Study of interacting dark energy model in holographic Kaluza_Klein universe with Garnda-Oliveros cutoff (LGO).

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Abstract:

In this work we study the interacting dark energy (DE) models with Garnda-Oliveros cutoff (LGO) within the work of Kaluza-Klein (KK) universe. We established mathematical formulas for the equation of state parameter $\omega_\Lambda$the decelaration parameter q and the geometrical statefinder parameters r, s. The results show that the universe is in an expansion mode for the model under study.

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I. *Introduction:*

The problems that faces the discovery of DE [1-10] are the fine tuning problem [11] and the cosmic coincidence problem [11]. In this paper we will try describe the properties of dark energy that will deeply explain its problems and give notice about its solutions, we define a holographic Kaluza-Klein universe, where the holographic principle [12, 13] is merged in Kaluza-Klein universe [14], holographic principle will be useful in defining the limit of dark energy density as gives [15, 16], where $L^3\rho_\Lambda \leq L{M_p}^2$, as ρ_Λ energy density of dark energy DE, M_p is the reduce Planck's mass and L the size of universe, this inequality led to a very useful relation $\rho_\Lambda \propto L^{-2}$, the horizon size R in hologram universe could be described through $R_{Hologram} = \int_0^t \frac{dt}{a(t)}$, the function $a(t)$ is also be described in any dimension $a(t) \sim t^{2/d(\omega_\Lambda+1)}$ the DE density is related to the acceleration of universe through $p = \rho_\Lambda\omega_\Lambda$. Also it describe why $\omega_\Lambda < 1$.

This paper is arranged to define and study some cosmological parameters for our model.

II. *Basic equations and KK model:*

According to Kaluza-Klein universe the Friedmann–Lemaître–Robertson–Walke (FLRW) metric [17, 18] for flat space is given by:

$$dS^2 = dt^2 - a(t)^2(dr^2 + r^2(d\theta^2 + \sin\theta^2\, d\varphi^2) + d\psi^2). \tag{1}$$

where r radial component, θ, φ angular coordinates, ψ extra spatial dimension and t is the cosmic time. The KK universe is filled with perfect fluid [17, 18] defined by the following energy-momentum tensor:

$$T_{\mu\gamma} = (P + \rho)U_\mu U_\gamma - g_{\mu\gamma}P, \tag{2}$$

where $\mu, \gamma = 0,1,2,3,4$, $P = P_\Lambda + P_m$the pressure, $\rho = \rho_\Lambda + \rho_m$ density, ρ_m energy density of dark matter (DM), Λ denote dark energy (vacuum energy), m the dark matter (DM) and U_μ, U_γ the five velocity such that $U^\gamma U_\gamma = 1$. The Einstein field equations [17, 18] are given by:

$$R_{\mu\gamma} - \frac{1}{2} g_{\mu\gamma} R = (\kappa) T_{\mu\gamma}, \tag{3}$$

where $R_{\mu\gamma}$ Ricci tensor, R Ricci scalar, $g_{\mu\gamma}$ metric tensor, $T_{\mu\gamma}$ the energy-momentum tensor and κ the coupling constant that taken $\kappa = 1$ according to KK metric, 1st Friedman equation in 5D space [17, 18]:

$$1 + H^2 = \frac{\rho}{6{M_p}^2}. \tag{4}$$

2nd Friedman equation in 5D space [17, 18]:

$$\dot{H} + 2H^2 = -\frac{2p}{6{M_p}^2}. \tag{5}$$

For KK universe we could define $\Omega_\Lambda$fractional energy density of DE and Ω_m fractional energy density of DM, Ω_k fractional energy density for universe curvature and $\rho_c$critical energy density [19]:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{6{M_p}^2 H^2}, \Omega_m = \frac{\rho_m}{6{M_p}^2 H^2}, \Omega_\Lambda = \frac{k}{a^2 H^2} \text{ and } \rho_c = 6H^2{M_p}^2, \tag{6}$$

where H Hubble parameter, which is a unit of measurement used to describe the expansion of universe, k the Curvature parameter describe the in details$(-1,0,1)$, a dimensionless scale factor measuring the expansion of universe, M_p Reduced Planck's mass $\left(\sqrt{\frac{1}{8\pi G}}\right)$, G the gravitational constant from continuity equation [14]:

