LHC 750 GeV Diphoton Resonance and Flavor Mixing from Majorana Higgs Bosons

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Abstract

We propose a Clifford algebra based model, which includes local gauge symmetries $SO(1,3) \otimes SU_L(2) \otimes U_R(1) \otimes U(1) \otimes SU(3)$. There are two sectors of Higgs fields as Majorana and electroweak Higgs bosons. The Majorana Higgs sector, is responsible for the 750 GeV diphoton resonance, flavor mixing, and right-handed neutrino Majorana masses. The electroweak Higgs sector, which induces Dirac masses, is composed of scalar, pseudoscalar, and antisymmetric tensor components.

Keywords. Clifford algebra, diphoton resonance, Higgs bosons, flavor structure, gravity.

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1 Introduction

The experiments at LHC recently indicated a diphoton resonance at about 750 Gev[1, 2], in addition to the earlier finding of Higgs boson with $m_h = 125$ Gev[3, 4]. Scenarios with either an isospin singlet state or an isospin doublet state can not accommodate the observed signal and an extended particle content is necessary[5, 6, 7, 8, 9, 10, 11, 12, 13].

We propose a Clifford algebra based model which encompasses Yang-Mills interactions as well as gravity. The 750 GeV diphoton resonance corresponds to color sextet Majorana Higgs boson loops. No further extended particle content is needed.

With the purpose of studying 3 generations of Standard Model fermions, a ternary Clifford vector is introduced alongside 6 binary Clifford vectors. The flavor projection operators facilitate flavor mixing via color singlet and sextet Majorana Higgs bosons.

The current paper is a continuation of our previous work[14, 15], which is based on three premises. Firstly, gauge gravity and Yang-Mills interactions should be integrated in a single overarching framework. The key is to take a page from effective field theory, where an infinite number of terms allowed by symmetry requirements should be included in a generalized action. Only the first order terms of the action are relevant in low-energy limit.

The second premise is that all idempotent projections of the original algebraic spinor should be realized as fermions of physical world. In other words, no spinor projection should be casually discarded. Hence, finding the right Clifford algebra turns out to be a simple process of counting numbers of fermion species. There are 16 Weyl fermions (including right-handed neutrino) with $16 \times 4 = 64$ real components in one generation. Clifford algebra $C\ell_{0,6}$, with $2^6 = 64$ degrees of freedom, seems to be a natural choice.

The third premise is that rotations should be generalized. As well known in Clifford algebra approaches, a rotation is realized by a rotor, which is an exponential of bivectors. It rotates a vector into another vector. However, a rotor could be defined to be an exponential of any multivectors. It could rotate a vector into a multivector, generalizing definition of rotations. Hence, one can entertain large symmetry groups with lower dimensional Clifford algebras, whereas the same symmetry groups would otherwise require higher Clifford dimensions within the conventional framework. While the conventional Dirac matrix operators $\gamma_1, \gamma_2, \gamma_3$ correspond to vectors in $C\ell_{0,6}$, the matrix operator γ_0 corresponds to a trivector. Lorentz boost rotations are represented as exponentials of Clifford 4-vectors $\gamma_0\gamma_1$, $\gamma_0\gamma_2$, $\gamma_0\gamma_3$.

This paper is structured as follows: Section 2 introduces binary Clifford algebra, gauge symmetries, and the action of the world. In section 3, an additional ternary Clifford algebra is defined. The Majorana Higgs sector, flavor mixing, and 750 Gev diphoton resonance are discussed. In section 4, we study electroweak Higgs sector. In section 5, we touch upon the topic of grand unification. In the last section we draw our conclusions.

2 Action of the World

2.1 6D Clifford Algebra

We begin with a review of orthogonal Clifford algebra $C\ell_{0,6}$. It is defined by anticommutators of orthonormal vector basis $(\gamma_j, \Gamma_j; j = 1, 2, 3)$

$$[\gamma_j, \gamma_k] = \frac{1}{2}(\gamma_j\gamma_k + \gamma_k\gamma_j) = -\delta_{jk}, \tag{1}$$
$$[\Gamma_j, \Gamma_k] = -\delta_{jk}, \tag{2}$$
$$[\gamma_j, \Gamma_k] = 0, \tag{3}$$

where $j, k = 1, 2, 3$. All basis vectors are space-like. There are $\binom{6}{k}$ independent k-vectors. The complete basis for $C\ell_{0,6}$ is given by the set of all k-vectors. Any multivector can be expressed as a linear combination of $2^6 = 64$ basis elements.

Two trivectors

$$\gamma_0 = \Gamma_1\Gamma_2\Gamma_3, \tag{4}$$
$$\Gamma_0 = \gamma_1\gamma_2\gamma_3 \tag{5}$$

square to 1, so they are time-like. The orthonormal vector-trivector basis $\{\gamma_a, a = 0, 1, 2, 3\}$ defines space-time Clifford algebra $C\ell_{1,3}$, with

$$\eta_{ab} = \langle\gamma_a\gamma_b\rangle = \begin{pmatrix} +1, 0, 0, 0 \\ 0, -1, 0, 0 \\ 0, 0, -1, 0 \\ 0, 0, 0, -1 \end{pmatrix}, \tag{6}$$

where $\langle\cdots\rangle$ means scalar part of enclosed expression. The reciprocal vectors $\{\gamma^a\}$ are defined by

$$\gamma^a\eta_{ab} = \gamma_b, \tag{7}$$

thus

$$\langle\gamma^a\gamma_b\rangle = \delta^a_b. \tag{8}$$

Here we adopt the summation convention for repeated indices.

The unit pseudoscalar

$$i = \Gamma_1\Gamma_2\Gamma_3\gamma_1\gamma_2\gamma_3 = \gamma_0\gamma_1\gamma_2\gamma_3 = \gamma_0\Gamma_0 \tag{9}$$

squares to -1, anticommutes with odd-grade elements, and commutes with even-grade elements.

Reversion of a multivector $M \in C\ell_{0,6}$, denoted $\tilde{M}$, reverses the order in any product of vectors. For any multivectors M and N, there are algebraic properties

$$(MN)\tilde{} = \tilde{N}\tilde{M}, \tag{10}$$
$$\langle MN\rangle = \langle NM\rangle\,. \tag{11}$$

The magnitude of a multivector M is defined as

$$|M| = \sqrt{\langle M^\dagger M \rangle}, \tag{12}$$

where

$$M^\dagger = -i\tilde{M}i, \tag{13}$$

is the Hermitian conjugate.

2.2 Algeraic Spinor

Algebraic spinor $\psi \in C\ell_{0,6}$ is a multivector , which is expressed as a linear combination (with Grassmann odd coefficients) of all $2^6 = 64$ basis elements.

