Rated Set Theory

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Definitions

Symbol	Description
$\mathcal{C}$	Count
$\mathbb{u}$	Unique Identifier
$\mathbb{Z}$	Set of Integers
$x\|condition$	x such that condition
$x \in Y$	x is an element of set Y

Symbol	Description
ρ_i	Probability of i occurance
σ	Standard deviation
μ	variance
ϕ_x	Probability density of x

A Rated Set

A rated set is defined as:

$$\Re = \left\{ \begin{pmatrix} member = \{ \mathbb{u}, \{charchateristics\} = \mathfrak{c} \} = m, \\ rating = r|(a|_{a\in\mathbb{Z}^+} \le r \le b|_{b\in\mathbb{Z}^+}, b > a), \\ rater = \{\mathbb{u}, \mathfrak{c}\} \; = \mathfrak{p} \end{pmatrix} \right\}$$

The unweighted rating for a member is: $R_m = \dfrac{\sum_{\Re|m} r}{\mathcal{C}_{\Re|m}}$

When comparing unweighted ratings we implicitly consider the following:

$$\rho_r = \frac{\mathcal{C}_{\Re|r}}{\mathcal{C}_{\Re}} \qquad \mu = \sum_{r=a}^{b} \rho_r r \qquad \sigma = \sqrt{\sum_{r=a}^{b} \rho_r (r-\mu)^2}$$

Assuming the rating distribution has a high correlation to a normal distribution in our minds we evaluate the difference between two ratings accordingly:

$$\phi_{\mathrm{m}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(R_m-\mu)^2}{2\sigma^2}}$$

$$\phi_{mn} = \iint \phi_{\mathrm{m}}\, dR_m - \phi_{\mathrm{n}}\, dR_n = \sqrt{\frac{2}{\pi}}\,\sigma e^{\frac{\mu}{2\sigma^2}} \left(R_m e^{-\frac{R_n}{2\sigma^2}} - R_n e^{-\frac{R_m}{2\sigma^2}} \right)$$

ϕ_{mn} effectively tells us the magnitude of the difference between two ratings with respect to data.

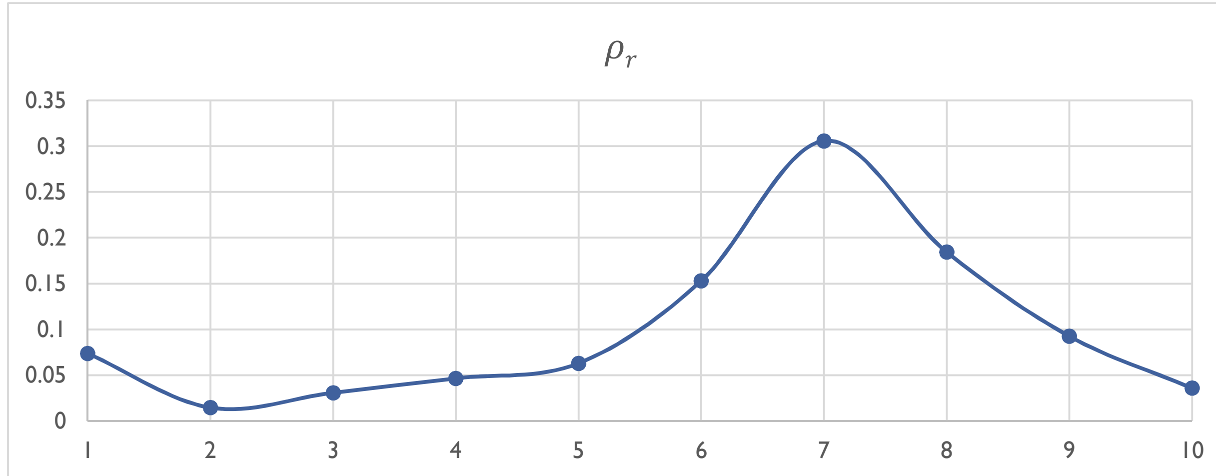
We then normalize this to the terms of the rating system to provide a weighted interpretation of the rating: ω_r.

$$\varphi_r = \sum_{\lambda=b}^{a} \rho_r \phi_{r\lambda} \qquad \omega_r = \begin{cases} r + \dfrac{1}{\varphi_r}\Big|\; except \\ r + \varphi_r | r = \dfrac{b}{2} \end{cases}$$

Example Data Set

For this example we will use my rating data from songs played on RadioParadise. http://www.radioparadise.com/rp_2.php?#name=Members&file=userinfo&u=6814

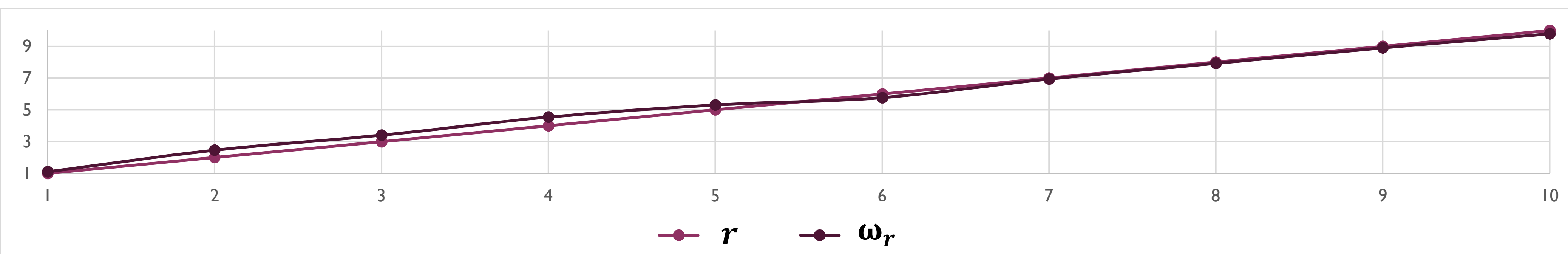
r	Occurrence	ρ	μ	σ
10	57	**0.035872**	0.358716	0.459483
9	147	**0.092511**	0.832599	0.615304
8	293	**0.184393**	1.475142	0.459724
7	486	**0.305853**	2.140969	0.102527
6	243	**0.152926**	0.917558	0.027107
5	100	**0.062933**	0.314663	0.12708
4	74	**0.04657**	0.186281	0.272963
3	49	**0.030837**	0.092511	0.360897
2	23	**0.014475**	0.028949	0.28291
1	117	**0.073631**	0.073631	2.163834
	1589		6.42102	2.207222