$$\dot{\rho_\Lambda} + 4H\rho_\Lambda(\omega_\Lambda + 1) = -Q, \tag{7}$$
$$\dot{\rho_m} + 4H\rho_m = Q, \tag{8}$$
$$\rho^{\cdot}_{ordenary\ matter} + 4H\rho_{ordenary\ matter} = \overline{Q}, \tag{9}$$

where Q the energy exchanges term (function describes the interaction between DE and DM), $\overline{Q}$ the energy exchange term (function describes the interaction between ordinary matter with DE and DM) and ω_Λ is the equation of state (it's a dimensionless number describe the flow of perfect fluid).

Multiply Eq.(2) with $\frac{-2\Omega_\Lambda}{4H\rho_\Lambda}$, we get:

$$-\Omega_\Lambda\omega_\Lambda = 2\left[\Omega_\Lambda + \frac{1}{4H}\times\frac{\dot{\rho}_\Lambda}{6{M_p}^2H^2} + \frac{Q}{24{M_p}^2H^3}\right]. \tag{10}$$

The 1st derivative for Ω_Λ is given by:

$$\dot{\Omega}_\Lambda = \frac{d}{dt}\Omega_\Lambda = \frac{d}{dt}\left(\frac{\rho_\Lambda}{6{M_p}^2H^2}\right) = \frac{\dot{\rho}_\Lambda}{6{M_p}^2H^2} - 2\Omega_\Lambda\frac{\dot{H}}{H}, \tag{11}$$

$$\frac{\dot{\rho}_\Lambda}{6{M_p}^2H^2} = \dot{\Omega}_\Lambda + 2\Omega_\Lambda\frac{\dot{H}}{H}. \tag{12}$$

By substituting Eq.(12) in Eq.(10) we get:

$$-2\Omega_\Lambda\omega_\Lambda = 2\left[\Omega_\Lambda + \frac{1}{4H}\times\left[\dot{\Omega}_\Lambda + 2\Omega_\Lambda\frac{\dot{H}}{H}\right] + \frac{Q}{24{M_p}^2H^3}\right]. \tag{13}$$

From 2nd Freidman equation for 5D flat space [17, 18]:

$$\dot{H} + 2H^2 = -\frac{\rho_\Lambda\omega_\Lambda}{3{M_p}^2}. \tag{14}$$

Divided this equation by H^2 gives:

$$\frac{\dot{H}}{H^2} + 2 = -\frac{2\rho_\Lambda\omega_\Lambda}{6{M_p}^2H^2}, \tag{15}$$

$$\frac{\dot{H}}{H^2} + 2 = -2\omega_\Lambda\Omega_\Lambda. \tag{16}$$

From Eq.(16) in Eq.(13), one finds:

$$\frac{\dot{H}}{H^2} + 2 = 2\left[\Omega_\Lambda + \frac{1}{4H}\times\left[\dot{\Omega}_\Lambda + 2\Omega_\Lambda\frac{\dot{H}}{H}\right] + \frac{Q}{24{M_p}^2H^3}\right], \tag{17}$$

$$\frac{\dot{H}}{H^2}(1-\Omega_\Lambda) = -2(1-\Omega_\Lambda) + \frac{\dot{\Omega}_\Lambda}{2H} + 2\frac{Q}{24{M_p}^2H^3}. \tag{18}$$

Multiply Eq.(18) with $\frac{-2H}{1-\Omega_\Lambda}$, we get:

$$-2\frac{\dot{H}}{H} = 4H - \frac{\dot{\Omega}_\Lambda}{1-\Omega_\Lambda} - 4H\frac{Q}{24{M_p}^2H^3(1-\Omega_\Lambda)}. \tag{19}$$

