The relation between CO_2, global temperature and world population

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Abstract
The present study reveals an almost two centuries lasting and surprisingly strong relation between the total world population and the CO_2-concentration in the atmosphere: an increase of 1 billion people has always led to an increase of 20 ppm CO_2 and, resulting from that, an increase of 0.135 °C of the long-term global temperature.
The study also reveals a surprisingly precise sinusoidal variation of the global temperature, superimposed on the long-term trend. This yet mysterious phenomenon explains why the global temperature didn't increase anymore over the last 10 years, notwithstanding the steadily increasing CO_2-concentration.
The scientific results obtained here have been translated into a political message.

Introduction

The conclusions mentioned in the abstract are exclusively based on mathematically determined relationships between the three meant variables, not hindered by any climatic expertise. The only mathematical aid that is applied in this investigation is fitting of mathematical functions on the measurement data using polynomial and curve fitting techniques.

Polynomial fitting

Polynomial fitting means that the function $y = \Sigma_0^k a_i x^i$ is fitted on measurements as a function of x. The variable k is the so-called order of the polynomial, which in principle can be chosen arbitrarily. The mathematical background of the way in which such a function is fitted on the measurements is given in appendix 1.
The properties of a polynomial fitting are such that the higher the order is chosen the more tendencies in the original measurements will be retained, but the more unreliable extrapolations outside the measured area will become. For that reason it is advised not to use this technique for extrapolations, but exclusively in order to show tendencies inside this measured area. These tendencies, due to the random character of the measurements, would otherwise not or hardly be observable. In this article it is only applied to the measurements of the global temperature.

Curve fitting

In this article curve fitting means: the use of 3 measurements points out of a collection of measurement data (here as function of time), in which a clear tendency is visible. Unlike the higher order polynomial fitting described above, these fitted curves can be used for extrapolations to estimate the data prior, or predict it beyond, the measurement window. The 3 measurements are used for the solution of the 3 variables a, b and c in the function $y = c + a*t^b$
Appendix 2 shows the mathematical background for the solution of these variables.

The CO_2-concentration in the atmosphere as function of time

From here on, for simplicity's sake, this variable will be called: CO_2.
The CO_2-measurements used for this investigation are carried out by the Earth System Research Laboratory of the National Oceanic & Atmospheric Administration, abbreviated as NOAA-ESRL
The measurements are called: Monthly Mean Concentrations at the Mauna Loa Observatory, to be down loaded from http://www.esrl.noaa.gov/gmd/ccgg/trends/
For the purpose of this investigation their monthly registrations are transformed to yearly mean values. The measurements have been carried out since 1959.
The units are in Parts Per Million or PPM.
The graphics of these measurements show a very smooth tendency, hardly possessing any random deviations. Very suitable to apply curve fitting.
The outcome based on the measurements in the years 1962, 1986 and 2014 is:

$$y_C(t) = 259.1 + 1.84x10^{-106} * t^{32.65} \text{ (ppm)} \qquad [1]$$

In this function the variable t is the actual year number, which explains the extremely small value of the constant a, in combination with the extremely high value of b.

Based on the fact that the calculated curve shows an excellent fit with the measurements, it is considered justifiable to extrapolate the values back to 1850. This is the year in which the recording of the global temperature, to be considered hereafter, was started. See figure 1.

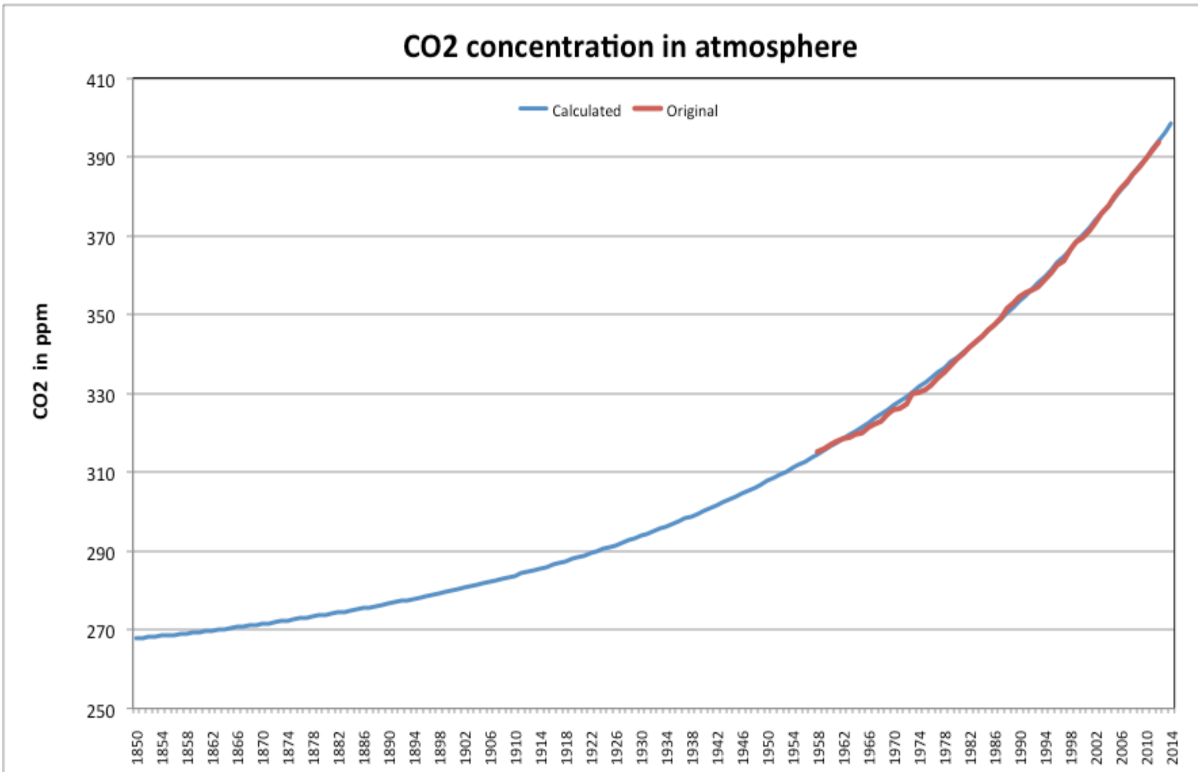

Figure 1: Measured CO_2 since 1959 and fitted curve extrapolated back to 1850

In the next chapter the derivative of this function with respect to time will be used.

