Tuning of a Feedforward Lag-Lead Second-Order Compensator used with a Highly Oscillating Second-order Process

Galal A. Hassaan
Emeritus Professor, Department of Mechanical Design & Production, Faculty of Engineering, Cairo University, Giza, EGYPT
galalhassaan@ymail.com

Abstract
Lag-lead compensators are well known in automatic control engineering. They have 4 parameters to be adjusted (tuned) for proper operation. The frequency response of the control system or the root locus plot are traditionally used to tune the compensator in a lengthy procedure.

A highly oscillating second-order process has a time response to a unit step input of 85.4 % maximum overshoot and about 6 seconds settling time is controlled using a lag-lead compensator (through simulation). The lag-lead compensator is tuned by minimizing the sum of time multiplied by the absolute error (ITAE) of the control system using MATLAB. Three functional constrains are used to control the performance of the lag-lead compensated control system. The result was reducing the process oscillation to 6.926 % overshoot and a 1.413 seconds settling time. The steady-state characteristics of closed-loop control system using the lag-lead compensator were excellent. It is possible reduce the steady-state error to any desired value through one of the compensator parameters.

Keywords: Feedforward Compensators, Lag-lead Second-order Compensator, Compensator Tuning, Control System Performance.

1 Introduction
Lag-lead compensators can improve the performance of linear control systems through the proper tuning of the compensator parameters. There are two schools in designing lag-lead compensators. One of them uses the frequency response specifications of the compensated control system. The other uses its time response specifications. In the present work we follow the research school using the time response specifications. Still the subject is interested to automatic control researchers.

Panda and Padhy (2007) used the genetic algorithm optimization technique to design thyristor controlled series compensator-based controller to enhance the power system stability [6]. Nassirharand (2008) developed an educational software utility for designing linear compensators based on the Youla parameterization technique and an exact model matching criterion [7]. Wang (2009) provided an approach for phase-lead/lag compensators to achieve the desired specifications of gain and phase margins for all-pole stable plant with time-delay [8]. Cao, Watkins and O’Brien (2009) discussed a compensator graphical user interface implemented by MATLAB to enable the user to design a continuous time compensator using the root-locus and Bode plot [9]. Li, Sheng and Chen (2010) derived the impulse response of the distributed order lead-lag compensator using MATLAB and compared with the numerical inverse Laplace Transformation method [10].

Zanasi and Cuoghi (2011) presented three different methods for the synthesis of lead-lag compensator meeting the phase margin and the gain crossover frequency [11]. Nandar (2012) proposed a robot power system stabilizer using genetic algorithm and a first-order lead-lag compensator [12]. Ntogramatzidis, Zanasi and Cuoghi (2012) presented a range of design techniques for the analysis of standard compensators (lead, lag, PID) in terms of the steady-state performance, the stability margins and the crossover frequencies [13]. Hassaan, Al-Gamil and Lashin (2013) used a second-order feedforward lag-lead compensator to control a difficult process consisting of a first-order with integrator process in a unity feedback loop having 67.3 % maximum percentage overshoot and a 12 seconds settling time. Through tuning the compensator, they succeeded to reduce the overshoot to only 2.4 % and the settling time to only 0.65 seconds [14]. Rodriguez, Guzman, Brenenguel and Hagglund (2014) presented a shaping design procedure with rules to obtain optimal feedforward controller for the case when the ideal compensator is not realizable due to right-half plane zeros in the process dynamics [15].
2 Analysis

Process:
The process is a second-order process having the transfer function:

\[ G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]  \hspace{1cm} (1)

The process has a 10 rad/s natural frequency and an 0.05 damping ratio. It has a time response to a unit step input shown in Fig.1 as generated by MATLAB:

![Step response of the process to a unit step input.](image)

The severity of the output oscillation is measured by its maximum percentage overshoot. It has a maximum overshoot of 85.4% and about 6 seconds settling time.

Lag-lead compensator:
A lag-lead compensator has 4 parameters [16]:

- First time constant, \( T_1 \).
- Second time constant, \( T_2 \).
- Time constant parameter, \( b \).
- Gain, \( K_c \) used in controlling the steady state error of the closed loop control system.
The lag-lead compensator has a transfer function $G_c(s)$ given by [16]:

$$
G_c(s) = K_c(1 + T_1's)(1+T_2's) / [(1 + T_1's)(1+T_2's)]
$$

(2)

Where:

$T_1' = (1/b)T_1$

$T_2' = bT_2$

**Control System Transfer Function:**

Assuming that the control system is a unit feedback one, its transfer function with $G_c(s)$ and $G_p(s)$ in its forward path is:

$$
M(s) = (b_0s^2 + b_1s + b_2) / (a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4)
$$