Spinors with left/right chirality correspond to odd/even multivectors

$$\psi = \psi_L + \psi_R, \tag{14}$$
$$\psi_L = \frac{1}{2}(\psi + i\psi i) \tag{15}$$
$$\psi_R = \frac{1}{2}(\psi - i\psi i). \tag{16}$$

A projection operator squares to itself. Idempotents are a set of projection operators

$$P_0 = \frac{1}{4}(1 + iJ_1 + iJ_2 + iJ_3) = \frac{1}{4}(1 + 3iJ), \tag{17}$$
$$P_1 = \frac{1}{4}(1 + iJ_1 - iJ_2 - iJ_3), \tag{18}$$
$$P_2 = \frac{1}{4}(1 - iJ_1 + iJ_2 - iJ_3), \tag{19}$$
$$P_3 = \frac{1}{4}(1 - iJ_1 - iJ_2 + iJ_3), \tag{20}$$
$$P_q = P_1 + P_2 + P_3 = \frac{3}{4}(1 - iJ), \tag{21}$$
$$P_\pm = \frac{1}{2}(1 \pm \Gamma_0\Gamma_3), \tag{22}$$

where

$$J_1 = \gamma_1\Gamma_1, J_2 = \gamma_2\Gamma_2, J_3 = \gamma_3\Gamma_3, \tag{23}$$
$$J = \frac{1}{3}(J_1 + J_2 + J_3), \tag{24}$$
$$P_0 + P_1 + P_2 + P_3 = P_0 + P_q = 1, \tag{25}$$
$$P_a P_b = \delta_{ab}, \quad (a, b = 0, 1, 2, 3), \tag{26}$$
$$P_+ + P_- = 1. \tag{27}$$

Here P_0 is lepton projection operator, P_q is quark projection operator, and P_j are color projection operators. The bivectors J_j appearing in the color projectors P_j suggest an interesting duality between 3 space dimensions and 3 colors of quarks.

Now we are ready to identify idempotent projections of spinor

$$\psi = (P_+ + P_-)(\psi_L + \psi_R)(P_0 + P_1 + P_2 + P_3) \tag{28}$$

with left-handed leptons, red, green, and blue quarks

$$\begin{cases} \nu_L = P_+\psi_L P_0, \\ e_L = P_-\psi_L P_0, \\ u_L = P_+\psi_L P_1 + P_+\psi_L P_2 + P_+\psi_L P_3 = P_+\psi_L P_q, \\ d_L = P_-\psi_L P_1 + P_-\psi_L P_2 + P_-\psi_L P_3 = P_-\psi_L P_q, \end{cases} \tag{29}$$

and right-handed leptons, red, green, and blue quarks

$$\begin{cases} \nu_R = P_-\psi_R P_0, \\ e_R = P_+\psi_R P_0, \\ u_R = P_-\psi_R P_1 + P_-\psi_R P_2 + P_-\psi_R P_3 = P_-\psi_R P_q, \\ d_R = P_+\psi_R P_1 + P_+\psi_R P_2 + P_+\psi_R P_3 = P_+\psi_R P_q. \end{cases} \tag{30}$$

2.3 Symmetries

Spinors transformation as

$$\begin{aligned} \psi_L &\to e^{\Theta_{LOR}+\Theta_{WL}}\psi_L e^{\Theta_J - \Theta_{STR}}, \\ \psi_R &\to e^{\Theta_{LOR}+\Theta_{WR}}\psi_R e^{\Theta_J - \Theta_{STR}}. \end{aligned} \tag{31}$$

It worth noting that all gauge transformations are with Grassmann even rotation angles, so that the transformed spinors remains to be Grassmann odd.

There are Lorentz $SO(1,3)$ gauge transformations

$$\{\gamma_a\gamma_b\} \in \Theta_{LOR}, (a, b = 0, 1, 2, 3, a \neq b), \tag{32}$$

weak isospin $SU(2)_L$ gauge transformations acting on left-handed fermions

$$\{\frac{1}{2}\Gamma_2\Gamma_3, \frac{1}{2}\Gamma_1\Gamma_3, \frac{1}{2}\Gamma_1\Gamma_2\} \in \Theta_{WL}, \tag{33}$$

weak $U(1)_R$ gauge transformation acting on right-handed fermions

$$\{\frac{1}{2}\Gamma_1\Gamma_2\} \in \Theta_{WR}, \tag{34}$$

$J\ U(1)$ gauge transformation

$$\{\frac{1}{2}J\} \in \Theta_J, \tag{35}$$

and color $SU(3)$ gauge transformations

$$\left\{ \begin{array}{l} T_1, T_2, T_3, \\ T_4, T_5, \\ T_6, T_7, \\ T_8 \end{array} \right\} = \left\{ \begin{array}{l} \frac{1}{4}(\gamma_1\Gamma_2 + \gamma_2\Gamma_1), \frac{1}{4}(\Gamma_1\Gamma_2 + \gamma_1\gamma_2), \frac{1}{4}(\Gamma_1\gamma_1 - \Gamma_2\gamma_2), \\ \frac{1}{4}(\gamma_1\Gamma_3 + \gamma_3\Gamma_1), \frac{1}{4}(\Gamma_1\Gamma_3 + \gamma_1\gamma_3), \\ \frac{1}{4}(\gamma_2\Gamma_3 + \gamma_3\Gamma_2\gamma_3), \frac{1}{4}(\Gamma_2\Gamma_3 + \gamma_2\gamma_3), \\ \frac{1}{4\sqrt{3}}(\Gamma_1\gamma_1 + \Gamma_2\gamma_2 - 2\Gamma_3\gamma_3) \end{array} \right\} \in \Theta_{STR}. \tag{36}$$

It is remarkable that the gauge groups contain both gravitational (Θ_{LOR}) and internal gauge transformations.

Because the product of lepton projector P_0 with any generator in color algebra (36) is zero $P_0 T_k = 0$, leptons are invariant under color gauge transformation.

After symmetry breaking of Θ_{WR}, Θ_{WL}, and Θ_J via Majorana and electroweak Higgs bosons, which will be detailed in later sections, the remaining electromagnetic $U(1)$ symmetry is a synchronized double-sided gauge transformations

$$\psi \to e^{\frac{1}{2}\epsilon_E \Gamma_1\Gamma_2} \psi e^{\frac{1}{6}\epsilon_E J}. \tag{37}$$

where a shared rotation angle ϵ_E synchronizes the double-sided gauge transformation.

Thanks to the properties

$$\begin{aligned} JP_0 &= -iP_0, \\ JP_j &= \frac{1}{3} i P_j, \\ \Gamma_1\Gamma_2 P_\pm &= \mp i P_\pm, \end{aligned} \tag{38}$$

electric charges q_k as in

$$e^{\frac{1}{2}\Gamma_1\Gamma_2} \psi_k e^{\frac{1}{2}J} = \psi_k e^{q_k i} \tag{39}$$

are calculated as $q_k = 0, -1, \frac{2}{3}$, and $-\frac{1}{3}$ for neutrino, electron, up quarks, and down quarks, respectively.

2.4 Gauge Field 1-Forms, Gauge-Covariant Derivatives, and Curvature 2-Forms

Gauge fields are Clifford-valued 1-forms (Cliffforms with Grassmann even coefficients) on 4-dimensional space-time manifold (x_μ, $\mu = 0, 1, 2, 3$)

$$e = e_\mu dx^\mu = e_\mu^a \gamma_a dx^\mu, \tag{40}$$

$$\omega = \omega_\mu dx^\mu = \frac{1}{4}\omega_\mu^{ab}\gamma_a\gamma_b dx^\mu \quad \in \quad \Theta_{LOR}, \tag{41}$$

$$W_L = W_{L\mu} dx^\mu = \frac{1}{2}(W_{L\mu}^1 \Gamma_2\Gamma_3 + W_{L\mu}^2 \Gamma_1\Gamma_3 + W_{L\mu}^3 \Gamma_1\Gamma_2) dx^\mu \quad \in \quad \Theta_{WL}, \tag{42}$$

$$W_R = W_{R\mu} dx^\mu = \frac{1}{2} W_{R\mu}^3 \Gamma_1\Gamma_2 dx^\mu \quad \in \quad \Theta_{WR}, \tag{43}$$

$$C = C_\mu dx^\mu = \frac{1}{2} C_\mu^J J dx^\mu \quad \in \quad \Theta_J, \tag{44}$$

$$G = G_\mu dx^\mu = G_\mu^k T_k dx^\mu \quad \in \quad \Theta_{STR}, \tag{45}$$

where e is vierbein, ω is gravity spin connection, G is strong interaction, and the rest are electroweak related interactions.

