Introduction:

In the usual Compton effect, coming photon (have momentum $P_1$) and colliding with electron which have zero initial momentum (and direction of electron momentum according with direction of coming photon momentum), because we are neglected the kinetic energy of this electron of agreeing with the equation:

$$E = \frac{P^2}{2m}$$

And as a result of this colliding, the photon is loses some of energy for the electron, and the photon is scattering with the angle $\theta$ for the coming path as the figure (1):

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{meC^2}(1 - \cos \theta)}$$

Figure (1): the usual Compton effect.

And the scattered electron acquire energy (and momentum) and formation the $\Phi$ angle with the coming photon axis.

The energy of the scattered photon is:
But in this work, we are consider the initial kinetic energy of the electron (so the
electron have initial momentum), and we supposed the initial momentum ray make
the angle $\phi$ with the coming photon axis (or with initial photon momentum ray axis)
And we find new results for this correction.
It is; the photon can acquire energy from the electron at some values for the $\phi$ angle
(this is reverse the usual Compton effect), and for this new effect (anti Compton
effect) there are many interesting results and new applications in many fields.

Anti Compton and Compton effects relation derivation:

When the photon coming which have initial momentum ($P_1 = h\nu/c$) on electron which
have initial momentum ($P_2$) and kinetic energy ($T_e$) and make the $\phi$ angle with
photon momentum direction, this photon is scattering of the electron and then it's new
momentum become ($P'_1 = h\nu'/c$) and also the electron momentum will becomes after
scattering ($P'_2$) and it's new kinetic energy is ($T'_e$), figure (2):

\[
\begin{align*}
\text{Figure (2).}
\end{align*}
\]

From principle of total energy conservation:

\[
h\nu + m_e c^2 + T_e = h\nu' + m_e c^2 + T'_e \quad \text{.................(1)}
\]

So we find:

\[
T'_e = T_e + h\nu - h\nu' \quad \text{.........(2)}
\]
And we have also:
\[ E^2 = p_e^2C^2 + m_e^2C^4 \].........(4)  \[ E'^2 = p'_e^2C^2 + m_e^2C^4 \].........(3)

In which:
\[ E^2 = (m_eC^2 + T_e)^2 = m_e^2C^4 + 2m_eC^2 \cdot T_e + T_e^2 \].........(5)
\[ E'^2 = (m_eC^2 + T'_e)^2 = m_e^2C^4 + 2m_eC^2 \cdot T'_e + T'_e^2 \].........(6)

And from the equations (4), (5) we find:
\[ p_e^2C^2 = T_e \left(T_e + 2m_eC^2\right) \].........(7)

And from the equations (4), (5) we find:
\[ p'_e^2C^2 = \left[T_e + hu - hu'\right]\left[T_e + hu - hu'+2m_eC^2\right] \].........(8)

From principle of momentum conservation:
\[ p_1 + p_2 = p'_{1} + p'_{2} \].........(9)

And the projection of the momentum on the X axis is:
\[ \frac{hu}{C} - p_e \cdot \cos \phi = \frac{hu'}{C} \cdot \cos \theta + p'_e \cdot \cos \phi \].........(10)

From the equation (10) we find:
\[ \cos \phi = \frac{hu}{p'_eC} - \frac{P_eC}{p'_eC} \cdot \cos \phi - \frac{hu'}{p'_eC} \cdot \cos \theta \].........(11)

And the projection of the momentum on the Y axis is:
\[ p_e \cdot \sin \phi = p'_e \cdot \sin \phi - \frac{hu'}{C} \cdot \sin \theta \].........(12)
From the equation (12) we find:
\[
\cos\phi = \sqrt{1 - \left[ \left( \frac{P'c}{P'eC} \cdot \sin\phi \right)^2 + \left( \frac{h\nu'}{P'eC} \cdot \sin\theta \right)^2 + \left( \frac{2P'eC \cdot h\nu'}{(P'eC)^2} \cdot \sin\phi \cdot \sin\theta \right) \right]}
\].................................(13)

From equations (11), (13) and squaring the both sides, we find:
\[
\left( \frac{h\nu}{P'eC} \right)^2 + \left( \frac{h\nu'}{P'eC} \right)^2 + \left( \frac{P'eC}{P'eC} \right)^2 - 1 - \frac{2h\nu \cdot P'eC}{(P'eC)^2} \cdot \cos\phi - \frac{2h\nu \cdot h\nu'}{(P'eC)^2} \cdot \cos\theta + \phi
+ \frac{2h\nu' \cdot P'eC}{(P'eC)^2} \cdot \cos\phi \cdot \cos\theta + \frac{2h\nu' \cdot P'eC}{(P'eC)^2} \cdot \sin\phi \cdot \sin\theta = 0..............................(14)
\]

