On the relation between the spontaneous and explicit breaking of global symmetries in the presence of the Higgs mechanism

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Abstract

One common way to define spontaneous symmetry breaking involves necessarily explicit symmetry breaking. Thus, how could we have spontaneous without explicit symmetry breaking? We study the concept of Hidden symmetries, which are not spontaneously broken by the Higgs potential, and its representations on the (multi-)Higgs field ϕ. Suppose ϕ is a direct sum of irreducible equivalent representations of the compact group G_a. If we impose a symmetry G_p (compact) to the most general potential of the multi-Higgs-doublet Higgs potential we show that:

1) if no explicit term exists violating G_a, then there is no spontaneous breaking of G_a (the symmetry G_a is hidden in the Higgs potential).

2) if explicit symmetry breaking of G_a, then G_a is allowed to break spontaneously;

Then we explain a recent and related conjecture related with the charge-parity symmetry.

1 Introduction

There are several definitions of spontaneous breaking of global symmetries [1, 2]. In the following common definition[1], spontaneous symmetry breaking is defined as a particular case of explicit symmetry breaking via the external source J.

Let $\mathcal{A}$ be an algebra of operators, the global symmetry β is a bijective map $\beta : \mathcal{A} \to \mathcal{A}$.

The system's expectation value ω_J is a positive linear functional $\omega_J : \mathcal{A} \to \mathbb{R}$,
$J \geq 0$ is the intensity of an external source breaking the symmetry β. The system has infinite size $\omega_J = \lim_{N\to\infty} \omega_{J,N}$.

For finite size N, the system is well behaved with continuous expectation values $J > 0$: $\omega_{J,N}(A - \beta(A)) = a_{J,N} \neq 0$ $J = 0$: $\omega_{0,N}(A) = \omega_{0,N}(\beta(A))$ $\lim_{J\to 0} \omega_{J,N}(A - \beta(A)) = 0$ for some A.

1) The spontaneous symmetry breaking happens when: $\lim_{J\to 0}\{\lim_{N\to\infty} \omega_{J,N}(A-\beta(A))\} \to a_0 \neq 0$ and it is possible due to the fact that the limits are non-commutative if $\omega_{J,N}$ is continuous but not uniformly continuous.

Other definitions do not consider an external source [1], at least not explicitly.

In statistical mechanics, it is more or less clear that these definitions should be all equivalent (e.g. in the Ising model [1]), although it is not easy to mathematically prove it as the systems with or without external source are physically different [3].

When it comes to quantum non-abelian gauge field theories, the theories themselves lack a non-perturbative mathematical definition [4], so it is even more difficult to relate these different definitions. By analogy with statistical mechanics, we expect that they are related. In the presence of the Higgs mechanism, there is yet another definition of spontaneous symmetry breaking, most common in the context of perturbation theory:

2) After a suitable perturbative non-abelian gauge fixing, the vacuum expectation value of the Higgs field is determined (up to quantum corrections) by one of the possible minima of the Higgs potential. The symmetries broken by the vacuum expectation value of the Higgs field are spontaneously broken symmetries.

It is not at all obvious that the last definition (2) is physically (not to mention mathematically) equivalent to the first definition (1) in the context of the Electroweak theory, since spontaneous symmetry breaking is a non-perturbative phenomenon.

However, the fact is that the perturbative predictions from the Electroweak theory seem to be a very good approximation to the existing experimental data in high-energy physics[5], and the lattice simulations so far agree with this picture [6–8] (also for two-Higgs-doublet models [9]). Therefore, for consistency these definitions should be related. While we cannot give a solid proof that this is so, we can check in concrete models that the perturbative definition is consistent with the non-perturbative ones.

There is a further ingredient to take into account [10]: a spontaneous breaking of local gauge symmetry without gauge fixing may be impossible in a gauge theory such as the Electroweak theory. The argument is based on the fact that local gauge transformations affect only a small sized system near each space-time point and so the non-commutativity of the limits seen above does not applies (under some assumptions on the analiticity of $\omega_{J,N}$). It can be argued that the Higgs mechanism avoids the presence of Nambu-Goldstone bosons precisely because the local gauge symmetry is not spontaneously broken [11, 12]. Moreover, there is a group-theory correspondence between gauge-invariant composite operators and the gauge-dependent elementary fields in the Electroweak theory [8, 12, 13].