$$dy_C(t)/dt = 6.0x10^{-105} * t^{31.65} \qquad [2]$$

The relation between CO_2 and the global temperature.

Short-term-trends
The measurements of the global temperature are downloaded from: http://climate.nasa.gov/vital-signs/global-temperature/ of the Goddard Institute for Space Studies, inside the organisation: National Aeronautics and Space Administration.

These measurements have been fitted to an 8th and 9th order polynomial.
The matrix X, as defined in appendix 1, has the dimension (165 X 10) for the 9th order polynomial.

Considering the mutual rather divergent behaviour, between the 8th and 9th order curves, during the last five years, the mean value of these two polynomials has been calculated too. See figure 2.

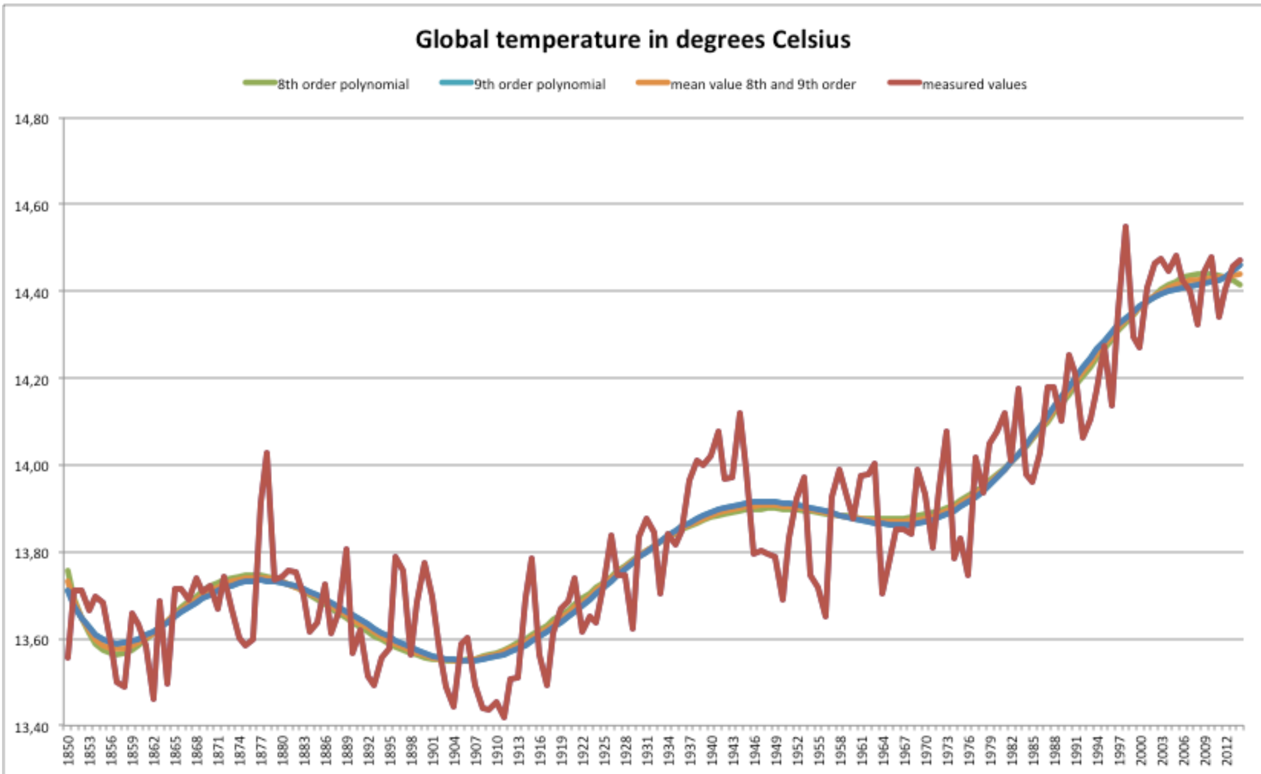

Figure 2: Global temperature, measured and polynomial fitted, as function of time

Over the past 10 years, the temperature of the earth no longer significantly increased (0.03 °C), despite the ongoing increase of the CO_2-concentration, as presented above.
This means that, assuming the green house theory is correct, seemingly other processes determine the temperature of the earth too.
This conclusion is supported by the observation that during the periods 1945-1965, 1875-1905 and ?- 1855 this temperature even decreased notwithstanding the always increasing CO_2-concentration.
So it might even be that the green house theory is not correct, with the consequence that the CO_2-concentration in the atmosphere is not responsible for the increase of the global temperature.

Long-term-trend

As a result of carrying out some exercises with the trend line program of Excel (of which the maximum order is 6) the necessity to also investigate a possible long-term-trend in the global temperature became clear as well.