The vierbein field e acts like space-time frame field, which is essential in building all actions as diffeomorphism-invariant integration of 4-forms on 4-dimensional space-time manifold. The space-time manifold is initially without metric. It's the vierbein field which gives notion to metric

$$g_{\mu\nu} = \langle e_\mu e_\nu \rangle = e_\mu^a e_\nu^b \eta_{ab}. \tag{46}$$

Local gauge transformations are coordinate-dependent gauge transformations. Gauge fields obey local gauge transformation laws

$$e(x) \quad \to \quad e^{\Theta_{LOR}(x)} e(x) e^{-\Theta_{LOR}(x)}, \tag{47}$$

$$\omega(x) \quad \to \quad e^{\Theta_{LOR}(x)} \omega(x) e^{-\Theta_{LOR}(x)} - (d e^{\Theta_{LOR}(x)}) e^{-\Theta_{LOR}(x)}, \tag{48}$$

$$W_L(x) \quad \to \quad e^{\Theta_{WL}(x)} W_L(x) e^{-\Theta_{WL}(x)} - (d e^{\Theta_{WL}(x)}) e^{-\Theta_{WL}(x)}, \tag{49}$$

$$W_R(x) \quad \to \quad W_R(x) - (d e^{\Theta_{WR}(x)}) e^{-\Theta_{WR}(x)}, \tag{50}$$

$$C(x) \quad \to \quad C(x) - e^{-\Theta_J(x)} (d e^{\Theta_J(x)}), \tag{51}$$

$$G(x) \quad \to \quad e^{\Theta_{STR}(x)} G(x) e^{-\Theta_{STR}(x)} + e^{\Theta_{STR}(x)} (d e^{-\Theta_{STR}(x)}) \tag{52}$$

where $d = dx^\mu \partial_\mu$.

It's worth emphasizing that gravity related fields $e(x)$ and $\omega(x)$ are treated as gauge fields with local gauge transformation properties, as the rest Yang-Mills gauge fields.

Gauge-covariant derivatives of spinor fields $\psi_{L/R}(x)$ are defined by

$$D\psi_L = (d + \omega + W_L)\psi_L + \psi_L(C - G), \tag{53}$$

$$D\psi_R = (d + \omega + W_R)\psi_R + \psi_R(C - G). \tag{54}$$

Here the connection fields are defined to absorb the coupling constants. The gravitational spin connection ω is essential in maintaining *local* Lorentz covariance of $D\psi_{L/R}$.

We introduce gauge curvature 2-forms by applying the covariant derivative to the 0-form spinor ψ and then to the 1-form spinor $D\psi$

$$\begin{aligned} D(D\psi_{L/R}) &= (d+\omega+W_{L/R})D\psi_{L/R} - D\psi_{L/R}(C-G) \\ &= (R+F_{WL/WR})\psi_{L/R}(F_J - F_{STR}), \end{aligned} \tag{55}$$

where gravity, left/right weak, J, and Strong force curvature 2-forms are

$$R = d\omega + \omega^2 = \frac{1}{2}R_{\mu\nu}dx^\mu dx^\nu, \tag{56}$$

$$F_{WL} = dW_L + W_L^2 = \frac{1}{2}F_{WL\mu\nu}dx^\mu dx^\nu, \tag{57}$$

$$F_{WR} = dW_R = \frac{1}{2}F_{WR\mu\nu}dx^\mu dx^\nu, \tag{58}$$

$$F_J = dC = \frac{1}{2}F_{J\mu\nu}dx^\mu dx^\nu, \tag{59}$$

$$F_{STR} = dG + G^2 = \frac{1}{2}F_{STR\mu\nu}dx^\mu dx^\nu. \tag{60}$$

$F^{\mu\nu k}$ is defined by

$$F^{\mu\nu k}\eta_{\mu\alpha}\eta_{\nu\beta} = F^k_{\alpha\beta}, \tag{61}$$

where k enumerates the Clifford components of each gauge field.

2.5 Gauge- and Diffeomorphism-Invariant Action of the World

The local gauge- and diffeomorphism-invariant action of the world is

$$\begin{aligned} S_{World} =& S_{Spinor-Kinetic} \\ &+ S_{Majorana-Yukawa} + S_{Majorana-Higgs} \\ &+ S_{Electroweak-Yukawa} + S_{Electroweak-Higgs} \\ &+ S_{Gravity} + S_{Yang-Mills}. \end{aligned} \tag{62}$$

The spinor kinetic action is now written down as

$$S_{Spinor-Kinetic} \sim \int \langle \bar{\psi}_L i e^3 D\psi_L + \bar{\psi}_R i e^3 D\psi_R \rangle, \tag{63}$$

where e^3 is vierbein 3-form, and $\bar{\psi}_{L/R}$ is defined as

$$\bar{\psi}_{L/R} = \psi^\dagger_{L/R}\gamma_0 = -i\tilde{\psi}_{L/R} i\gamma_0 = \mp\tilde{\psi}_{L/R}\gamma_0. \tag{64}$$

Here outer products between differential forms are implicitly assumed.

One can write down the action for gravity as

$$S_{Gravity} \sim \int \left\langle ie^2(R + \frac{\Lambda}{24}e^2) \right\rangle, \tag{65}$$

where e^2 is vierbein 2-form, $R = d\omega + \omega^2$ is spin connection curvature 2-form, and Λ is cosmological constant.

The Yang-Mills action is written as

$$\begin{aligned} &S_{Yang-Mills} = S_{WL} + S_{WR} + S_J + S_{STR}, \\ &S_{WL} \sim \int \left\langle (e^2 F_{WL})^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &S_{WR} \sim \int \left\langle (e^2 F_{WR})^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &S_J \sim \int \left\langle (e^2 F_J)^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &S_{STR} \sim \int \left\langle (e^2 F_{STR})^2 \right\rangle / \left\langle ie^4 \right\rangle, \end{aligned} \tag{66}$$

where e^4 is vierbein 4-form.

The Clifford algebra elements, which are related to left-(e, ω, W_L, W_R) and right-(C, G)sided gauge fields, are formally assigned to two sets of Clifford algebras in Yang-Mills action (and other actions without spinor fields). Elements from different sets formally commute with each other. Here $\langle \cdots \rangle$ means scalar part of both sets.

It's understood that 4-form factor d^4x in one of e^2F in each Yang-Mills term should be canceled out by 4-form factor d^4x in the denominator before any further outer multiplication of differential forms as

$$\int \left\langle \frac{e^2 F}{\langle ie^4 \rangle} e^2 F \right\rangle, \tag{67}$$

In this way, the Yang-Mills action is a diffeomorphism-invariant integration of 4-form on 4-dimensional space-time manifold.

There is no explicit Hodge dual in Yang-Mills action. Vierbein plays the role of Hodge dual, when it acquires non-zero vacuum expectation value (VEV) in the case of flat space-time, which will be discussed in next section.

Higgs field related portion of the actions will be subjects of later chapters.