The above discussion implies that there must exist specific relations between the gauge-dependent minima of the Higgs potential and the gauge-invariant operators appearing in the Lagrangian, for consistency reasons. That is, relations between explicit and spontaneous symmetry breaking. Some of these relations were noted recently in the context of the study of the CP symmetry in multi-Higgs-doublet models [14] and were summarized in the form of a conjecture. We will study the concept of Hidden symmetries, which are not necessarily spontaneously broken, and its real representations in Higgs fields. Then we will generalize and try to understand the recent conjecture mentioned above [14].

2 Hidden symmetries and minima

An important remark is that the minimization of the potential is done classically. Then quantum corrections may be added, but evaluating if spontaneous symmetry breaking is allowed or not is always done purely classically and more importantly, only Higgs fields and the potential are considered.

Another remark is that the Higgs potential is a real function, so the parameters of the Higgs potential are an irreducible representation over the real numbers of the group of background symmetries (which admit more operators than the corresponding irreducible complex representations, in this sense they are more general than the complex representations) [15]. However, since only bi-linears enter the Higgs potential, we can consider the Higgs field ϕ to be a complex field, this has the advantage that the representation of the direct product of two groups can be done by tensor products of the corresponding representations. Since only one Higgs field ϕ enter the potential, the bilinears are all real. Therefore, there is always a $U(1)$ phase that gets absorbed, such phase cannot correspond to the $U(1)_Y$ group, otherwise we could not have the charged $W^{\pm}$ for instance since by definition we cannot have transformations which to not commute with the imaginary unit.

Suppose we have a compact group of symmetries G. We consider a complex irreducible faithful representation of G, ϕ.

We call to the sub-groups G_l the lagrangian group and to G_h the hidden group. The representation space ϕ is a direct sum of irreducible equivalent representations of G_h.

Now we assume that every irreducible subspace of ϕ conserved by G_l is also conserved by G_h. This implies that every hermitian H operator acting on ϕ that is invariant under G_l is also invariant under G_h. (the converse is also true: every hermitian operator that is conserved by G_l is proportional to the identity in each irreducible representation of G_l— by the Schur's lemma— if all of them are conserved by G_h that implies that G_l leaves invariant the projection operators defining the subspace of each irreducible representation of G_l).

We write the most general quartic potential symmetric under G_l, $V(\phi)$ and therefore verify that it is also invariant under G_h. Then the group of background symmetries G_b is the subgroup of G such that G_h is a normal sub-group. That is because we cannot tell the difference, whether we wanted or not for the symmetry G_h to be conserved by the bilinears, as the potential is exactly the same. The parameters of the potential will be an irreducible representation of G_b, with G_h acting trivially.

Then the Higgs basis will necessarily conserve G_h. These symmetries are hidden in the problem of minimization. They are not broken by the bilinears in the Higgs basis. This is what lies behind the fact that despite the Higgs mechanism in the SM involves a minimization problem, that does not lead necessarily to the breaking of the $SU(2)_R \times SU(2)_L \to SO(3)$ [11–13].

The coordinates of a vector space covariant under a group G are not meaningful mathematically (neither physically). The only relevant information we can extract from a group representation are its invariants, in the case of the Higgs potential which only depends on Higgs fields in one space-time point, these are the bilinears which always absorb the G_h group, thus these symmetries are hidden at least at the Higgs potential which determines what are the spontaneously broken symmetries.

Even if we assume that there is spontaneous symmetry breaking of G_h, the fact is that the background transformations relating the Lagrangian basis with the Higgs basis leave G_h intact. Thus the representation of G_h is the same in the Higgs basis or in the Lagrangian basis. Note that when we impose the $U(1)_Y$ symmetry to the Higgs potential, such symmetry can be spontaneously broken by the minima of the potential, because the two-dimensional representations where it acts are reducible for $U(1)_Y$, in order to become irreducible we need to include also the CP symmetry in G_h, as we will see along the paper.

3 The case when explicit symmetry breaking is allowed

For the most general Lagrangian invariant under G_l, suppose that explicit symmetry breaking of G_h is possible. Then, in the Higgs basis we can have one such term breaking G_h. That implies spontaneous symmetry breaking if in the Lagrangian basis the symmetry G_h is conserved. Note that the quadratic terms of the Lagrangian are an irreducible representation of the group of background symmetries G_b so there are no invariant subspaces under G_b, therefore for arbitrary parameters of the potential we cannot exclude the spontaneous symmetry breaking of G_h. Of course that we can always play with the particular values of the parameters so to avoid spontaneous symmetry breaking, an extreme case would be for instance setting all the quadratic terms to zero we would avoid any spontaneous symmetry breaking.