By assuming the interaction term Q can take the form:

$$Q = 24{M_p}^2H^3(1-\Omega_\Lambda)(1+\frac{f(t)}{H}) \tag{20}$$

We introduce $f(t)$ function to study the behavior of Q, from Eq.(20) in Eq.(19), we find:

$$-2\frac{\dot{H}}{H} = 4H - \frac{\dot{\Omega}_\Lambda}{1-\Omega_\Lambda} - 4H(1+\frac{f(t)}{H}) \tag{21}$$

$$-2\frac{\dot{H}}{H} = -4f(t) - \frac{\dot{\Omega}_\Lambda}{1-\Omega_\Lambda} \tag{22}$$

By integrate Eq.(22) according to time t, one finds:

$$-2\int\frac{dH}{H} = \int\frac{d(1-\Omega_\Lambda)}{1-\Omega_\Lambda} + \int[-4f(t)]dt, \tag{23}$$

$$-2\ln H = \ln(1-\Omega_\Lambda) - \ln F, \tag{24}$$

where $\ln F = \int 4f(t)dt$, according to Eq.(24), we get:

$$\Omega_\Lambda = 1 - H^{-2}F(t). \tag{25}$$

From Eq.(16), one finds:

$$\omega_\Lambda = -\frac{\frac{\dot{H}}{H^2}+2}{2\Omega_\Lambda}. \tag{26}$$

From KK model the, DE density define in universe size [20, 21] as:

$$\rho_\Lambda = 3m^2{M_p}^2L^{-2}, \tag{27}$$

where m constant (to reconcile the theoretical results to observed results) and L is the future event horizon (the event horizon of the observable universe is the largest comoving distance from which light emitted now can ever reach the observer in the future). Divided Eq.(27) by $6{M_p}^2H^2$, one finds:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{6{M_p}^2H^2} = \frac{m^2}{2}H^{-2}L^{-2}. \tag{28}$$

III. *Garnda-Oliveros in KK theory:*

Let the future event horizon equal the red shift horizon equals the Garnda-Oliveros in flat space[21] ($L^{-2} = {L_{Go}}^{-1} = \beta\dot{H} + \alpha H^2$as β, α arbitrary constants):

$$\Omega_\Lambda = \frac{m^2}{2}H^{-2}{L_{Go}}^{-1} = \frac{m^2}{2}H^{-2}\left(\beta\dot{H}+\alpha H^2\right) = \frac{m^2}{2}\left(\beta\frac{\dot{H}}{H^2}+\alpha\right). \tag{29}$$

By substituting Eq.(29) in Eq.(25), we get:

$$\frac{m^2}{2}\left(\beta\frac{\dot{H}}{H^2}+\alpha\right)=1-H^{-2}F(t), \tag{30}$$

$$\left(\frac{m^2}{2}\right)^{-1}\left(\beta\dot{H}+\alpha H^2\right)^{-1}(H^2-F(t))=1. \tag{31}$$

Eq.(31) is a 1st order differential equation can be divided into two equations:

$$\left(\frac{m^2}{2}\left(\beta\dot{H}+\alpha H^2\right)\right)^{-1}=H^{-2}X(t)^{-1}\ \ and\ (H^2-F(t))=H^2X(t). \tag{32}$$

The 1st equation can be solved as 1st order differential equation gives:

$$\frac{m^2}{2}\left(\beta\dot{H}+\alpha H^2\right)=H^2X(t). \tag{33}$$

Divided Eq.(33) with H^2, we get:

$$\frac{\dot{H}}{H^2}=-\frac{\alpha}{\beta}+\left(\frac{\beta m^2}{2}\right)^{-1}X(t). \tag{34}$$

By integrate Eq.(34) according to t we get:

$$\int\frac{dH}{H^2}=\int\left(-\frac{\alpha}{\beta}+\left(\frac{\beta m^2}{2}\right)^{-1}X(t)\right)dt. \tag{35}$$

$$H=\left(\int\left[\frac{\alpha}{\beta}-\left(\frac{\beta m^2}{2}\right)^{-1}X(t)\right]dt\right)^{-1} \tag{36}$$