2.6 Local Lorentz Symmetry Breaking and Minkowskian space-time

Up to this point, the action of the world is constructed in *curved* space-time, with space-time dependent vierbein and spin connection. In a vacuum with zero cosmological constant $\Lambda = 0$, vierbein field e acquires a non-zero Minkowskian flat space-time VEV

$$< 0|e|0 >= \delta^a_\mu \gamma_a dx^\mu, \tag{68}$$

while VEV of spin connection is zero

$$< 0|\omega|0 >= 0. \tag{69}$$

The soldering form $\delta^a_\mu \gamma_a dx^\mu$ breaks *local* Lorentz gauge invariance and diffeomorphism invariance. The action of the world is left with a residual *global* Lorentz symmetry, with synchronized Clifford space and x coordinate space Lorentz rotations.

With the substitution of vierbein and spin connection with their VEVs, the spinor kinetic action(63) in flat Minkowskian space-time can be rewritten as

$$S_{Spinor-Kinetic} = \int \left\langle \bar{\psi}_L \gamma^\mu D_\mu \psi_L + \bar{\psi}_R \gamma^\mu D_\mu \psi_R \right\rangle d^4x, \tag{70}$$

where

$$D_\mu \psi_{L/R} = (\partial_\mu + W_{L/R\mu})\psi_{L/R} + \psi_{L/R}(C_\mu - G_\mu). \tag{71}$$

Similarly, the Yang-Mills action(66) can be rewritten as

$$\begin{aligned} S_{Yang-Mills} = & -\frac{1}{4g_{WL}^2}\int F^k_{WL\mu\nu} F^{\mu\nu k}_{WL} d^4x \\ & -\frac{1}{4g_{WR}^2}\int F_{WR\mu\nu} F^{\mu\nu}_{WR} d^4x \\ & -\frac{1}{4g_J^2}\int F_{J\mu\nu} F^{\mu\nu}_J d^4x \\ & -\frac{1}{4g_{STR}^2}\int F^k_{STR\mu\nu} F^{\mu\nu k}_{STR} d^4x, \end{aligned} \tag{72}$$

where g_{WL}, g_{WR}, g_J, and g_{STR} are dimensionless gauge coupling constants.

In the following chapters, we will stay with local Lorentz gauge invariant curved space-time formulation.

2.7 Relation to Conventional Matrix Formulation

A map [14] can be constructed by placing the Dirac column spinor $\hat{\psi}$ in one-to-one correspondence with the algebraic spinor. And the mappings for the operators are

$$\hat{\gamma}^\mu \hat{\psi} \leftrightarrow \gamma^\mu \psi, (\mu = 0, 1, 2, 3) \tag{73}$$

$$\hat{i}\hat{\psi} \leftrightarrow \psi i, \tag{74}$$

$$\hat{\gamma}^5 \hat{\psi} \leftrightarrow -i\psi i \tag{75}$$

where $\hat{i}$ is the conventional unit imaginary number, and $\hat{\gamma}^\mu$ and $\hat{\gamma}^5$ are the Dirac matrix operators.

We will not go into the details of further mappings in this paper.

3 Flavor Structure, Majorana Higgs, and 750 Gev Diphoton Resonance

3.1 Ternary Clifford Algebra and Flavor Projection Operators

With the purpose of studying 3 generations of fermions, we turn to another kind of Clifford algebra involving ternary communication relationships rather than the usual binary ones. Let's consider ternary $C\ell_{T1}$, which is defined by

$$[\zeta, \zeta, \zeta] = \zeta^3 = 1, \tag{76}$$

with ζ commuting with $C\ell_{0,6}$

$$\zeta\gamma_j - \gamma_j\zeta = 0, \tag{77}$$

$$\zeta\Gamma_j - \Gamma_j\zeta = 0. \tag{78}$$

Flavor projection operators are define by

$$P_{G1} = \frac{1}{3}(1 + e^{\theta' + \theta}\zeta + e^{-\theta' - \theta}\zeta^2) \tag{79}$$

$$= \frac{1}{3}P_0(1 + \zeta + \zeta^2) + \frac{1}{3}P_q(1 + e^{-\theta}\zeta + e^{\theta}\zeta^2), \tag{80}$$

$$P_{G2} = \frac{1}{3}(1 + e^{\theta'}\zeta + e^{-\theta'}\zeta^2) \tag{81}$$

$$= \frac{1}{3}P_0(1 + e^{-\theta}\zeta + e^{\theta}\zeta^2) + \frac{1}{3}P_q(1 + e^{\theta}\zeta + e^{-\theta}\zeta^2), \tag{82}$$

$$P_{G3} = \frac{1}{3}(1 + e^{\theta' - \theta}\zeta + e^{-\theta' + \theta}\zeta^2) \tag{83}$$

$$= \frac{1}{3}P_0(1 + e^{\theta}\zeta + e^{-\theta}\zeta^2) + \frac{1}{3}P_q(1 + \zeta + \zeta^2), \tag{84}$$

where

$$P_{G1} + P_{G2} + P_{G3} = 1, \tag{85}$$

$$P_{Gj}P_{Gk} = \delta_{jk}, \quad (j, k = 1, 2, 3), \tag{86}$$

$$\theta = \frac{2\pi}{3}i, \theta' = \frac{2\pi}{3}I, \tag{87}$$

$$I = \frac{1}{2}(i + 3J), I^2 = -1, \tag{88}$$

and P_0 and P_q are lepton and quark projection operators, respectively.

We label 3 generations of spinors as $\psi_{L/Rj}$ valued in $C\ell_{0,6}$. The spinor kinetic action involves 3 families of fermions as

$$S_{Spinor-Kinetic} \sim \int \langle \bar{\psi}_{Lj} i e^3 D\psi_{Lj} P_{Gj} + \bar{\psi}_{Rj} i e^3 D\psi_{Rj} P_{Gj} \rangle, \tag{89}$$

without flavor-mixing cross terms. Here $\langle \cdots \rangle$ means scalar part of both $C\ell_{0,6}$ and $C\ell_{T1}$.

Flavor-mixing is induced via Majorana Higgs fields, which is the subject of next section.

3.2 Majorana Higgs, Majorana Yukawa Action, and Flavor Mixing

We define Higgs fields in the general sense that they are 0-form (not gauge connection 1-form fields) boson fields with possible symmetry-breaking VEVs. They may or may not be invariant under local Lorentz gauge transformations.

The isospin singlet and Lorentz scalar Majorana Higgs field is

$$\phi_{MAJ} = \phi_{MAJ}^{\dagger} = \phi_{+} + \phi_{-}, \tag{90}$$

where $\phi_{\pm}$ are $C\ell_{0,6}$ trivectors obeying gauge transformation rules

$$\phi_{\pm} \quad \to \quad e^{\mp\check{\Theta}_{WR}-\Theta_J+\Theta_{STR}}\phi_{\pm}e^{\pm\check{\Theta}_{WR}+\Theta_J-\Theta_{STR}}, \tag{91}$$

and

$$\check{\Theta}_{WR} = \frac{1}{2}\epsilon_{WR}i \tag{92}$$

shares rotation angle ϵ_{WR} with

$$\Theta_{WR} = \frac{1}{2}\epsilon_{WR}\Gamma_1\Gamma_2. \tag{93}$$

Each Majorana Higgs field can be broken down into two sectors as

$$\phi_{+} = \phi^{\nu} + \phi^{u}, \tag{94}$$

$$\phi_{-} = \phi^{e} + \phi^{d}, \tag{95}$$

with color singlets ϕ^{ν} and ϕ^{e} valued in Clifford space spanned by 2 trivectors

$$\{\Gamma_0 P_0, i\Gamma_0 P_0\}, \tag{96}$$

and color sextets ϕ^{u} and ϕ^{d} valued in Clifford space spanned by $6 * 2 = 12$ trivectors

$$\{\Gamma_0 P_q, i\Gamma_0 P_q, \Gamma_0 T_1, i\Gamma_0 T_1, \Gamma_0 T_3, i\Gamma_0 T_3, \Gamma_0 T_4, i\Gamma_0 T_4, \Gamma_0 T_6, i\Gamma_0 T_6, \Gamma_0 T_8, i\Gamma_0 T_8\}, \tag{97}$$

where T_k are define in eq. 36.

