Studies on the Accuracy of the Results Predicted by the Simple Beam Theory in the case of Short Beams that are Fixed at Both the Ends

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***Abstract*—The simple beam theory can predict the deflection of beams that are fixed at both the ends, with a high level of accuracy if the length-to-diameter ratios for the beams are high. Hence the readily available formula (which is based on the simple beam theory) for beams may be used with confidence when the length-to-diameter ratios are high. However, studies on the accuracy of the results predicted by the simple beam theory are not common when the length-to-diameter ratios are small. Hence the present work compares the results obtained through the use of the well-known formula (based on the simple bending theory) with the results obtained by making use of the well-known commercial finite element software ANSYS, for beams of different length-to-diameter ratios and circular and square cross section, in order to get an idea of the accuracy of the results that are obtained by making use of the well-known formula. All these results are tabulated. The present study is restricted to beams that are fixed at both the ends and only those cases where a single concentrated load is applied at the midpoint between the two ends. The present study would be of use to designers who are concerned over the accuracy of the results that are obtained through the use of the well-known analytical formula. The present study is a continuation of the work presented in the author's earlier work "Kirana Kumara P, Studies on the Accuracy of the Results Predicted by the Simple Beam Theory in the case of Short Cantilever Beams, National Conference on: Role of Mechanical Engineering and Computational Science for a Sustainable Future (NCAME-2015), September 25th & 26th, 2015, Dayananda Sagar College of Engineering, Bengaluru, India", and hence the present work — wherever useful — borrows ideas and sentences from the said reference.**

Keywords*—*beam; short; accuracy; deflection; bending

I. Introduction

Predicting the deflection at the centre of a beam that is fixed at both the ends has wide applications in the design of mechanical components. The formula that is based on the simple (pure) bending theory is widely used for the prediction. Literature (e.g., [1] and [2]) tells that the analytical formula gives reasonably accurate results when the length-to-diameter ratios for beams are high (>10, say). However, extensive and exhaustive studies on the deflection of short beams are rarely available, although such studies could have practical applications too (e.g., in the design and analysis of crank pins). Hence the present work is an attempt to study the deflection of short beams, beams that are fixed at both the ends in particular. Of course, a few sources in the literature (e.g., [1]) do contain material on this topic. But many a times the coverage of the topic is not comprehensive and exhaustive; for example, error in the prediction of stresses might have been addressed well but error in the prediction of deflection might have been neglected. So the present work aims to make a fresh attempt towards the quantification of the error when the analytical formula that is based on the pure bending theory is used for the prediction of the deflection of the beam that is fixed at both the ends, when the length-to-diameter ratio for the beam is low (i.e., beam is short).

Of course, all the problems considered in the present study are linear elastostatic problems, and body forces are not considered. Moreover, the present work deals with only those cases where the load is applied only at the centre of the beam (a single concentrated load that is applied at the midpoint between the two ends).

A study that is very similar to the present study is presented in the author's earlier work [3]. In fact, the present work is a continuation of the author's work [3]. Wherever useful, the present work borrows ideas and sentences (but not the results) from the reference [3]. The reference [3] is concerned about cantilever beams whereas the present study is focused on beams that have both the ends fixed.

II. Methodology

The first geometry considered is a solid cylinder of 1 *mm* diameter. The length of the cylinder could be 1 *mm*, 3 *mm*, 10 *mm*, or 20 *mm*; these correspond to length-to-diameter ratio (L/d) equal to 1, 3, 10, or 20 respectively. The Poisson's ratio is assumed to be equal to 0.33, and the Young's modulus (E) is assumed to be equal to 200000 N/mm^2.

The second geometry that is considered in the present work is a block of 1 *mm* by 1 *mm* cross section (a square cross section). The length of the block could be 1 *mm*, 3 *mm*, 10 *mm*, or 20 *mm*; these correspond to length-to-diameter ratio (L/d) equal to 1, 3, 10, or 20 respectively. Just as earlier, the

Poisson's ratio is assumed to be equal to 0.33, and the Young's modulus (*E*) is assumed to be equal to 200000 N/mm^2. The models are oriented such that the edges of the block are always parallel to the coordinate axes. Of course, in the present work, in the case of beams with square cross section, the term "length-to-diameter ratio (*L/d*)" refers to length-to-thickness ratio in fact.