The 2nd equation gives a representation for $F(t)$, that:

$$H^2-F(t)=H^2X(t), \tag{37}$$

$$F(t)=H^2\left(1-X(t)\right). \tag{38}$$

By substitute the present values of $H=H_0$, $\Omega_\Lambda=\Omega_{\Lambda_0}$ and $a=a_0$[22] with:

$$H_0=77.6\left({}^{km}/_{S}\right)/Mpc, \tag{39}$$

$$\Omega_{\Lambda_0}=0.7, \tag{40}$$

$$a_0=1. \tag{41}$$

For the present time t_0, we have ($\Omega_{\Lambda_0}=\frac{\rho_{\Lambda_0}}{6{M_p}^2{H_0}^2}$), then from Eq.(6) we get:

$$\Omega_\Lambda = \Omega_{\Lambda_0} * \frac{\rho_\Lambda}{\rho_{\Lambda_0}} * \frac{{H_0}^2}{H^2}. \tag{42}$$

Same way for Eq.(27), one finds:

$$\frac{\rho_\Lambda}{\rho_{\Lambda_0}} = \frac{{L_0}^2}{L^2}. \tag{43}$$

Using Eq.(43) in Eq.(42), we get:

$$\Omega_\Lambda = \Omega_{\Lambda_0} * \frac{{L_0}^2}{L^2} * \frac{{H_0}^2}{H^2}. \tag{44}$$

By using the concept of future event horizon [20, 21]:

$$L = a(t) \int \frac{dt}{a(t)}, \tag{45}$$

$$\dot{L} = LH + 1, \tag{46}$$

$$LH = \dot{L} - 1. \tag{47}$$

Divided Eq.(47) by $L_0 H_0$, we find:

$$\frac{LH}{L_0 H_0} = \frac{\dot{L} - 1}{L_0 H_0}, \tag{48}$$

$$\frac{L_0 H_0}{LH} = \frac{L_0 H_0}{\dot{L} - 1}. \tag{49}$$

By using the proper distance concept [23, 24]:

$$j(t) = a(t) * j_0 \tag{50}$$

where $j(t)$ the proper distance (Proper distance roughly corresponds to where a distant object would be at a specific moment of cosmological time) at time t and j_0 the proper distance at time t_0, suppose that the proper distance equivalent to the future event horizon.

We can get:

$$L = a(t) * L_0, \tag{51}$$

$$\dot{L} = \dot{a}(t) * L_0. \tag{52}$$

From Eq.(52) in Eq.(49), we get:

$$\frac{L_0 H_0}{LH} = \frac{L_0 H_0}{\dot{a}(t) * L_0 - 1}, \tag{53}$$

$$\frac{L_0H_0}{LH}=\frac{L_0H_0}{\frac{a(t)}{a(t)}\dot{a}(t)*L_0-1}, \tag{54}$$

$$\frac{L_0H_0}{LH}=\frac{L_0H_0}{\frac{\dot{a}(t)}{a(t)}*a(t)*L_0-1}. \tag{55}$$

For $\frac{\dot{a}(t)}{a(t)}=H$, Eq.(55) takes the form:

$$\frac{L_0H_0}{LH}=\frac{L_0H_0}{L_0Ha(t)-1}, \tag{56}$$

By using the particle horizon concept [23, 24], one finds:

$$(1+z)^{\upsilon}=\frac{H_0}{H}=\frac{a_0}{a}, \tag{57}$$

where z is the redshift parameter $1+z=\frac{\lambda_{observed}}{\lambda_{emitted}}$, from Eq.(57) in Eq.(56), we find:

$$\frac{L_0H_0}{LH}=\frac{L_0H_0}{L_0H_0a_0(1+z)^{-2\nu}-1}. \tag{58}$$

Now by applying Eq.(58) in Eq.(44), we get:

$$\Omega_\Lambda=\Omega_{\Lambda 0}*\left(\frac{L_0H_0}{L_0H_0a_0(1+z)^{-2\nu}-1}\right)^2. \tag{59}$$

As from Eq.(42) and Eq.(43):

$$(L_0H_0)^2=\frac{m^2/2}{\Omega_{\Lambda 0}}, \tag{60}$$

By substituting Eq.(60) in Eq.(59), we get:

$$\Omega_\Lambda=m^2/2\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu}-1}\right)^2. \tag{61}$$

By substituting Eq.(61) in Eq.(25), one finds:

$$1 - H^{-2}F(t) = {m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2, \tag{62}$$

$$F(t) = H^2\left[1 - {m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2\right] \tag{63}$$