In first instance a second order polynomial fitting has been carried out. When the trend of this was surprisingly much like that of the CO_2 as calculated with the function $y = c + a*t^b$, this fitting has been applied here too. For this purpose the already calculated mean value of the 8th and 9th order polynomial fitting has been used, because the original observations do not lend themselves for this due to their excessive arbitrary character. The result depends on the three points that will be chosen. For the sets: (1891/13.62), (1957/13.89) and (2014/14.44), being as close as possible to the centre of the mentioned polynomial fitting, the result is: (See figure 3)

$$y_T(t) = 13.50 + 1.80x10^{-102} * t^{30.79} \qquad [3]$$

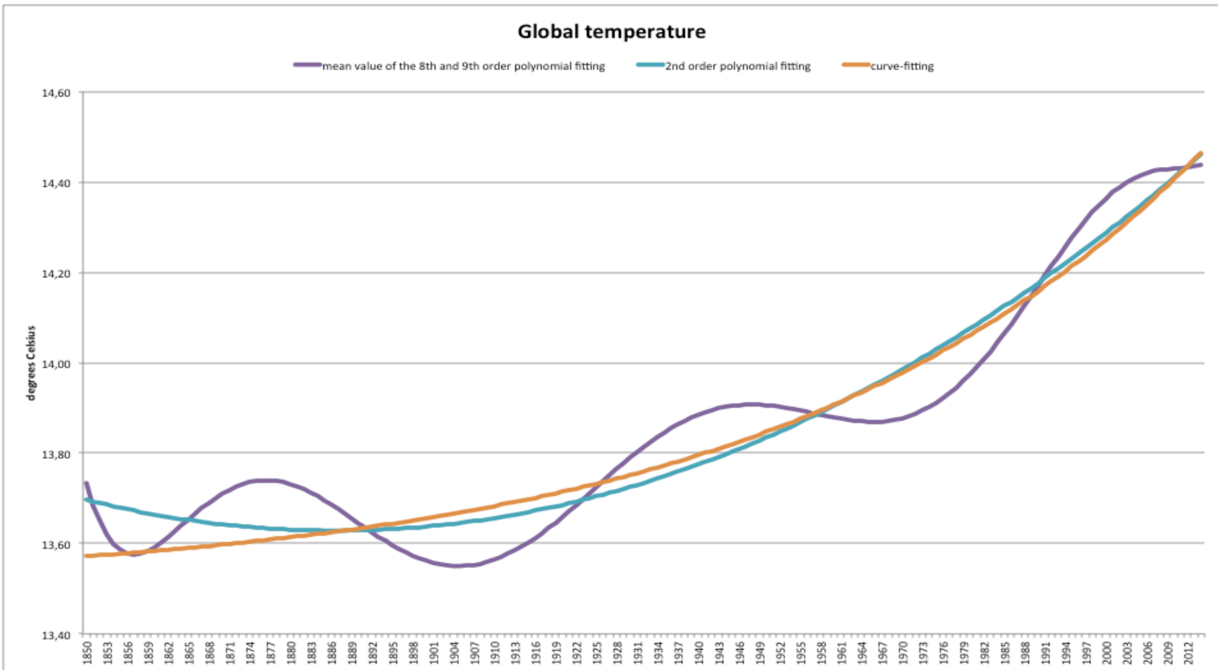

Figure 3: Global temperature, polynomial and curve fitted, as function of time

The derivative of $y_T(t)$ with respect to time is:

$$dy_T(t)/dt = 55x10^{-102} * t^{29.79}$$

This divided by the derivative of the CO_2-curve [2] results in:

$$dy_T(t)/dy_C(t) = 9.22x10^3 * t^{-1.86}$$

The term $t^{-1,86}$ has as *ratio* for the years 1850 and 2014: $(1850/2014)^{-1.86} = 1.17$.

This number is close enough to 1 in order to calculate whether the curve fitting for the temperature, starting from $t^{32.65}$, thus in conformity with the power of t for the CO_2-curve, will lead to an acceptable fitting. Now given the constant b, in this case it is sufficient to take two sets of values out of the previous curve fitting. For example: (1850/13.62) and (2014/14.44)

In this way it is found: $$y_T(t) = 13.51 + 1.22x10^{-108} * t^{32.65} \qquad [4]$$

The mutual differences between the graphics for [3] and [4] are that small that showing the last mentioned one explicitly doesn't make sense. This observation indicates that closely equal graphical curves are obtained with mutual very different terms $a*t^b$!

Given this expression for $y_T(t)$, the following relation can be drawn:

$$\text{global temperature}(t) = 13.51 + 6.65x10^{-3} * \{CO_2(t) - 259.1\} \qquad [5]$$

Important spin-off

If the long-term-trend curve is subtracted from the total curve a very precise periodical curve results! See figure 4. Possibly the deviations in the period 1850-1857 are caused by an error in the observations, like the original graphics already suggests.
The graphics in figure 4 shows 2.5 periods in 160 years. That is exactly 64 years per period.
Interesting stuff for relevant specialists to figure out what might be causing this.
See also: http://adsabs.harvard.edu/abs/2014EGUGA..1611025P
The amplitude of this periodic phenomenon seems to decrease somewhat as a function of time, but the next decades it will certainly be 0.1 °C. With the long-term curve of [4] the zero line of the sinusoidal phenomenon increased a bit with increasing time. In order to obtain a perfectly balanced curve around the zero line the long-term curve has been corrected by only 2‰. Instead of the value 14.439 in 2014 the value 14.465 has been taken.
Doing so the expressions [4] and [5] don't change in the applied number of decimals.