The color singlet Majorana Higgs fields ϕ^{ν} and ϕ^{e} transform as

$$\phi^{\nu} \quad \to \quad e^{-\check{\Theta}_{WR}-\Theta_J}\phi^{\nu}e^{\check{\Theta}_{WR}+\Theta_J}, \tag{98}$$

$$\phi^{e} \quad \to \quad e^{+\check{\Theta}_{WR}-\Theta_J}\phi^{e}e^{-\check{\Theta}_{WR}+\Theta_J}, \tag{99}$$

while color sextet Majorana Higgs fields ϕ^u and ϕ^d transform as

$$\phi^u \rightarrow e^{-\check{\Theta}_{WR}-\Theta_J+\Theta_{STR}}\phi^u e^{\check{\Theta}_{WR}+\Theta_J-\Theta_{STR}}, \quad (100)$$

$$\phi^d \rightarrow e^{+\check{\Theta}_{WR}-\Theta_J+\Theta_{STR}}\phi^d e^{-\check{\Theta}_{WR}+\Theta_J-\Theta_{STR}}. \quad (101)$$

We can write Majorana Yukawa action of right-handed fermions as

$$\begin{aligned} S_{Majorana-Yukawa} \sim\ & y^{\nu}_{MAJjk}\int \left\langle \bar{\nu}_{Rj}e^4\Gamma_2\Gamma_3\nu_{Rk}P_{Gk}\phi^{\nu}P_{Gj}\right\rangle, \\ & +y^{u}_{MAJjk}\int \left\langle \bar{u}_{Rj}e^4\Gamma_2\Gamma_3u_{Rk}P_{Gk}\phi^{u}P_{Gj}\right\rangle, \\ & +y^{e}_{MAJjk}\int \left\langle \bar{e}_{Rj}e^4\Gamma_2\Gamma_3e_{Rk}P_{Gk}\phi^{e}P_{Gj}\right\rangle, \\ & +y^{d}_{MAJjk}\int \left\langle \bar{d}_{Rj}e^4\Gamma_2\Gamma_3d_{Rk}P_{Gk}\phi^{d}P_{Gj}\right\rangle, \end{aligned} \quad (102)$$

where $y^{\nu}_{MAJjk}, y^{u}_{MAJjk}, y^{e}_{MAJjk}$, and y^{d}_{MAJjk} are Majorana Yukawa coupling constants. Any potential phase factor $e^{\theta\Gamma_1\Gamma_2}$ as in

$$e^{\theta\Gamma_1\Gamma_2}\Gamma_2\Gamma_3 = cos(\theta)\Gamma_2\Gamma_3 + sin(\theta)\Gamma_3\Gamma_1 \quad (103)$$

can be absorbed into redefined Higgs fields.

Since Majorana Higgs fields commute with $P_{0/q}$ and anticommute with i, there are properties

$$\begin{aligned} P_{G1}\phi^{\nu/e} &= \phi^{\nu/e}P_{G1}, \\ P_{G2}\phi^{\nu/e} &= \phi^{\nu/e}P_{G3}, \\ P_{G3}\phi^{\nu/e} &= \phi^{\nu/e}P_{G2}, \\ P_{G1}\phi^{u/d} &= \phi^{u/d}P_{G2}, \\ P_{G2}\phi^{u/d} &= \phi^{u/d}P_{G1}, \\ P_{G3}\phi^{u/d} &= \phi^{u/d}P_{G3}, \end{aligned} \quad (104)$$

according to the definition of flavor projection operators (80, 82, 84). Therefore, there are flavor-mixing terms in the Majorana Yukawa action between 2nd and 3rd generation right-handed leptons as well as between 1st and 2nd generation right-handed quarks.

As ϕ^{ν} acquires a non-zero VEV, which will be investigated in next section, flavor mixing between 2nd and 3rd generation right-handed neutrinos is more salient. Higher order processes can introduce further effective mixing between generations. One may potentially couple above effects with appropriate choices of Majorana and electroweak Yukawa coupling constants to explain the quite different patterns of CKM and PMNS matrices.

3.3 Majorana Higgs Action, Symmetry Breaking, and Majorana Mass

Majorana Higgs action reads

$$S_{Majorana-Higgs} = S_{Majorana-Higgs-Kenetic} - V_{Majorana-Higgs}, \tag{105}$$

with

$$\begin{aligned} S_{Majorana-Higgs-Kenetic}(\phi^\nu) &\sim \int \left\langle (e^3 D\phi^\nu)^2 \right\rangle / \left\langle ie^4 \right\rangle \\ V_{Majorana-Higgs}(\phi^\nu, -\mu_\nu^2, \lambda_\nu) &\sim \int (-\mu_\nu^2 |\phi^\nu|^2 + \lambda_\nu |\phi^\nu|^4) \left\langle ie^4 \right\rangle, \end{aligned} \tag{106}$$

and

$$\begin{aligned} &S_{Majorana-Higgs-Kenetic}(\phi^u), \quad V_{Majorana-Higgs}(\phi^u, +\mu_u^2, \lambda_u), \\ &S_{Majorana-Higgs-Kenetic}(\phi^e), \quad V_{Majorana-Higgs}(\phi^e, +\mu_e^2, \lambda_e), \\ &S_{Majorana-Higgs-Kenetic}(\phi^d), \quad V_{Majorana-Higgs}(\phi^d, +\mu_d^2, \lambda_d), \end{aligned} \tag{107}$$

where

$$D\phi^\nu = (d - \check{W}_R - C)\phi^\nu + \phi^\nu(\check{W}_R + C), \tag{108}$$

$$D\phi^u = (d - \check{W}_R - C + G)\phi^u + \phi^u(\check{W}_R + C - G), \tag{109}$$

$$D\phi^e = (d + \check{W}_R - C)\phi^e + \phi^e(-\check{W}_R + C), \tag{110}$$

$$D\phi^d = (d + \check{W}_R - C + G)\phi^d + \phi^d(-\check{W}_R + C - G), \tag{111}$$

$$\check{W}_R = \check{W}_{R\mu} dx^\mu = \frac{1}{2} W_{R\mu}^3 i dx^\mu \tag{112}$$

Notice that ϕ^ν has negative $-\mu_\nu^2$, while the rest have positive $+\mu_u^2$, $+\mu_e^2$, and $+\mu_d^2$. It means that ϕ^ν acquires a non-zero VEV as

$$< 0|\phi^\nu|0 >= \frac{1}{\sqrt{2}} \upsilon_\nu e^{\alpha i} \Gamma_0 P_0 = \frac{1}{\sqrt{2}} \frac{\mu_\nu}{\sqrt{\lambda_\nu}} e^{\alpha i} \Gamma_0 P_0, \tag{113}$$

while there is no spontaneous symmetry breaking for ϕ^u, ϕ^e, and ϕ^d. As a result, the symmetry related to gauge field

$$Z'_\mu = W_{R\mu}^3 - C_\mu^J, \tag{114}$$

is spontaneously broken.

