

# The proof that Bell's theorem is scientifically unfounded

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## Abstract.

We prove that the Bell's theorem and the nonlocality of quantum mechanics are scientifically unfounded statements.

## 1. Introduction.

The Bell's theorem says that assuming the locality of quantum mechanics (QM) one can derive Bell inequalities (BI). There are two forms of the Bell's theorem

- (i) The strong form: (locality of QM  $\rightarrow$  BI)
- (ii) The weak form: ((locality and realism of QM)  $\rightarrow$  BI)

In this paper we shall consider mainly the Bell's theorem in the strong form (i) but in the last sections we shall consider also the weak form (ii).

We shall show that the strong form of the Bell's theorem must be considered as the scientifically unfounded statement. Then we show that the so-called Bell nonlocality cannot be considered as a true statement. (This paper is a continuation of the paper [1].)

Our main argument in this paper can be presented in the schematic form.

Let QM denote the standard quantum mechanics. Let us assume that there exists a theory T which has the following properties

- (i) Empirical predictions of QM and T are the same.
- (ii) The Bell's theorem cannot be proved in T (i.e. no known derivation of BI can be applied in T).

Since QM and T are empirically indistinguishable we cannot be sure that the "right" version is QM and not T. Let us assume that the "right" version of QM is T – then the Bell's theorem cannot be proved. Since such a possibility cannot be excluded we have to state that Bell's theorem is scientifically unfounded.

Thus the main point of the proof is to find the theory T satisfying properties (i) and (ii). We show that the good candidate for T is the modified QM introduced in [2, 3].

In the part 2 we shall introduce the basic concepts of the truth in QM in the situation where there exists a theory T satisfying (i) and (ii).

In the part 3 we shall show that (assuming locality) BI cannot be derived in the modified QM, i.e. that the strong form of the Bell's theorem cannot be derived in the modified QM. This is the main part of the proof.

In the part 4 we shall consider the status of the weak form of the Bell's theorem and its consequences.

In the part 5 we shall show that the Bell's nonlocality (i.e. the nonlocality of QM based on the Bell's theorem) is the scientifically unfounded statement.

The part 6 contains the discussion and conclusions.

## **2. A concept of truth in QM.**

We shall remind the basic notions from [1].

**Definition 1.** Two theories are empirically equivalent if their empirical predictions are identical.

**Definition 2.** The QMversion is each theory which is empirically equivalent to QM.

It is important to understand that both definitions do not use the concept of the experimental verification. Only the concept of the empirical predictions of a given theory is used. This means that the concept of empirically equivalent theories depends only on these theories and does not depend on experiments.

QMversions are empirically indistinguishable so that we can define a new concept of truth in QM.

**Definition 3.**

- (i) The statement S is true in QM if S is true in each QMversion.
- (ii) The statement S is undecidable in QM if S is true in some QMversion and S is false in some other QMversion.
- (iii) The statement S is false in QM if S is false in each QMversion.

It is evident that every empirical prediction of QM is the true statement in QM.

There are following basic consequences of these concepts.

**Definition 4.**

- (i) The statement  $S$  is well-founded in QM if it is true in every QMversion.
- (ii) The statement  $S$  is unfounded in QM if it false in some QMversion.

**Theorem 1.**

- (i) The statement  $S$  is well-founded in QM if and only if  $S$  is true in QM.
- (ii) The statement  $S$  is unfounded in QM if and only if either  $S$  is undecidable in QM or  $S$  is false in QM

**Proof.** (i) The statement  $S$  is true in all QMversions so that  $S$  cannot be undecidable in QM and  $S$  cannot be false in QM. (ii) In this case the statement  $S$  is false in some QMversion so  $S$  cannot be true in all QMversions.

This means that only well-founded statements can be considered as really true.

This means that we are not able to choose the “right” QMversion (i.e. the choice based on the empirical evidence is impossible) since all QMversions are empirically equivalent.

**The principle of objectivity.**

- (i) No undecidable statement in QM can be considered as a true statement or as a false statement.
- (ii) The idea that there exists certain (currently unknown) QMversion which is “really true” should be rejected (i.e. the idea of the preferred QMversion should be rejected).

We think that from the principle of objectivity it follows that all QMversions must be considered as equivalent and no QMversion can be preferred.

It can be mentioned the so-called Occam`s razor saying that the simplest variant should be chosen. But in science Occam`s razor cannot be used: the Wikipedia says “In science, Occam's razor is used as a heuristic technique (discovery tool) to guide scientists in the development of theoretical models, rather than as an arbiter between published models.” – see [3] and also the discussion in [2]. (In general, it is by no means clear that Nature prefers the simplest variant.)

**3. In the modified QM any known derivation of the Bell’s theorem is invalid.**

This is the central part of the paper and will be divided into some parts.