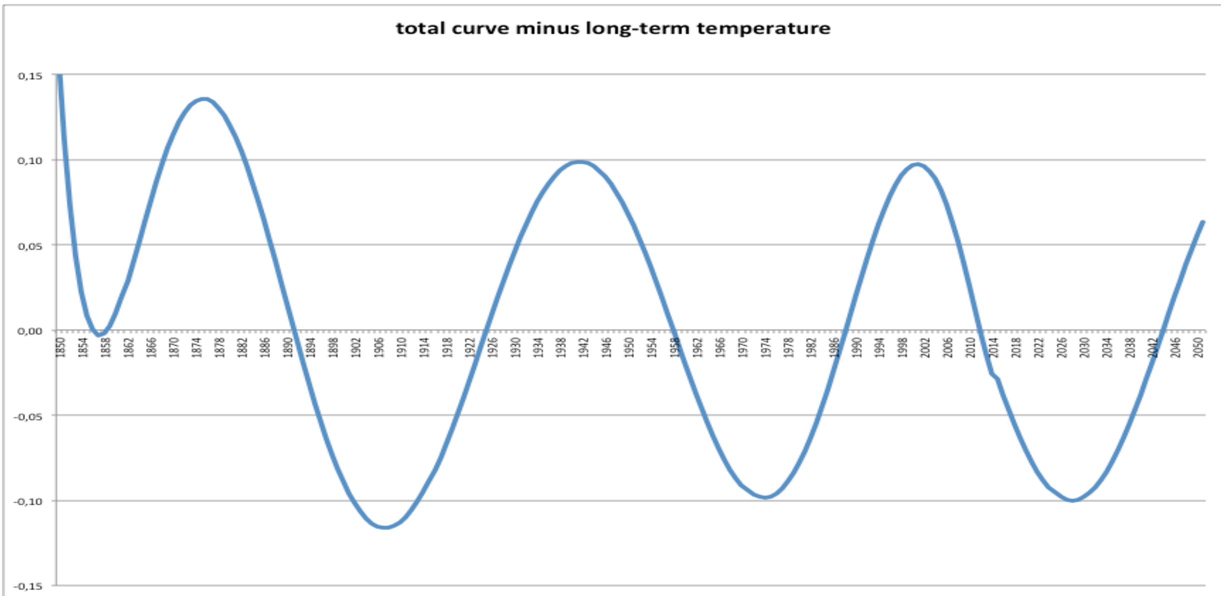

Figure 4: High order polynomial fitted global temperature minus the curve fitted one in °C

With this periodic function, to be written as: A sin{ω(t – 2011)}, with ω= 2π/64, and A=-0.1, it is possible to calculate precisely the temperature for the next decades, *assuming that the CO_2 will keep on growing in the same way as it did up to now.* See Figure 5.

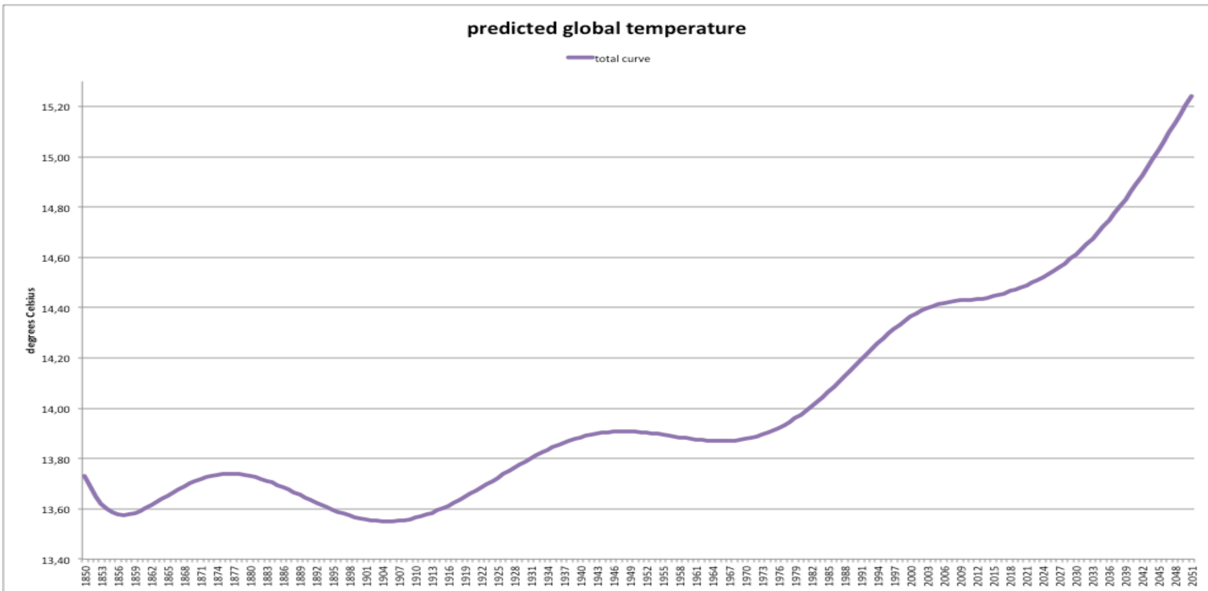

Figure 5: Predicted global temperature until 2050, including periodical variation in ° C

The world population as function of time

There are several sources at the Internet informing about this subject.
The world population as shown in the following link has been taken as the first approximation: https://en.wikipedia.org/wiki/World_population
As the first approximation, because the graphics show such an unnatural character that it is impossible to qualify this as correct:

- -an artificial nod in 1950 as well as in 1925,
- -in between the two nods and from 1925 to 1800 a straight line,
- -regarding the last mentioned line it has to be concluded that, according to this data,

no people at all lived on earth round 1660.
The legend tells that the values from 1950 up to now have been registered, while the other values are estimated. In order to obtain a more credible, which means: as belonging to a natural process, some exercises have been carried out. A difficult additional problem is that there is no indication about the accuracy of the records.
The graphics shown in the same source under: https://en.wikipedia.org/wiki/World_population#/media/File:Human_population_growth_from_1800_to_2000.png is, in the context of the foregoing, very informative!