From Eq.(57) in Eq.(63), we get:

$$F(t) = \left(\frac{H_0}{(1+z)}\right)^2\left[1 - {m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2\right]. \tag{64}$$

By substituting Eq.(63) in Eq.(38), we establish:

$$H^2\big(1 - X(t)\big) = H^2\left[1 - {m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2\right], \tag{65}$$

$$H^2X(t) = H^2\,{m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2, \tag{66}$$

$$X(t) = {m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2 = \Omega_\Lambda, \tag{67}$$

From Eq.(67) in Eq.(34), we get:

$$\frac{\dot{H}}{H^2} = -\frac{\alpha}{\beta} + \frac{1}{\beta}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2. \tag{68}$$

From Eq.(57), we calculate $\frac{\dot{H}}{H^2}$ as:

$$\frac{d}{dt}(1+z)^{\nu} = \frac{d}{dt}\left(\frac{H_0}{H}\right), \tag{69}$$

$$\frac{\dot{H}}{H^2} = -\nu\frac{(1+z)^{\nu-1}z^{\cdot}}{H_0}. \tag{70}$$

From Eq.(70) in Eq.(68), we get:

$$z^{\cdot} = \frac{1}{\nu}(1+z)\left(\frac{\alpha}{\beta} - \frac{1}{\beta}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1}\right)^2\right). \tag{71}$$

We could calculate $\Omega_{\Lambda}{}^{\backslash}$, which is given by:

$$\Omega_{\Lambda}{}^{\backslash} = \frac{d}{d\ln a}(\Omega_{\Lambda}), \tag{72}$$

where $\frac{d}{d\ln a}(\Omega_{\Lambda}) = \frac{dz}{d\ln a}\left(\frac{d}{dz}\Omega_{\Lambda}\right) = \frac{a\dot{z}}{\dot{a}}\left(\frac{d}{dz}\Omega_{\Lambda}\right)$, we get:

$$\Omega_{\Lambda}{}^{\backslash} = \frac{\dot{z}}{H}\left(\frac{d}{dz}\Omega_{\Lambda}\right). \tag{73}$$

By substituting from Eq.(57), Eq.(61) and Eq.(71) in Eq.(72), we get:

$$\Omega_{\Lambda}{}^{\backslash} = 4a_0\sqrt{\frac{\left(m^2/2\right)^3}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu}\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1\right)^{-3}\left(\frac{\alpha}{\beta} - \frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu} - 1\right)^{-2}\right). \tag{74}$$

In Fig.(1) we plot $\Omega_{\Lambda}^{\backslash}$ parameter which is mainly used to study the evolution of universe as function of z for $m = 0.7, 0.9$ and 1.

IV. *Calculations of same cosmological parameters:*

By substituting Eq.(61) and Eq.(68) in Eq.(16), one finds:

$$\omega_{\Lambda} = \frac{\left(2 - \frac{\alpha}{\beta}\right) + \frac{1}{\beta}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}(1+z)^{-2\nu} - 1}}\right)^2}{-2\,{m^2}/{2}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}(1+z)^{-2\nu} - 1}}\right)^2}, \tag{75}$$

$$\omega_{\Lambda} = \frac{1}{m^2}\left(\left(\frac{\alpha}{\beta} - 2\right)\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}(1+z)^{-2\nu} - 1}\right)^2 - \frac{1}{\beta}\right). \tag{76}$$

In Fig.(2), we plot ω_{Λ} as function of z for $m = 0.7, 0.9$ and 1. We observe that $\omega_{\Lambda} < -1$ for all values that indicates our universe according to our model is described by phantom evolution. The deceleration parameter q [25], that defines as a dimensionless measure of the cosmic acceleration of the expansion of space in a Friedmann–Lemaître–Robertson–Walker universe is given by:

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{77}$$

By substituting Eq.(61) in Eq.(77), we get:

$$q = 1 + \frac{\alpha}{\beta} - \frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}(1+z)^{-2\nu} - 1}\right)^{-2}. \tag{78}$$

In Fig.(3), we plot q as function of z for $m = 0.7, 0.9$ and 1. The plot shows, for all the values of m, we took into account the deceleration parameter has an increasing behavior, always staying at negative level.