As you can see I favor a 7 rating, which is because I tend to like what RadioParadise plays. I also heavily weight 1's with respect to the set in general, so the distribution isn't exactly normal, however it leads to a much higher μ, σ than a standard normal distribution. I contend that people tend to assume a standard distribution when interpreting raw ratings with the absence of an understanding of the distribution.

$\phi_{r\lambda}$	10	9	8	7	6	5	4	3	2	1
10	0.0	-2.5	-5.2	-8.1	-11.1	-14.3	-17.7	-21.4	-25.3	-29.5
9	2.5	0.0	-2.7	-5.5	-8.4	-11.6	-14.9	-18.5	-22.2	-26.3
8	5.2	2.7	0.0	-2.8	-5.7	-8.8	-12.1	-15.5	-19.2	-23.1
7	8.1	5.5	2.8	0.0	-2.9	-6.0	-9.2	-12.5	-16.1	-19.8
6	11.1	8.4	5.7	2.9	0.0	-3.0	-6.2	-9.5	-13.0	-16.6
5	14.3	11.6	8.8	6.0	3.0	0.0	-3.1	-6.4	-9.8	-13.3
4	17.7	14.9	12.1	9.2	6.2	3.1	0.0	-3.2	-6.6	-10.0
3	21.4	18.5	15.5	12.5	9.5	6.4	3.2	0.0	-3.3	-6.7
2	25.3	22.2	19.2	16.1	13.0	9.8	6.6	3.3	0.0	-3.4
1	29.5	26.3	23.1	19.8	16.6	13.3	10.0	6.7	3.4	0.0

$\rho_r\phi_{r\lambda}$	10	9	8	7	6	5	4	3	2	1	ω_r
10	0.00	-0.09	-0.19	-0.29	-0.40	-0.51	-0.63	-0.77	-0.91	-1.06	**9.79**
9	0.09	0.00	-0.25	-0.51	-0.78	-1.07	-1.38	-1.71	-2.06	-2.43	**8.90**
8	0.19	0.49	0.00	-0.51	-1.06	-1.63	-2.23	-2.86	-3.54	-4.26	**7.94**
7	0.29	1.67	0.85	0.00	-0.89	-1.82	-2.80	-3.83	-4.92	-6.07	**6.94**
6	0.40	1.29	0.88	0.45	0.00	-0.46	-0.95	-1.45	-1.98	-2.54	**5.77**
5	0.51	0.73	0.55	0.38	0.19	0.00	-0.20	-0.40	-0.62	-0.84	**5.31**
4	0.63	0.69	0.56	0.43	0.29	0.15	0.00	-0.15	-0.31	-0.47	**4.55**
3	0.77	0.57	0.48	0.39	0.29	0.20	0.10	0.00	-0.10	-0.21	**3.40**
2	0.91	0.32	0.28	0.23	0.19	0.14	0.10	0.05	0.00	-0.05	**2.46**
1	1.06	1.94	1.70	1.46	1.22	0.98	0.74	0.49	0.25	0.00	**1.10**



Correcting for Non-Normal Distributions

Unverified system equations:

$$\mathrm{P} = \{\{r, \rho_r\}\}$$

$$\mathrm{P}' = \bigcup_{\mathrm{P}} \bigwedge_{r|\rho_r - \rho_{r+n} > 0}^{0<n\le b-r} \left\{ \{r\} = R', \left(\sum_{k=r}^{r+n} \rho_k\right) \Big/ (r+n) = \rho^{\odot} \quad , \left\{\frac{\rho_k}{\rho^{\odot}} = \rho'\right\} \right\}$$

$$\mathrm{M}' = \bigcup_{\mathrm{P}'} \left\{ R', \{\rho'\}, \sum_{r\in R'} \rho' r = \mu' \right\}$$

$$\Xi = \bigcup_{\mathrm{M}'} \left\{ R', \mu', \{\rho'\}, \sqrt{\sum_{r\in R'} \rho'(r-\mu')^2} = \sigma' \right\}$$

$$\phi'_{mn} = -\sqrt{\frac{2}{\pi}} R'_n \sigma'_m e^{\frac{\mu'_m - R'_m}{2\left(\sigma'_m\right)^2}} + \sqrt{\frac{2}{\pi}} R'_m \sigma'_n e^{\frac{\mu'_n - R'_n}{2\left(\sigma'_n\right)^2}}$$

$$\Phi = \bigcup_{\Xi} \bigcup_{R'} \bigwedge_{r\in R'}^{n|r-n\in R'} \left\{ \{r, r-n = \lambda\}, \sum_{\lambda} \rho'_r \phi'_{r\lambda} = \varphi'_r \right\}$$

$$\Omega = \bigcup_{\Phi} \left\{ r, \left. \begin{matrix} r + \left.\dfrac{1}{\varphi'_r}\right| except \\ r + \varphi'_r | r = \dfrac{\max(r)}{2} \end{matrix} \right\} = \omega'_r \right\}$$