The finite element models are constructed in the commercial finite element software ANSYS. A fine mesh is used always, so that the solutions given by ANSYS are accurate. The element type used is *Tet 10node 187* always, and the geometry is always divided into tetrahedral elements using the *free* meshing option. Both the ends (the entire surfaces) of the beam are completely fixed (no displacement is allowed along any of the directions). A node that is located somewhere close to the plane that is located in the middle of the two fixed ends (distances from both the ends of the beam to this plane should be the same; also, this plane should be parallel to both the fixed ends), is subjected to a point load of 1 *N*, along any of the coordinate axes excluding the direction that is parallel to the length of the beam. The problem is to find the deflection at the loaded (by 1 *N*) node, along the direction of the load. Fig. 1 to Fig. 8 show the finite element models together with loads and boundary conditions, as displayed in ANSYS.

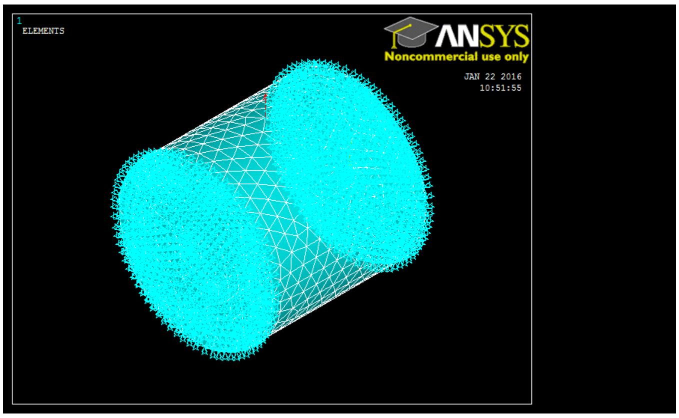

Fig. 1. Circular cross section ($L/d = 1$).

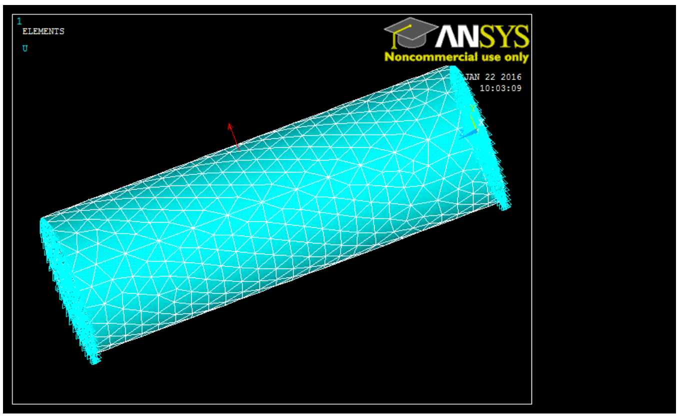

Fig. 2. Circular cross section ($L/d = 3$).

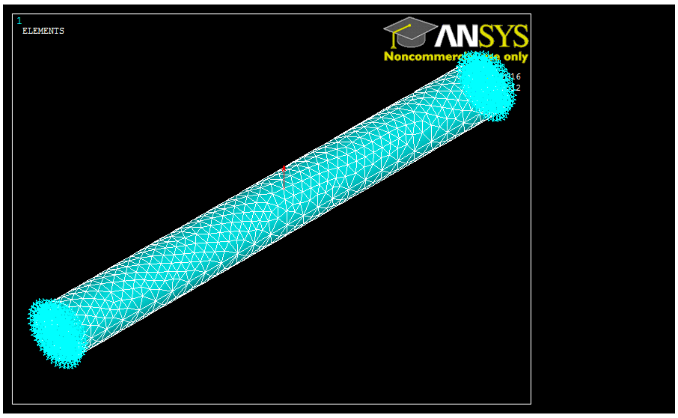

Fig. 3. Circular cross section ($L/d = 10$).