(3)

where:

$b_0 = \omega_n^2K_cT_1T_2$

$b_1 = \omega_n^2K_c(T_1 + T_2)$

$b_2 = \omega_n^2K_c$

$a_0 = T_1'T_2'$

$a_1= T_1'+T_2' + 2\zeta\omega_n T_1'T_2'$

$a_2 = 1 + 2\zeta\omega_n (T_1'+T_2') + \omega_n^2T_1'T_2'+ \omega_n^2K_cT_1T_2$

$a_3 = 2\zeta\omega_n + \omega_n^2(T_1 + T_2) + \omega_n^2K_c(T_1+T_2)$

$a_4 = \omega_n^2 + \omega_n^2K_c$

**System Step Response:**

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response $c(t)$ as function of time [17].

### 3 Compensator Tuning

The sum of time multiplied by the absolute error (ITAE) is used as an objective function, $F$ of the optimization process. Thus:
\[ F = \int t |c(t) - c_{ss}| \, dt \]  

where \( c_{ss} \) = steady state response of the system.

The performance of the control system is controlled using four functional constraints:

(a) The maximum percentage overshoot constraint, \( c_1 \):

\[ c_1 = OS_{max} - OS_{des} \]  

Where \( OS_{des} \) is the desired maximum percentage overshoot of the control system.

(b) The settling time constraint, \( c_2 \):

\[ c_2 = T_s - T_{sdes} \]  

Where \( T_{sdes} \) is the desired settling time of the control system.

The steady-state error constraint, \( c_3 \):

This constraint is written in the form:

\[ c_3 = e_{ss} - e_{ssdes} \]  

where: \( e_{ss} \) is the steady-state error of the control system and \( e_{ssdes} \) is the desired value for the steady state error.

(c) The stability constraint:

The control system is a fourth order one which depending on the transfer function parameters may be unstable. Therefore, a constraint or more is required to be imposed to guarantee a stable system during the optimization process. The constraints for the case study in hand are assigned using Routh-Hurwitz criterion of the system characteristic equation:

- Positive compensator parameters.
- Certain function relating the compensator parameters. This function gives the third functional constraint of the optimization process. That is:

\[ C_4 = a_0a_3 - a_1a_2 \]  

\( c_4 \) has to be positive for a stable control system.

**Parameters Limits:**

- A lower limit of 0.001 is set for the compensator parameters: \( T_1, T_2, b \) and \( K_c \).
- An upper limit of 100 is set for the compensator parameters \( T_1 \) and \( T_2 \) and \( K_c \).
- An upper limit of 10 is set for the compensator parameter \( b \) (this was because with all guessing values of the compensator parameters, \( b \) was less than 1).
4 Tuning Results

The MATLAB command "fmincon" is used to minimize the optimization objective function given by Eq.4 subjected to the functional inequality constraints given by Eqs. 5 through 8 to provide the lag-lead compensator parameters subjected to the limits mentioned in section 6 [18]. The results are as follows:

Controller parameters:

\[ \begin{align*}
T_1 &= 0.332706 \text{ s} \\
T_2 &= 0.394846 \text{ s} \\
b &= 48.57566 \\
K_c &= 100
\end{align*} \]

The time response of the compensated system to a unit step input is shown in Fig.2.

![Fig.2 Step response of the lag-lead compensated system.](image)
Characteristics of the control system using the tuned lag-lead compensator:

- Maximum percentage overshoot: 6.926 %
- Maximum percentage undershoot: 14.700 %
- Settling time: 1.413 s

5 Conclusions

- The suggested tuning technique of lag-lead compensators used with a highly oscillating process is straightforward and simple compared with techniques based on classical parameter identification such as frequency response or root locus plot techniques.
- Through using the proposed tuning technique, it was possible to reduce the maximum percentage overshoot of uncompensated process with unit feedback from 85.4 % to only about 7 %.
- Using the proposed tuning technique, it was possible to reduce the settling time from about 6 seconds to only 1.413 seconds.
- The undershoot was relatively high (14.7 %). In other compensating techniques, it is possible to eliminate completely both overshoot and undershoot.
- The ITAE objective function has given good characteristics compared with other functions tried by the author.

References


Biography

Galal Ali Hassaan

- Professor of System Dynamics and Automatic Control.
- Faculty of Engineering, Cairo University, EGYPT
- Research on Automatic Control, Mechanical Vibrations and Mechanism Synthesis.
- Author of 10’s of research papers.
- Author of a book on Experimental Systems Control.