After replacing ϕ^ν, ϕ^u, ϕ^e, and ϕ^d with their VEVs, the Majorana Yukawa action reduces to

$$S_{Majorana-Yukawa} \sim \int \left\langle \bar{\nu}_{Rj} e^4 \Gamma_2 \Gamma_3 \nu_{Rk} P_{Gj} M_{jk} \Gamma_0 P_0 P_{Gk} \right\rangle, \tag{115}$$

with Majorana masses

$$M_{jk} = y^\nu_{MAJjk} \frac{1}{\sqrt{2}} \upsilon_\nu, \tag{116}$$

where the $e^{\alpha i}$ phase factor is canceled out via a global rotation of spinor

$$\psi \quad \rightarrow \quad \psi e^{-\frac{1}{2}\alpha i}. \tag{117}$$

Neutrino Majorana masses are much heavier than Neutrino Dirac masses, if we assume

$$y^{\nu}_{MAJ}\upsilon_{\nu} >> y\upsilon \tag{118}$$

where constants y and υ are electroweak Higgs section counterparts, which will be defined in later section. Because of the hierarchy, very small effective masses are generated for neutrinos, known as seesaw mechanism.

Now we express gauge fields W^3_R and C^J in terms of B and Z'

$$\begin{aligned} W^3_{R\mu} &= B_\mu + (cos\theta'_W)^2 Z'_\mu, \\ C^J_\mu &= B_\mu - (sin\theta'_W)^2 Z'_\mu, \end{aligned} \tag{119}$$

where

$$\begin{aligned} cos\theta'_W &= \frac{g_{WR}}{g_{Z'}}, \\ sin\theta'_W &= \frac{g_J}{g_{Z'}}, \\ g_{Z'} &= \sqrt{g^2_{WR} + g^2_J}. \end{aligned} \tag{120}$$

Gauge field B remains massless with an effective coupling of

$$g_B = \frac{g_{WR}g_J}{g_{Z'}}, \tag{121}$$

while gauge field Z' acquires a mass from neutrino part of the Majorana Kinetic action

$$M_{Z'} = \frac{1}{2}\upsilon_\nu g_{Z'}. \tag{122}$$

3.4 LHC 750 Gev Diphoton Resonance as Majorana Higgs Loops

If we assume that $\upsilon_\nu >> \upsilon$, gauge boson Z' would be too heavy to be detected at electroweak energy scale. The rest gauge fields interacting with Higgs fields (via 109, 110, and 111) are B (interacting with ϕ^u, ϕ^e, and ϕ^d) and G (interacting with ϕ^u and ϕ^d).

The LHC 750 GeV diphoton resonance[1, 2] may be explained by color sextet Majorana Higgs boson loops, with the involvement of ϕ^u or both ϕ^u and ϕ^d. Through four Higgs interaction term $\lambda|\phi|^4$ in the Higgs potential, the Higgs loops are connected together as repeated loop diagrams in a random phase approximation (RPA) fashion.

Since ϕ^u and ϕ^d directly interacts with B and gluons, no further extended intermediary particle content is needed. The first Higgs loop is produced in gluon fusion. It then propagates as repeated RPA loops, and finally decays to two B bosons. There can also be numbers of internal B and G interaction lines within each loop and between loops. Gauge field B contains massive gauge field Z (upon electroweak symmetry breaking) and massless electromagnetic gauge field A. Thus we are expecting the detection of resonance decaying to Z bosons as well, in addition to decaying to photons.

Top quark is the heaviest quark, hence with the largest electroweak Yukawa coupling constant. If Majorana Yukawa coupling constant of top quark y^u_{MAJ33} is large as well, it means that flavor-preserving processes like

$$y^u_{MAJ33}\bar{t}_R\Gamma_2\Gamma_3 t_R\phi^u \tag{123}$$

are dominant, while other allowed processes like flavor-changing

$$y^u_{MAJ12}\bar{u}_R\Gamma_2\Gamma_3 c_R\phi^u \tag{124}$$

are suppressed, assuming

$$y^u_{MAJ12} << y^u_{MAJ33}. \tag{125}$$

The flavor-preserving processes induce repeated top quark RPA bubbles within each of the two lines of a given Higgs loop. The composite Higgs scenario discussed in next section also support the view of regarding Higg propagator as collective excitation of top quark-antiquark pairs. Therefore the diphoton resonance is a collective excitation phenomena involving four underlying top quarks in a double RPA schema. It suggests that the diphoton resonance mass should be close to 4 times the top quark mass 173 Gev, which is not far off. The effective Majorana Higgs mass is roughly twice of top quark mass or half of the diphoton resonance mass, which is around 360 Gev.

3.5 Composite Majorana Higgs and Dynamical Symmetry Breaking

The entire Higgs sector might be just an effective, Ginzbrug-Landay-type, description of the low energy physics represented by composite Higgs fields. One approach is to assume effective four-quark interactions strong enough to induce top quark-antiquark condensation into composite electroweak Higgs fields[16, 17, 18], via dynamical symmetry breaking mechanism.

Likewise, the Majorana Higgs fields might also be collective excitations of underlying composite spinors. For example, $\phi_\pm$ could be effective representation of 4-form fields

$$\begin{aligned}\check{\phi}_+ &= \sqrt{y^\nu_{MAJjk}}P_{Gj}\bar{\nu}_{Rj}e^4\Gamma_2\Gamma_3\nu_{Rk}P_{Gk} + \sqrt{y^u_{MAJjk}}P_{Gj}\bar{u}_{Rj}e^4\Gamma_2\Gamma_3 u_{Rk}P_{Gk},\\ \check{\phi}_- &= \sqrt{y^e_{MAJjk}}P_{Gj}\bar{e}_{Rj}e^4\Gamma_2\Gamma_3 e_{Rk}P_{Gk} + \sqrt{y^d_{MAJjk}}P_{Gj}\bar{d}_{Rj}e^4\Gamma_2\Gamma_3 d_{Rk}P_{Gk}.\end{aligned} \tag{126}$$

And four-spinor interactions are

$$\int \langle(\check{\phi}_+)^2\rangle / \langle ie^4\rangle + \int \langle(\check{\phi}_-)^2\rangle / \langle ie^4\rangle. \tag{127}$$

4 Electroweak Higgs

4.1 Electroweak Higgs and Electroweak Yukawa Action

Electroweak Higgs field ϕ_{EW} spans the whole 32 component $C\ell_{0,6}$ even space. It obeys gauge transformation rules

$$\phi_{EW} \quad \rightarrow \quad e^{\Theta_{LOR}+\Theta_{WL}}\phi_{EW}e^{-\Theta_{LOR}-\Theta_{WR}}. \tag{128}$$

Electroweak Higgs field can be broken down into three sectors as

$$\phi_{EW} = \phi_S + \phi_P + \phi_{AT}, \tag{129}$$

with scalar ϕ_S valued in Clifford space spanned by 4 multivectors

$$\{1, \Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k\}, \tag{130}$$

pseudoscalar ϕ_P valued in Clifford space spanned by 4 multivectors

$$\{i, i\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k\}, \tag{131}$$

and antisymmetric tensor ϕ_{AT} valued in Clifford space spanned by $4*6 = 24$ multivectors

$$\{\gamma_a\gamma_b, \gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k, a,b = 0,1,2,3, a \neq b\}. \tag{132}$$

The scalar and pseudoscalar electroweak Higgs fields ϕ_S and ϕ_P transform as

$$\phi_{S/P} \quad \rightarrow \quad e^{\Theta_{WL}}\phi_{S/P}e^{-\Theta_{WR}}, \tag{133}$$

while up antisymmetric tensor electroweak Higgs field ϕ_{AT} transforms as

$$\phi_{AT} \quad \rightarrow \quad e^{\Theta_{LOR}+\Theta_{WL}}\phi_{AT}e^{-\Theta_{LOR}-\Theta_{WR}}. \tag{134}$$

Notice that ϕ_{AT} is *not a Lorentz scalar*, since it's not invariant under local Lorentz gauge transformations.