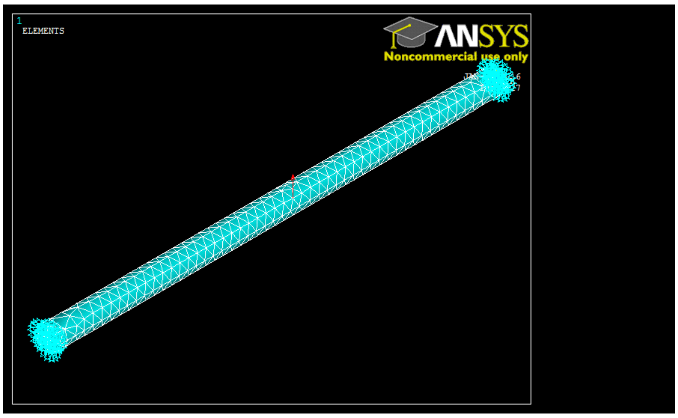

Fig. 4. Circular cross section ($L/d = 20$).

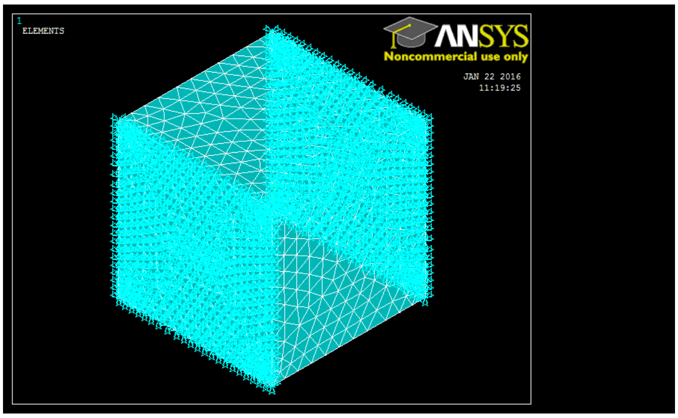

Fig. 5. Square cross section ($L/d = 1$).

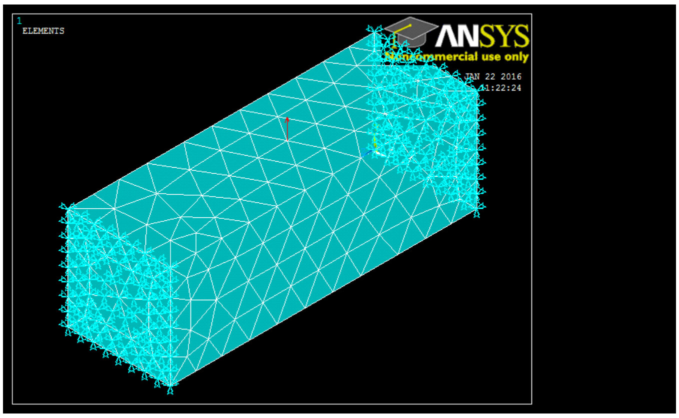

Fig. 6. Square cross section ($L/d = 3$).

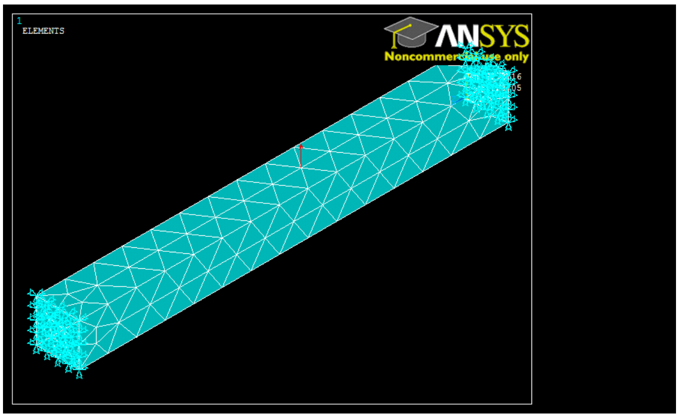

Fig. 7. Square cross section ($L/d = 10$).