**Part 1.** We shall describe the main property of the modified QM which will be used. Let us start with the concept of an individual state. It is assumed that to each system (at a given time) there is associated a pure state called the individual state of this system. The state of an ensemble is called homogeneous, if all its elements are in the same individual state (see [4]).

The pure state is called the individual state if it is a possible individual state of some individual system. In the standard QM it is assumed that each pure state is an individual state (the “von Neumann’s axiom”).

In the modified QM the set of individual states of a given system is a certain orthogonal base so that each two different individual states are orthogonal.

If the ensemble is in the individual state then all its elements are in the same individual state. If the experiment is repeated then the relative frequency of the given individual state is equal to the standard QM probability given by the Born’s rule. Otherwise the individual states are random.

### **Part 2.**

Each proof of each Bell’s theorem is based on the analysis of individual states. Already the principle of the proof of any Bell’s theorem is to consider the Alice’s system in some bases and the Bob’s system in some bases. By this analysis certain inequality is obtain and this inequality is then integrated over the parameter  $\lambda$  and the final inequality (BI) is obtained.

The origin of the proof is always based on the properties of individual states. It is supposed that each wave function describes the possible state of an individual system.

### **Part 3.**

Any proof of some Bell’s theorem requires either at least two different individual bases on the Alice’s side or at least two individual bases on the Bob’s side. Under the notion “individual base” we mean the base composed exclusively from individual states.

The lowest number of bases is required in the original Bell’s proof, where there must exist two individual bases at the Alice’s side ( $A, A'$ ) and one individual base at the Bob’s side ( $B$ ). In the standard proof (the CHSH inequality) there are required two individual bases at Alice ( $A, A'$ ) and two individual bases at Bob ( $B, B'$ ). In the well-known Mermin’s proof there are required three different bases at Alice and three different bases at Bob.

The spirit of the Bell’s proof consists in the fact that more than one individual base is used.

### **Part 4.**

The conclusion that Bell’s proof cannot be reproduced in the modified QM. In the modified QM only one individual base is available on the Alice’s side and only one individual base on the Bob’s side. In fact, the step “Part 3” cannot be realized. Thus Bell’s proof cannot be applied in the modified QM.

### **Theorem.**

No known proof of the Bell's theorem can be realized in the modified QM. I.e. each proof of the Bell's theorem is invalid in the modified QM. Then the Bell's theorem is unfounded in QM.

**Remark.**

There is a discussion among specialists if the strong form of the Bell's theorem can be proved in the standard QM. But in our results we do not need this proof since we do not assume that there exists a QM version such that Bell's theorem is true in it. We only assert that the Bell's theorem is either undecidable or false, i.e. unfounded.

**4. The Bell nonlocality is the scientifically unfounded statement.**

The standard argument is based on the strong form of the Bell's theorem and on the contradiction of the Bell inequality with the QM and it implies the nonlocality of QM. This argument cannot be applied here since the Bell's theorem cannot be proved in the modified QM. Thus the Bell nonlocality cannot be proved using the strong form of the Bell's inequality.

The weak form of the Bell's theorem uses as assumptions the locality and the realism. In fact, the assumption of the realism (each wave function describes a possible state of the system) is the scientifically unfounded statement since it is true in the standard QM but it is evidently false in the modified QM (where not every wave functions describes a possible individual state of the system). Thus the realism is an unfounded statement.

In the case of the weak form of the Bell's theorem the fact that Bell inequality contradicts to QM implies only that either locality is false or realism is false. Since realism is not the well-founded statement (e.g. it is not true in the modified QM) we cannot deduce the nonlocality.

**Theorem.**

The Bell nonlocality is not the scientifically well-founded statement.

Since the Bell's theorem is the unique way how to prove the nonlocality of QM we can say that the proof of the nonlocality of QM is not valid.

**5. Discussion and conclusions.**

The essence of our proof is the following: since there is a theory empirically equivalent to the QM where the Bell's theorem cannot be proved (the modified QM), then the Bell's theorem is scientifically unfounded.

There are at least the following consequences of the principle of objectivity:

- The individual superposition principle is scientifically unfounded (see[1])
- The Bell's theorem in the strong form is scientifically unfounded
- The nonlocality of QM is scientifically unfounded

These findings have own consequences:

- The Schroedinger's cat paradox is scientifically unfounded since it is based on the use of the individual superposition principle
- The basic argument in the measurement problem is based on the use of the individual superposition principle and thus it is unfounded

The main part of the proof consists in the impossibility to reproduce the Bell's proof in the modified QM (see Section 3, Part 3).

**Remark.**

The situation described above is analogous to the situation in the axiomatic set theory. In this theory there are many undecidable statements, e.g. the continuum hypothesis. In fact, there exists a Goedel's model where the continuum hypothesis is true and there exists the Cohen's model where the continuum hypothesis is false. After many years of the research (at least from 1964) there is no intuitive principle which could resolve this dilemma.

**References**

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