We can write electroweak Yukawa action of fermions as

$$\begin{aligned} &S_{Electroweak-Yukawa} \quad \sim \\ &\int \langle \bar{\psi}_{Lj} i e^4 \phi_{EW}(y_j^\nu \nu_{Rj} + y_j^e e_{Rj} + y_j^u u_{Rj} + y_j^d d_{Rj}) i P_{Gj}\rangle \\ &+ \int \langle (y_j^\nu \bar{\nu}_{Rj} + y_j^e \bar{e}_{Rj} + y_j^u \bar{u}_{Rj} + y_j^d \bar{d}_{Rj}) i e^4 \bar{\phi}_{EW}\psi_{Lj} i P_{Gj}\rangle, \end{aligned} \tag{135}$$

where

$$\bar{\phi}_{EW} = \gamma_0\phi_{EW}^\dagger\gamma_0 = \gamma_0\tilde{\phi}_{EW}\gamma_0 \tag{136}$$

and y_j^ν, y_j^e, y_j^u, and y_j^d are electroweak Yukawa coupling constants.

4.2 Electroweak Higgs Action, Symmetry breaking, and Dirac Mass

Electroweak Higgs action reads

$$S_{Electroweak-Higgs} = S_{Electroweak-Higgs-Kenetic} - V_{Electroweak-Higgs}, \tag{137}$$

with

$$\begin{aligned} &S_{Electroweak-Higgs-Kenetic}(\phi_S) \sim \int \left\langle (e^3(\bar{D\phi_S}))(e^3 D\phi_S) \right\rangle / \left\langle ie^4 \right\rangle \\ &V_{Electroweak-Higgs}(\phi_S, -\mu_S^2, \lambda_S) \sim \int (-\mu_S^2|\phi_S|^2 + \lambda_S|\phi_S|^4) \left\langle ie^4 \right\rangle, \end{aligned} \tag{138}$$

and

$$\begin{aligned} &S_{Electroweak-Higgs-Kenetic}(\phi_P), V_{Electroweak-Higgs}(\phi_P, -\mu_P^2, \lambda_P), \\ &S_{Electroweak-Higgs-Kenetic}(\phi_{AT}), \\ &V_{Electroweak-Higgs}(\phi_{AT}, +\mu_{AT}^2, \lambda_{AT}) \sim \int (\mu_{AT}^2 \left\langle \bar{\phi}_{AT}\phi_{AT} \right\rangle + \lambda_{AT} \left\langle \bar{\phi}_{AT}\phi_{AT} \right\rangle^2) \left\langle ie^4 \right\rangle, \end{aligned} \tag{139}$$

where

$$D\phi_{P/S} = (d + W_L)\phi_{P/S} - \phi_{P/S}(W_R), \tag{140}$$

$$D\phi_{AT} = (d + \omega + W_L)\phi_{AT} - \phi_{AT}(\omega + W_R), \tag{141}$$

Notice that ϕ_S and ϕ_P have negative $-\mu_S^2$ and $-\mu_P^2$. It means that ϕ_S and ϕ_P acquire non-zero VEVs via spontaneous symmetry breaking

$$< 0|\phi_S|0 >= \frac{1}{\sqrt{2}}\upsilon_S = \frac{1}{\sqrt{2}}\frac{\mu_S}{\sqrt{\lambda_S}}, \tag{142}$$

$$< 0|\phi_P|0 >= \frac{1}{\sqrt{2}}\upsilon_P i = \frac{1}{\sqrt{2}}\frac{\mu_P}{\sqrt{\lambda_P}}i. \tag{143}$$

The situation of ϕ_{AT} is a bit complicated, and will be discussed in later section. Let's for the moment assume that its VEV is zero.

After replacing ϕ_S, ϕ_P, and ϕ_{AT} with their VEVs, the electroweak Yukawa action gives rise to complex Dirac masses

$$m_j = y_j \frac{1}{\sqrt{2}}(\upsilon_S + \upsilon_P i) = y_j \frac{1}{\sqrt{2}}\upsilon e^{\beta i}, \tag{144}$$

with

$$\begin{aligned} &\upsilon = \sqrt{\upsilon_S^2 + \upsilon_P^2}, \\ &tan(\beta) = \frac{\upsilon_P}{\upsilon_S}. \end{aligned} \tag{145}$$

However the $e^{\beta i}$ phase factor can be canceled out via a global rotation of spinor

$$\psi \quad \rightarrow \quad e^{-\frac{1}{2}\beta i}\psi, \tag{146}$$

so that the fermion Dirac masses are real valued.

Since the experiments at LHC indicated only one Higgs boson with $m_h = 125$ Gev[3, 4], there could be two scenarios. Case one is that both scalar and pseudoscalar Higgs contributes to the electroweak symmetry breaking and their masses are degenerate

$$m_h = m_S = m_P. \tag{147}$$

Case two is that only one of them acquires a non-zero VEV (with negative $-\mu^2$), which is the $m_h = 125$ Gev Higgs. The other maintains a zero VEV (with positive μ^2), which is still waiting to be detected at LHC.

Now we express gauge fields W_L^3 and B in terms of A and Z

$$\begin{aligned} W_{L\mu}^3 &= A_\mu + (cos\theta_W)^2 Z_\mu, \\ B_\mu &= A_\mu - (sin\theta_W)^2 Z_\mu, \end{aligned} \tag{148}$$

where

$$\begin{aligned} cos\theta_W &= \frac{g_{WL}}{g_Z}, \\ sin\theta_W &= \frac{g_B}{g_Z}, \\ g_Z &= \sqrt{g_{WL}^2 + g_B^2}. \end{aligned} \tag{149}$$

Electromatic field A remains massless with an effective coupling of

$$g_A = \frac{g_{WL}g_B}{g_Z} = \frac{g_{WL}g_{WR}g_J}{\sqrt{g_{WL}g_{WR} + g_{WL}g_J + g_{WR}g_J}}, \tag{150}$$

while gauge field Z acquires a mass

$$M_Z = \frac{1}{2}\upsilon g_Z. \tag{151}$$

4.3 Antisymmetric Tensor Higgs and Dark Spin

As stated earlier, the antisymmetric tensor electroweak Higgs field ϕ_{AT} is not invariant under local Lorentz gauge transformations. Hence, its Higgs potential should involve Lorentz invariant

$$\langle \bar{\phi}_{AT}\phi_{AT} \rangle = \left\langle \gamma_0 \phi_{AT}^\dagger \gamma_0 \phi_{AT} \right\rangle, \tag{152}$$

as opposed to

$$|\phi_{AT}|^2 = \left\langle \phi_{AT}^\dagger \phi_{AT} \right\rangle, \tag{153}$$

which is not Lorentz invariant.