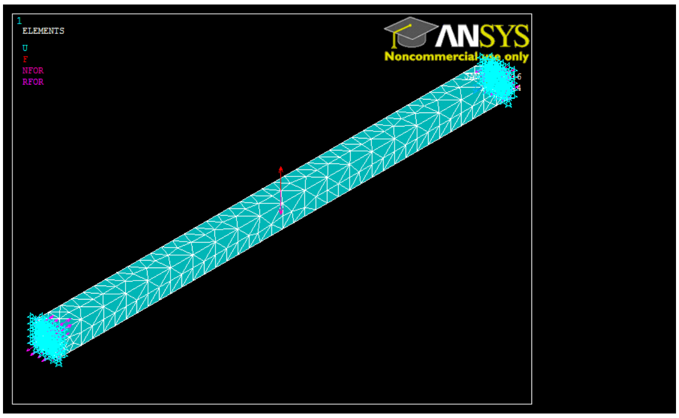

Fig. 8. Square cross section (L/d = 20).

Just for the sake of completeness, the analytical formula used in the present work is given by:

$$y = \frac{FL^3}{192EI} \quad (1)$$

where y is the deflection at the centre of the beam

F is the load at the centre of the beam

L is the length of the beam

E is the Young's modulus

I is the area moment of inertia of the cross section of the beam

Again just for the sake of completeness, the area moment of inertia for a beam with circular cross section is given by:

$$I = \frac{\pi d^4}{64} \quad (2)$$

where d is the diameter of the beam

And the area moment of inertia for a beam with square cross section is given by:

$$I = \frac{h^4}{12} \quad (3)$$

where h is the thickness of the beam

Results are tabulated for each of the (three dimensional) finite element simulations (ANSYS is used for the simulations). It might be important to mention that all the finite element simulations in the present work use three dimensional finite elements, not the beam elements. These results are presented in Table I. Results obtained by using the (one dimensional) analytical formula (the formula neglects the effect of shear forces) are also tabulated and these results are presented in Table II.

III. Results

As mentioned previously, the results from ANSYS are presented in Table I whereas the results from the analytical formula are presented in Table II, for different length-to-diameter ratios (L/d), and for circular cross section and square cross section.

The percentage error is also calculated for each of the cases, and this is presented in Table III. Error is calculated here by taking the results from ANSYS as the reference values.

TABLE I. Results Obtained by using ANSYS

	Circular Cross Section	**Square Cross Section**
***L/d* = 1**	0.000119 *mm*	0.000127 *mm*
***L/d* = 3**	0.000112 *mm*	0.000060 *mm*
***L/d* = 10**	0.000590 *mm*	0.000337 *mm*
***L/d* = 20**	0.004382 *mm*	0.002541 *mm*

TABLE II. RESULTS OBTAINED BY USING THE ANALYTICAL FORMULA

	Circular Cross Section	Square Cross Section
***L/d* = 1**	0.000001 *mm*	0.000000 *mm*
***L/d* = 3**	0.000057 *mm*	0.000008 *mm*
***L/d* = 10**	0.000531 *mm*	0.000313 *mm*
***L/d* = 20**	0.004244 *mm*	0.002500 *mm*

TABLE III. PERCENTAGE ERROR

	Circular Cross Section	Square Cross Section
***L/d* = 1**	99.160	100.000
***L/d* = 3**	49.107	86.67
***L/d* = 10**	10.000	7.122
***L/d* = 20**	3.149	1.614

IV. CONCLUDING REMARKS

The present work is a continuation of the author's work [3]. Of course, all that is mentioned as future work in [3] has not been presented here (as all those tasks are not yet complete).

Future work could be to generate and tabulate more results, by considering more number of length-to-diameter ratios and more number of cross sections (e.g., rectangular cross section), for different types of loads and boundary conditions. This would result in a study that is more extensive and exhaustive.

REFERENCES

[1] Warren C. Young, Richard G. Budynas, Roark's Formulas for Stress and Strain, Seventh Edition, McGraw-Hill: New York, 2002.

[2] K. Mahadevan, K. Balaveera Reddy, Design Data Handbook, Fourth Edition, CBS Publishers & Distributors: New Delhi, 2013.

[3] Kirana Kumara P, Studies on the Accuracy of the Results Predicted by the Simple Beam Theory in the case of Short Cantilever Beams, National Conference on: Role of Mechanical Engineering and Computational Science for a Sustainable Future (NCAME-2015), September 25th & 26th, 2015, Dayananda Sagar College of Engineering, Bengaluru, India. Available from: http://vixra.org/abs/1509.0169 (accessed January 22, 2016)