The experiences with the CO_2 measurements and with the measurements of the global temperature learned that the function $y = c + a*t^b$ lends itself superbly for the fitting of measurements of natural processes, even if it is the prevailing opinion that these processes are caused by human being. Human being is somehow part of our nature. This consideration therefore does not prevent the use of this feature to create a credible curve for the world population as a function of time.

Exercise 1
Drawing a straight line in between the points 1950 and 2014, learns that the graphics, as shown in https://en.wikipedia.org/wiki/World_human_population#/media/File:World-Population-1800-2100.svg deviates at the most 9% downwards from this straight line. The curve, therefore, does not look like one of a natural process. The counting around 1980 is supposedly too high, assumed that the counts in 1950 and 2014 are sufficient accurate regarding this conclusion.

Exercise 2
Laying the function $y = c + a*t^b$ through the points 1950, 1980 and 2014 results in a graphics of which it can be seen already superficially that it will pass the zero line between 1800 and 1900. The reason for this is the same as mentioned under exercise 1

Exercise 3
Fitting the same function trough the points 1950, 2014 and a downwards adapted value at 1880, leads to curves which deviate more at 1980 with respect to the original value the closer the value at 1880 is chosen to the original value. For that reason exercise 4 is carried out.

Exercise 4
In order to keep somewhat closer to the original value at 1980 as well as at 1880, the value at 1950 has been adapted upwards with 10% and the value at 1880 brought to 1.3 billion instead of 1.6. The function then becomes:

$$y_P(t) = 8.1x10^8 + 3.8x10^{-115} * t^{37.6} \qquad [6]$$

The curve, as shown in figure 6, obtained in this way shows a satisfying resemblance with the original curve and at the same time a credible shape.

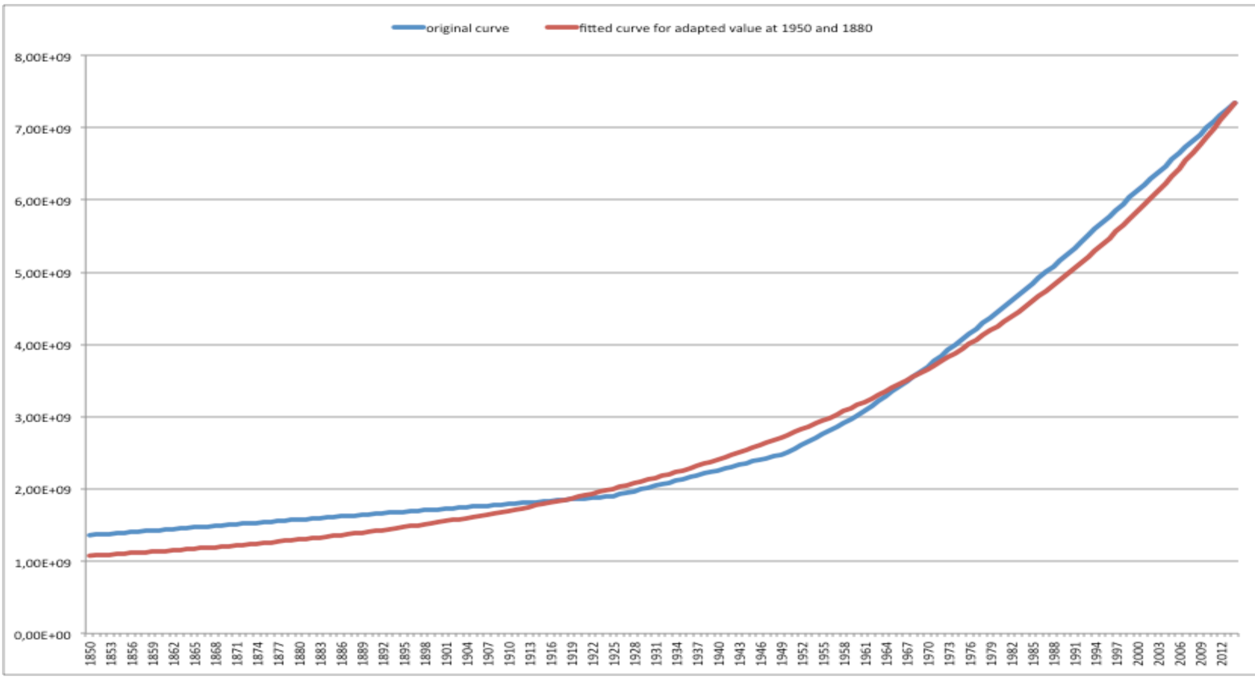

Figure 6: The world population as function of time: counted and adapted to a more realistic curve

Formula [6] will further be investigated in relation to the CO_2 curve. For that purpose the derivative to time of both functions are considered:

$$dy_C(t)/dt = 6.00x10^{-105} * t^{31.6} \qquad [2]$$
$$dy_B(t)/dt = 14.3x10^{-113} * t^{36,6}$$

This leads to: $dy_C(t)/dy_B(t) = 4.2x10^8 * t^{-5}$, being the increase of CO_2 per human.