By using the two equations (7), (8) and put their in equation (14), then solving the result equation for \( h\nu' \), then we find the final equation:
\[
h\nu' = \frac{h\nu \left[ T_e + meC^2 + \sqrt{T_e \left( T_e + 2meC^2 \right)} \cdot \cos\phi \right]}{\left[ h\nu + T_e + meC^2 - h\nu \cdot \cos\theta + \sqrt{T_e \left( T_e + 2meC^2 \right)} \cdot \cos\left( \phi - \theta \right) \right]}
\].................................(15)

Discussion of the equation (15) and it's results:

This equation is giving the photon energy after scattering and it is describe the deficiency of photon energy after scattering (the usual Compton effect) and also it is describe the increase of the photon energy after scattering in some cases according with the angles (\( \phi \) and \( \theta \)) (anti Compton effect).

This equation is giving the equation that is giving the photon energy after scattering in usual Compton effect:
\[
h\nu' = \frac{h\nu}{1 + \frac{h\nu}{meC^2} \cdot (1 - \cos\theta)}
\].................................(16)

That is in the special case, when:
\[
\phi = 180\ deg \quad \text{and} \quad T_e = 0
\]
- in the usual Compton effect, the photon is loses the energy for the electron, and the max of loses is occur when the photon scattering at the angle (θ = 180°), the max of energy acquired for the electron is given by the equation:

\[ T'^e_{\text{max}} = \frac{4 \cdot E^2_\gamma}{4 \cdot E_\gamma + 1.022} \] ..........................(17)

From this relation we find that the electron can't take all of photon energy.
- but in the anti Compton effect the opposite of is occur; the electron is loss the energy for the photon, and the max loses of electron energy is:

\[
(T_e - T'^e)_{\text{max}} = \frac{2h\nu\left[\sqrt{T_e(T_e+2meC^2)} - h\nu\right]}{2h\nu + T_e + meC^2 - \sqrt{T_e(T_e+2meC^2)}}
\] ..........................(18)

That is in the following condition:

\[ \phi = 0 \quad \text{and} \quad \theta = 180 \text{ deg} \]

From the equation (18) we find the electron can loses all it's energy, and the scattered photon can take all of electron energy (T'e = 0) only in the next case:

\[ h\nu = \frac{1}{2} \left[\sqrt{T_e(T_e + 2meC^2)} - T_e\right] \] ..........................(19)

That's mean; if we have coming photon which has energy according with the equation (19) on electron which has kinetic energy (T_e) and the following condition:

\[ \phi = 0 \quad \text{and} \quad \theta = 180 \text{ deg} \]

Is materialized, so the electron will loses all of it's energy and give it to the photon.
Example: if the photon is coming which have energy \( h\nu = 2205.27971682437 \text{ eV} \) on the hydrogen atom's electron which have energy \( T_e = 19.2 \text{ eV} \) and the following condition:

\[ \phi = 0 \quad \text{and} \quad \theta = 180 \text{ deg} \]
Is materialized, so this electron will lose all of its energy and then this electron will merge with the proton and give a neutron.

We can abstract the results of the equation (15) as follows:

The anti Compton effect only occurs in the two following cases:

1- when:

\[-45 < \varphi < +45 \text{ deg}\]

And in the all values of the angle \(\theta\) (that is in the back scattering and in the frontal scattering).

2- when:

\[+45 \leq \varphi < +90 \quad \text{and} \quad -90 < \varphi \leq -45\]

And only in the back scattering for the photon, but without the values \(\theta = \pm 90 \text{ deg}\).

So for in the another ranges for the angle \(\varphi\) only the usual Compton effect is occurs.

The meaning of back scattering is: scattering of photon with reversal direction of the coming direction.

The meaning of frontal scattering is: scattering of photon with the same direction of the coming direction.

The next figure is explaining the ranges of anti Compton effect for the angles \(\theta\) and \(\varphi\):

\[+\theta \Leftrightarrow \cos \theta \geq 0 \quad \text{and} \quad -\theta \Leftrightarrow \cos \theta \leq 0\]
As a result of this new effect, we can find in the plasma (example) the following two important phenomena:

1- if the plasma was not perfect ionizing (including some neutral atoms) we can find one of the atom's electron exist in energy band which not exist in the natural cases.
2- we can find some of the photons has large energy bigger than the another medium particle's energy and we can find some of the electrons has large energy bigger than the another medium particle's energy.

But in the resultant; the usual Compton effect will cancel most of anti Compton effect (not all) in the continuous scattering interactions, so this effect (anti Compton) is not noticed in the usual cases, because the usual Compton effect has ranges bigger than (max 3 times) anti Compton's ranges to occur (from the figure (3)).

References:

QUANTUM ELECTRODYNAMICS, Third Edition. : W. Greiner • J. Reinhart