4 CP symmetry with neutral vacuum

In the context of multi-Higgs-doublet models, the vacuum may break $U(1)_{em}$. That is because we are not considering all the possible terms which are invariant under $SU(2)_L$ (note that the Higgs potential is a real function), we are setting some of the terms in the potential manually to zero, such that $U(1)_Y$ is conserved explicitly [16]. So, we have to work with the most general potential invariant under the groups corresponding to irreducible representations, i.e. either $SU(2)_L$ or $SU(2)_L \times U(1)_Y \rtimes Z_4$, where $U(1)_Y \rtimes Z_4 \subset SU(2)_R$ (the custodial group) and the Z_4 is related with the CP transformation.

So, whenever $U(1)_Y \rtimes Z_4$ is necessarily conserved, the CP symmetry is a hidden symmetry and so it is unaffected by the minimization of the Higgs potential. Even if we assume that there is spontaneous breaking of the hidden symmetry, the breaking $SU(2)_L \times U(1)_Y \rtimes Z_4 \to U(1)_{em} \rtimes Z_2$ conserves both CP and $U(1)_{em}$.

However, if it is possible to have terms in the potential breaking $U(1)_Y \rtimes Z_4$, then it is possible to have spontaneous breaking of both CP and/or $U(1)_Y$. If we additionally assume that we will choose parameters of the Higgs potential such that $U(1)_Y$ is conserved, then we have for look to the terms which verify 3 conditions: break CP, conserve $U(1)_Y$ and finally are non-null at the neutral minima. If no such term exists then there is no spontaneous breaking of CP. If such term exists then spontaneous breaking of CP is allowed.

Note that the terms in the Higgs potential which will be null for a neutral vacuum are irrelevant when evaluating spontaneous symmetry breaking at neutral minima, since changing such terms leaves the potential invariant at the minimum (they correspond to Lagrange multipliers in the bilinear formalism to minimize the potential). Dropping such terms is similar to consider a system where all the Higgs fields are neutral (the neutral Higgs sector [14]).

In order to apply the theorem to generalized CP, we need to consider the most general group that includes CP, i.e. $(SU(n) \times U(1)_{em}) \rtimes Z_2$ where n is the number of Higgs doublets. Therefore, there is a basis such that any generalized CP transformation is given by the standard CP transformation [17, 18]. Even if G_p is not conserved explicitly in such basis, selecting hermitian operators invariant under G_p is a basis invariant process and so the theorem is valid to generalized CP.

5 Conclusion

The coordinates of a vector space covariant under a group are not meaningful mathematically (neither physically). The only relevant information we can extract from a group representation are its invariants, in the context of the Higgs potential these are the Higgs bilinears which may hide a group.

Suppose the Higgs field ϕ is a direct sum of irreducible equivalent representations of the compact group G_h. If we impose a symmetry G_p (compact) to the most general potential of the multi-Higgs-doublet Higgs potential we show that:

1) if no explicit term exists violating G_h, then there is no spontaneous breaking of G_h (the symmetry G_h is hidden in the Higgs potential).

2) if explicit symmetry breaking of G_h is possible, then G_h is allowed to break spontaneously;

Using this proposition we explained a recent and related conjecture [14], related with the CP symmetry. Note that the theorem of this paper is not just valid for the breaking of the CP symmetry, it also considers restrictions from compact groups (not necessarily finite or abelian), unlike the mentioned conjecture [14].

Already for consistency reasons, the above proposition (modulo technical details) should be valid: probably the most popular way to define spontaneous symmetry breaking involves necessarily explicit symmetry breaking (def. 1 of the introduction). Thus, how could we have spontaneous without explicit symmetry breaking? (1st part of theorem). Also, since we are assuming that the spontaneous symmetry breaking is determined by the minima of the Higgs potential, then if we allow explicit symmetry breaking of the Higgs potential we necessarily allow spontaneous symmetry breaking, since we can redefine the field coordinates with respect to a particular minimum that breaks explicitly the symmetry (2nd part of theorem).

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