The function shows a continuously *decreasing* value of ΔCO_2/human.
Formulated in another way: ΔCO_2/human would have been a factor 1.5 larger in 1850 than in 2014. This is so contrary to the prevailing view that, given the correctness of the CO_2 curve, the world population as presented by [6] and in figure 6 must be investigated further.

In order to accommodate as much as possible to this contradiction with the prevailing insight, the power of t in [6] has to be decreased as much as possible.
Indeed, for b=32.65 in [6], ΔCO_2/human would be constant over the whole period 1850-2014.

Exercise 5
In this exercise b=32.65 has been taken as a starting point, calculating the constants a and c directly from the registrations in 1950 and 2014. Doing so, all estimated values have been ignored. For this configuration c, however, is negative. As a result the values at 1950 or 2014 can not be maintained. The most obvious solution is to enhance (afresh) the value at 1950. An increase of 15% is assessed as the most likely and results in closely resembling the same curve as the already well fitting curve in figure 6. Doing so a curve for the world population is realized that fits well with the registrations over the past 50 years and replaces the most unnatural and old part by an overall credible picture. See figure 7.

Precise outcomes are already not important, because it only concerns the finding of credible trends. Together with that it turns out that it does not matter much which power of t is chosen, within the boundaries considered here. And for comparison with the CO2 and temperature variables, it is, from a mathematical point of view, useful to choose this variable equal to 32.65.

The curve then becomes as shown in figure 7 with the function:

$$y_P(t) = 4.8x10^8 + 9.0x10^{-99} * t^{32.65} \qquad [7]$$

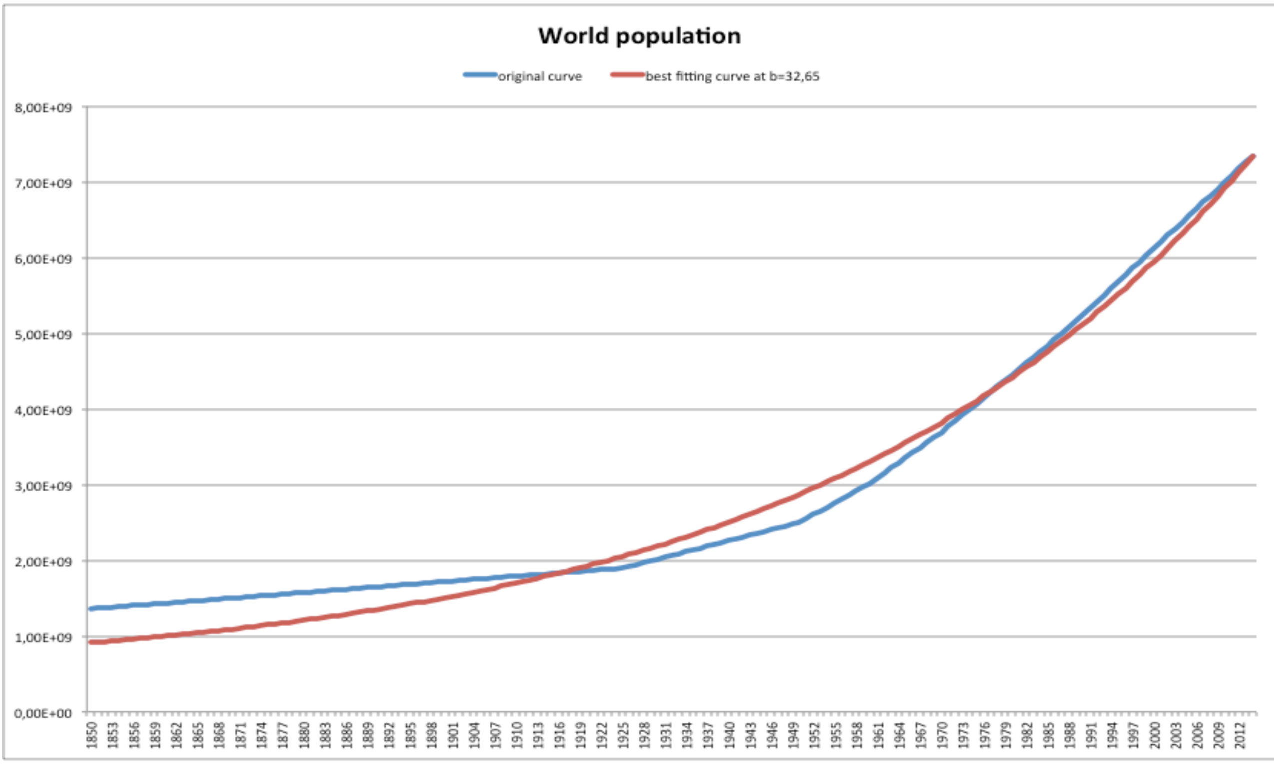

Figure 7: The world population as function of time: counted and adapted to the most likely curve

Mutual relations between the variables

Now that it is clear that the term $t^{32.65}$ can be applied very well for all of the three variables under investigation, their mutual relationship can be drawn mathematically in a very simple way.
The long-term global temperature as function of CO_2 has already be shown by [5]:

$$\text{global temperature}(t) = 13.5 + 6.65\text{x}10^{-3} * \{CO_2(t) - 259\}$$

The world population can be expressed as function of CO_2 by:

$$\text{population}(t) = 0.48\text{x}10^9 + 4.9\text{x}10^7 * \{CO_2(t) - 259\}$$

and as a result:

$$\text{global temperature}(t) = 13{,}5 + 1.35\text{x}10^{-10} * \{\text{population}(t) - 0.48\text{x}10^9\}$$

For each 1 billion humans the global temperature rises 0.135 °C.

