Observation on the numbers 4p^2+2p+1 where p and 2p-1 are primes

Abstract. In this paper I observe that many numbers of the form 4*p^2 + 2*p + 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d – n + m)/m; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d + n – m)/m, and I make few related notes.

Observation:

Many numbers of the form 4*p^2 + 2*p + 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d – n + m)/m; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d + n - m)/m.

Verifying the observation:
(true for the first 27 odd primes p for which 2*p – 1 is also prime)

Note that if d is equal to 7 is obviously respected condition (i) or condition (ii).

: for p = 3, N = 43, prime;
: for p = 7, N = 211, prime;
: for p = 19, N = 1483, prime;
: for p = 31, N = 3907, prime;
: for p = 37, N = 7*13*61 so d = 7;
: for p = 79, N = 7*37*97 so d = 7;
: for p = 97, N = 37831, prime;
: for p = 139, N = 77563, prime;
: for p = 157, N = 98911, prime;
: for p = 199, N = 158803, prime;
: for p = 211, N = 7*7*3643 so d = 7;
: for p = 229, N = 13*16171 and 16171 = (2695*13 – 2695 + 2)/2;
: for p = 271, N = 13*22639 and 22639 = (3773*13 – 3773 + 2)/2;
: for p = 307, N = 13*29047 and 29047 = (4841*13 – 4841 + 2)/2;
: for p = 331, N = 7*62701 so d = 7;
: for p = 337, N = 7*64993 so d = 7;

: for p = 367, N = 79*6829 and 6829 = (683*79 + 683 - 8)/8;
: for p = 379, N = 7*82189 so d = 7;
: for p = 439, N = 771763, prime;
: for p = 499, N = 7*7*20347 so d = 7;
: for p = 547, N = 7*171133 so d = 7;
: for p = 577, N = 43*30997 and 30997 = (738*43 – 738 + 1)/1;
: for p = 601, N = 1446007, prime;
: for p = 607, N = 31*47581 and 47581 = (1586*31 – 1586 + 1)/1;
: for p = 619, N = 13*117991 and 117991 = (19665*13 – 19665 + 2)/2;
: for p = 661, N = 13*134539 and 134539 = (22423*13 – 22423 + 2)/2.

Note:

: Some numbers of this form meet another condition, i.e. they are equal to d*Q, where d is the least prime factor and Q the product of the others, and the number Q – d + 1 is prime or respectively the number Q + d - 1 is prime. An example: for p = 691, N = 43*44449 and 44449 + 43 - 1 = 44491, prime.