-y_{i+1} + \mathcal{G}] \} = \sum_{i=0}^{N-1} p_i y_{i+1} + p_N \mathcal{G}
\]

\[
\begin{aligned}
\dot{y}_{j \neq N} &= \frac{\partial H}{\partial p_j} = f_j = y_{j+1} \\
\dot{y}_N &= \frac{\partial H}{\partial p_N} = f_N = \mathcal{G} \\
\dot{p}_{j \neq N} &= -\frac{\partial H}{\partial y_j} = -p_N \frac{\partial \mathcal{G}}{\partial y_j} - p_{j-1} \\
\dot{p}_N &= -\frac{\partial H}{\partial y_N} = -p_{N-1}
\end{aligned}
\]

(6)

the trajectories in the coordinates are ever the same, for each momentum initial condition; the volume of the phase space is an invariant in the space (coordinates,momenta) and the sum of the areas is invariant, because of there is a momenta compensation.

In this case, each quantum system is equal to the classical system:

\[
H = p_i f_i \\
i \hbar \partial_t \psi = -i \hbar \sum_i f_i \partial_i \psi \\
0 = \partial_t \psi + \sum_i f_i \partial_i \psi
\]

(7)

there are two classical solutions:

\[
\begin{aligned}
\dot{y}_1 &= f_1 \\
\vdots \\
\dot{y}_N &= f_N \\
\frac{dy_1}{f_1} &= \cdots = \frac{dy_N}{f_N} = dt \\
\partial_t \psi + \sum_i f_i \partial_i \psi &= 0
\end{aligned}
\]

(8)

that is a surface solution, in an \( N+1 \) dimensional space (coordinates and times), and \( \psi \) is the solution of the differential equation.

Another solution is the Hamilton-Jacobi equation, that give the classical solution of the Hamiltonian:

\[
\begin{aligned}
H &= p_i f_i \\
\partial_t \psi + H(p_i, \partial_i \psi, y_i) &= 0 \\
\partial_t \psi + \sum_i f_i \partial_i \psi &= 0
\end{aligned}
\]

(9)

in this case the function \( \psi \) permit to calculate the momenta values like a gradient of the \( \psi \) function. Also in this case the classical solution, and the quantum solution, coincide; and the equation for the amplitude, or the probability, are equal because of the linearity of the equation.