Finally:

$$CO_2(t) = 259 + 20\text{x}10^{-9} * \{\text{population}(t) - 0{,}48\text{x}10^9\}$$

For each 1 billion humans the CO_2 concentration rises 20 ppm.

N.B. All functions are only valid for $t \geq 0$.

Closer consideration of the variable 'ΔCO2/human'

In this article it has mathematically been made plausible that over the past 150 years the variable 'ΔCO2/human' surely did not increase. This is, as already mentioned, against all expectations. The question thus is what the background for this phenomenon might be.
Suppose that the total increase of the CO_2 is almost only a result of the total increase of the combustion of fossil fuels, hereafter shortly written as industrialisation. Then it follows that because of the, say for convenience, constant 'ΔCO2/human' the industrialisation per human also has been the same for so long. A possible exploitation for this phenomenon is the following.
Suppose we divide the world population into a poor and a prosperous group.

Suppose that exclusively within the prosperous group the industrialisation per human indeed increases continuously.
Suppose the poor group remained, regarding this industrialisation, at the level of 2 centuries ago. Then their contribution to the CO_2-concentration in the atmosphere remains limited to those caused by their breathing (and heating their shelter, where applicable).
Such a contribution may be considered as negligible w.r.t. the contribution of the prosperous group. However, if the size of the poor group increases, in an absolute sense, much more than that of the prosperous group, it is not unthinkable that, taking the mean value over the whole world, '$\Delta CO2$/human' remains the same. Maybe even decreases. The data on the world's population are not sufficiently reliable for a definitive conclusion.

Social consequences

In case the above mentioned model, no matter how black and white it is presented, is in essence correct, which counter measures are needed to stop the increase of the global temperature?
Such measures would obviously only be effective if they were applied to the prosperous group. That can only be accomplished in three ways: 1) ask/force them to reduce their level of wealth and comfort drastically, or 2) decrease their size significantly, or 3) a combination of both measures. But none of these measures will succeed. Remains that world leaders get convinced of the need to reduce the population worldwide.
The CO_2 curve proves clearly that up till now all other measures did have no effect at all over the past decades!
The already 150 years lasting consistent constant '$\Delta CO2$ / man' therefor compels man to fight the reduction of CO_2 concentration in the atmosphere by means of a reduction of the world population.

For consideration

The increase in global temperature is almost entirely due to the increase in the annual amount of fossil fuels burned mainly by the wealthy part of the world.
The curve of the CO2 concentration in the atmosphere clearly shows that the measures taken in recent decades to reduce this concentration have had no effect at all.

Telling in this context is the result of a recently conducted research, which shows that humans waste more energy as more durable energy is produced.

Another possibility is to lower the level of prosperity and comfort of this part of the world population. Because humans, in general, will not be inclined to give up such a level of prosperity once obtained, the only other political approach left will be a significant reduction of the total world population. Such a measure must eventually be put in motion by humankind to prevent that nature will do it itself. And it will certainly not do this gently!

Based on the evidence shown in this article it is concluded that mankind has primarily an overpopulation problem and that the climate problem is one of the consequences.

Noteworthy is the experience that about 130 Dutch climate scientists have no interest in the results presented in this article. After some moral insistence, through a newspaper article, about 10 scientists have responded. The arguments put forward by them, intended to degrade the value and contents of the article, were of such an irrelevant character that the question arises: what might inspire them to lower themselves to such a level. The author believes that the reason might be sought in the danger that threatens them if the political efforts would be aimed to stop, hopefully even reduce, the size of the world population. In that case climate scientists are threatened by unemployment!
This educated guess is based on a reaction of one of them: "Admittedly, it is indeed sometimes a bit taboo to speak in connection with climate problems about overpopulation." This scientist was also the only one not showing any irrelevant arguments!

Conclusions

1. In this article it has been shown that a clear mutual relation exists between the CO_2 concentration in the atmosphere, the long-term global temperature and the world population. Because of this strong correlation one can calculate one from the other as a function of time as well as extrapolate them for the long term forwards and backwards.

2. It is interesting that for the CO_2 concentration, the long-term global temperature, as well as for the world population, the term $t^{33\pm0.5}$ has been found as a good fitting function for all of these three variables. For the CO_2 curve $t^{32.65}$ is an excellent fitting term.

3. Superimposed on this long-term trend a very precise sinusoidal trend has been found, which allows one, together with the highly predictable long-term trend, to calculate accurately the behaviour of the global temperature for the next decades

4. The under 3 mentioned yet mysterious phenomenon seems to be the reason for the puzzling phenomenon that the global temperature over the last 10 years no longer increases, despite the increase of the CO_2 concentration in the atmosphere.

5. The annual increase of the CO_2 concentration in the atmosphere divided by the annual increase of the world population over the past 1.5 centuries appears to have remained the same, maybe even decreased. The data on the world's population are not sufficient reliable for a definitive conclusion.

6. Assuming that the increase in CO_2 concentration in the atmosphere is almost exclusively a consequence of the increase in the amount of fossil fuels burned, it follows from 5, against all expectations, that the world wide mean amount of fossil fuel burned *per human* has remained the same, or maybe even decreased, over the past 1.5 century.

7. The long-term trend (1850 to present) in the global temperature has an equally tight relationship with the CO_2 concentration in the atmosphere, as the world population has with it.

8. As a result the long-term increase in the global temperature and in the CO_2-concentration in the atmosphere can easily be calculated from the increase in the world population by means of:
for each billion humans the global temperature rises 0.135 °C and the CO_2 concentration 20 ppm.

9. Given the measurements of the CO_2-concentration it is not necessary anymore to count the world population. It simply equals $0.5 + 0.05*(CO_2 - 259)$ ppm, expressed in billions.