It's easy to see that $\langle\bar{\phi}_{AT}\phi_{AT}\rangle$ is not a positive definite quantity. Components of

$$\{\gamma_a\gamma_b, \gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k=1,2,3, j\neq k, a,b=1,2,3, a\neq b\}, \tag{154}$$

have positive 'metric' and components of

$$\{i\gamma_a\gamma_b, i\gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k=1,2,3, j\neq k, a,b=1,2,3, a\neq b\}, \tag{155}$$

have negative 'metric'.

A zero VEV $<0|\phi_{AT}|0>$ is allowed only if $\mu_{AT}^2=0$. On the other hand, non-zero VEV can be acquired for any value of μ_{AT}^2, including $\mu_{AT}^2=0$. Replacing ϕ_{AT} with non-zero $<0|\phi_{AT}|0>$ in the Higgs kinetic action, we have a Lorentz symmetry breaking term

$$\int\left\langle(e^3(\omega<0|\phi_{AT}|0>-<0|\phi_{AT}|0>\omega)\tilde{)}(e^3(\omega<0|\phi_{AT}|0>-<0|\phi_{AT}|0>\omega))\right\rangle/\left\langle ie^4\right\rangle \tag{156}$$

This spin connection ω related term can contribute to space-time torsion equation. We call it 'dark spin'. It is a counterpart of dark energy, with the former affecting space-time torsion and the later affecting space-time curvature.

Since we know that Lorentz symmetry breaking modifications to torsion could have gravitational and cosmological consequences[19], it's worth further research on the above Higgs-induced scenario.

5 Possible Grand Unification Symmetries

Embolden by the power of Clifford algebra, we now explore more symmetries allowed by an algebraic spinor. Let's begin with general gauge transformations

$$\psi \quad\rightarrow\quad e^{\Theta}\psi e^{\Theta'}, \tag{157}$$

where e^{Θ} and $e^{\Theta'}\in C\ell_{0,6}$ are independent gauge transformations. Spinor bilinear

$$\left\langle\tilde{\psi}\gamma_0\psi\right\rangle \tag{158}$$

is invariant if

$$e^{\tilde{\Theta}}\gamma_0 e^{\Theta}=\gamma_0, \tag{159}$$
$$e^{\Theta'}e^{\tilde{\Theta}'}=1, \tag{160}$$

where we restrict our discussion to gauge transformations continuously connected to identity. General solution of these equations includes $\Theta\sim so(4,4)$, which is a linear combination of 28 gauge transformation generators

$$\{\gamma_a, \gamma_a\gamma_b, \Gamma_a\Gamma_b, i\Gamma_j, \Gamma_0\gamma_j\Gamma_k; j,k=1,2,3, a,b=0,1,2,3, a>b\}\in\Theta, \tag{161}$$

and $\Theta' \sim sp(8)$, which is a linear combination of 36 gauge transformation generators of pseudoscalar, all bivectors, and all trivectors

$$\{i, \gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l, \gamma_0, \Gamma_0, \gamma_0\gamma_j\Gamma_k, \Gamma_0\gamma_j\Gamma_k; j, k, l = 1, 2, 3, k > l\} \in \Theta'. \tag{162}$$

The de Sitter algebra $\Theta_{DS} \sim so(1, 4)$

$$\{\gamma_a, \gamma_a\gamma_b\} \in \Theta_{DS} \tag{163}$$

is a subalgebra of Θ.

The Clifford odd parts of Θ and Θ' mix odd (left-handed ψ_L) and even (right-handed ψ_R) spinors. Since we know that left- and right-handed spinors transform differentially, only Clifford even subalgebras of Θ and Θ' are permitted, namely

$$\{\gamma_a\gamma_b, \Gamma_a\Gamma_b\} \in \Theta_{Even} \sim so(1, 3) \oplus so(1, 3), \tag{164}$$

$$\{i, \gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l\} \in \Theta'_{Even} \sim u(1) \oplus so(6) \sim u(1) \oplus su(4). \tag{165}$$

The gauge transformations $\{\Gamma_a\Gamma_b\}$ can be further decomposed into weak transformations $\{\Gamma_k\Gamma_l\}$ and weak boost transformations $\{\Gamma_0\Gamma_j\}$, which are counterparts of spacial rotation $\{\gamma_k\gamma_l\}$ and Lorentz boost transformations $\{\gamma_0\gamma_j\}$.

Unitary algebra $u(3)$ is embedded in $\{\gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l\} \sim su(4)$. Removing $u(1)$ $\{J\}$ from $u(3)$ defines the color algebra $su(3)$.

Since there are left-handed weak $su(2)_L$ and right-handed weak $u(1)_R$, one might expect left-right symmetric $su(2)_R$ as well. We can even go further and entertain the possibility of two exact copies of left-handed Θ_{EvenL} and right-handed Θ_{EvenR}.

Of course, the grand unification symmetries studied in this section are speculative in nature. If there is indeed grand unification scale physics involving Θ_{EvenL}, Θ_{EvenR} and Θ'_{Even}, either symmetry breaking or other mechanism is needed to prevent detection of gauge interactions related to pseudoscalar $\{i\}$, quark/lepton mixing $su(4) \ominus u(3)$, weak boost $\{\Gamma_0\Gamma_j\}$, right-handed $su(2)_R$, and differences between left-handed $\{\gamma_a\gamma_b\}_L$ and right-handed $\{\gamma_a\gamma_b\}_R$ Lorentz transformations. It's an interesting topic. Nevertheless, we leave grand unification to future research.

6 Conclusion

We propose a Clifford algebra based model. A ternary Clifford vector is introduced alongside 6 binary Clifford vectors. The model includes local gauge symmetries $SO(1, 3) \otimes SU_L(2) \otimes U_R(1) \otimes U(1) \otimes SU(3)$. Both gravitational and Yang-Mills interactions are treated as gauge fields.

There are two sectors of Higgs fields as Majorana and electroweak Higgs bosons. The Majorana Higgs sector causes flavor-mixing between 2nd and 3rd generation right-handed leptons as well as between 1st and 2nd generation right-handed quarks. Higher order processes can introduce further effective mixing between generations. One may potentially couple above effects with appropriate choices of Majorana and electroweak Yukawa coupling constants to explain the quite different patterns of CKM and PMNS matrices.

The color sextet part of Majorana Higgs sector may be responsible for the 750 GeV diphoton resonance via Higgs boson loops. Through four-Higgs interaction term, the Higgs loops are connected together as repeated loop diagrams in a random phase approximation fashion. Majorana Higgs fields directly interact with photons, Z bosons, and gluons. No further extended intermediary particle content is needed. We expect the detection of resonance decaying to Z bosons as well, in addition to decaying to photons. It is suggested that the diphoton resonance mass should be close to 4 times the top quark mass 173 Gev. The effective color sextet Majorana Higgs mass is roughly twice of top quark mass or half of the diphoton resonance mass, which is around 360 Gev.

The neutrino part of Majorana Higgs sector acquires a non-zero VEV via spontaneous symmetry breaking, inducing Majorana masses of right-handed neutrinos via Yukawa-like couplings.

The electroweak Higgs sector is composed of scalar, pseudoscalar, and antisymmetric tensor components. Scalar and/or pseudoscalar Higgs fields break the electroweak symmetry, contributing masses to fermions.

The antisymmetric tensor Higgs is not a Lorentz scalar. Its non-zero VEV would break Lorentz symmetry, giving rise to 'dark spin'. 'Dark spin' is a counterpart of dark energy, with the former affecting space-time torsion and the later affecting space-time curvature. Since we know that Lorentz symmetry breaking modifications to torsion could have gravitational and cosmological consequences[19], it's worth further research on the Higgs-induced scenario.

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