10. The same applies for the global temperature: $0.5 + 7.5*(T - 13.5)$ °C, expressed in billions.

Appendix 1

Mathematical background of the polynomial fitting with order k

Given the set measurements y_n as function of the variable x_n.

Requested: the function $y = \Sigma_0^k a_i x^i$, in such a way that the sum of the quadratic deviations between the measurements and y is minimal.

$$R = \Sigma \{y_i - (a_0 + a_1 x_i + \ldots\ldots + a_k x_i^k)\}^2$$

For minimization the following relations have to be fulfilled:

$$\partial R/\partial a_0 = -2\ \Sigma_1^n \{y_i - (a_0 + a_1 x_i + \ldots\ldots + a_k x_i^k)\} = 0$$
$$\partial R/\partial a_1 = -2\ \Sigma_1^n \{y_i - (a_0 + a_1 x_i + \ldots\ldots + a_k x_i^k)\}\ x = 0$$
$$\partial R/\partial a_2 = -2\ \Sigma_1^n \{y_i - (a_0 + a_1 x_i + \ldots\ldots + a_k x_i^k)\}\ x^2 = 0$$
$$.$$
$$.$$
$$\partial R/\partial a_k = -2\ \Sigma_1^n \{y_i - (a_0 + a_1 x_i + \ldots\ldots + a_k x_i^k)\}\ x^k = 0$$

Resulting in the equations:

$$a_0 + a_1 \Sigma_1^n x_i + \ldots + a_k \Sigma_1^n x_i^k = \Sigma_1^n y_i$$
$$a_0 \Sigma_1^n x_i + a_1 \Sigma_1^n x_i^2 + \ldots + a_k \Sigma_1^n x_i^{k+1} = \Sigma_1^n x_i y_i$$
$$.$$
$$.$$
$$a_0 \Sigma_1^n x_i^k + a_1 \Sigma_1^n x_i^{k+1} + \ldots + a_k \Sigma_1^n x_i^{2k} = \Sigma_1^n x_i^k y_i$$

In matrix format:

$$\begin{matrix} n & \Sigma_1^n x_i & \ldots & \Sigma_1^n x_i^k \\ \Sigma_1^n x_i & \Sigma_1^n x_i^2 & \ldots & \Sigma_1^n x_i^{k+1} \\ . & . & & . \\ . & . & & . \\ \Sigma_1^n x_i^k & \Sigma_1^n x_i^{k+1} & \ldots & \Sigma_1^n x_i^{2k} \end{matrix} \quad X \quad \begin{matrix} a_0 \\ a_1 \\ . \\ . \\ a_k \end{matrix} \quad = \quad \begin{matrix} \Sigma_1^n y_i \\ \Sigma_1^n x_i y_i \\ . \\ . \\ \Sigma_1^n x_i^k y_i \end{matrix}$$

This equation can, as follows, be written in a simple matrix notation.

The equation: $y_j = \Sigma_0^k a_i x_j^i$, with j =1 to n, equals the matrix multiplication X.a = y:

$$\begin{matrix} 1 & x_1 & \ldots & x_1^k \\ 1 & x_2 & \ldots & x_2^k \\ . & . & & . \\ . & . & & . \\ 1 & x_n & \ldots & x_n^k \end{matrix} \quad X \quad \begin{matrix} a_0 \\ a_1 \\ . \\ . \\ a_k \end{matrix} \quad = \quad \begin{matrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{matrix}$$

The matrix equation X.a = y can be written as $X^T. X . a = X^T. Y$.

Further elaborated:

$$(X^T. X)^{-1} . X^T. X . a = (X^T. X)^{-1} . X^T . Y$$
$$I.a = a = (X^T. X)^{-1} . X^T . Y$$

with a_i (i = 0 to k) the requested coefficients.

Note:

If the control calculation $(X^T. X). (X^T. X)^{-1} = I$ is executed in Excel, the result strongly departs from that unity matrix I for orders greater than 6. However the 8th and 9th order polynomial seem to be calculated good enough in the current investigation, given their strong mutual agreement.

Appendix 2

Mathematical background of the curve fitting $y=c + a.x^b$

This curve fitting may be applied only to a series of measurements that show already a smooth shape, so with small random deviations.

Given the measuring points: (x_1 , y_1) , (x_2 , y_2) en (x_3 , y_3) the solution of the constant c is as follows:

$y_1\text{-}c= a.x_1^b$ $\quad$ $y_2\text{-}c= a.x_2^b$ $\quad$ $y_3\text{-}c= a.x_3^b$

$(y_1\text{-}c)/(y_2\text{-}c) =x_{12}^b$ $\quad$ with $x_{12}=x_1/x_2$

Take the logarithm on both sides:

$$\log\{(y_1-c)/(y_2-c)\} = b.\log (x_{12})$$

And:

$\log\{(y_2-c)/(y_3-c)\} = b.\log (x_{23})$ $\quad$ with $x_{23}=x_2/x_3$

The quotient of both equations results in:

$$\log\{(y_1-c)/(y_2-c)\} / \log\{(y_2-c)/(y_3-c)\} = \log (x_{12})/ \log (x_{23})$$

c can only be solved numerically by means of an iteration process, applied to the function :

$$\log\{(y_1-c)/(y_2-c)\} / \log\{(y_2-c)/(y_3-c)\} - x_{123} = 0 \qquad \text{with: } x_{123} = \log (x_{12})/\log (x_{23})$$

Having calculated c, b follows from:

$$b=\log\{(y_1-c)/(y_2-c)\}/\log(x_{12})$$

And a from:

$$a=(y_2-c)/x_2^b$$