To study the properties of DE, we introduce the statefinder parameters r, s the statefinder [26, 27] is a geometrical diagnostic and allows us to characterize the properties of dark energy in a model independent manner. The statefinder is dimensionless and is constructed from the scale factor of the Universe and its time derivatives only. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters after the Hubble parameter H and the deceleration parameter q, while s is a linear combination of q and r chosen in such a way that it does not depend upon the dark energy density. The statefinder pair r, s is algebraically related to the equation of state of dark energy and its first time derivative, the state-finder operator r is given by:

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, \tag{79}$$

where:

$$\frac{d}{dt}\left(\frac{\dot{H}}{H^2}\right) = H\left(\frac{\ddot{H}}{H^3} - 2\left(\frac{\dot{H}}{H^2}\right)^2\right), \tag{80}$$

$$\frac{\ddot{H}}{H^3} = \frac{1}{H}\frac{d}{dt}\left(\frac{\dot{H}}{H^2}\right) + 2\left(\frac{\dot{H}}{H^2}\right)^2. \tag{81}$$

From Eq.(68), we could calculate $\frac{d}{dt}\left(\frac{\dot{H}}{H^2}\right)$ as:

$$\frac{d}{dt}\left(\frac{\dot{H}}{H^2}\right) = \frac{4a_0}{\beta}\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}\frac{(1+z)^{-2\nu}\left(\frac{\alpha}{\beta} - \frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}(1+z)^{-2\nu} - 1\right)^{-2}\right)}{\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}(1+z)^{-2\nu} - 1\right)^3} \tag{82}$$

By substituting Eq.(82) in Eq.(81), and find:

$$\frac{\ddot{H}}{H^3} = \frac{4a_0}{H_0\beta}\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}\frac{(1+z)^{-\nu}\left(\frac{\alpha}{\beta} - \frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}(1+z)^{-2\nu} - 1\right)^{-2}\right)}{\left(a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}(1+z)^{-2\nu} - 1\right)^3} + 2\left(-\frac{\alpha}{\beta} + \frac{1}{\beta}\left(\frac{1}{a_0\sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}}(1+z)^{-2\nu} - 1}\right)^2\right)^2. \tag{83}$$

By substituting Eq.(83) and Eq.(68) in Eq.(79), one finds:

$$r = 1 + \left(-\frac{\alpha}{\beta} + \frac{1}{\beta} \left(a_0 \sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}} (1+z)^{-2\nu} - 1 \right)^{-2} \right) \left(3 + \right.$$

$$\frac{4a_0}{H_0 \beta} \sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}} (1+z)^{-\nu} \left(-\frac{\alpha}{\beta} + \frac{1}{\beta} \left(a_0 \sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}} (1+z)^{-2\nu} - 1 \right)^{-2} \right)^{-3} + \tag{84}$$

$$\left. 2 \left(-\frac{\alpha}{\beta} + \frac{1}{\beta} \left(a_0 \sqrt{\frac{m^2/2}{\Omega_{\Lambda_0}}} (1+z)^{-2\nu} - 1 \right)^{-2} \right) \right). \tag{85}$$

Cutting-edge Fig.(4), we plot r as function of z for $m = 0.7, 0.9 \ and\ 1$. The state-finder operator s is given by:

$$s = \frac{3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}}{6\frac{\dot{H}}{H^2} + 9}. \tag{86}$$

By substituting Eq.(79) and Eq.(77) in Eq.(86), we establish:

$$s = -\frac{r-1}{3-6q}. \tag{87}$$

By substituting Eq.(85) and Eq.(78) in Eq.(87), we get:

$$s = -\frac{\left(3+\frac{4a_0}{H_0\beta}\sqrt{\frac{m^2/_2}{\Omega_{\Lambda 0}}}(1+z)^{-\nu}\left(-\frac{\alpha}{\beta}+\frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/_2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu}-1\right)^{-2}\right)^{-3}+2\left(-\frac{\alpha}{\beta}+\frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/_2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu}-1\right)^{-2}\right)\right)}{\left(-3\left(\frac{\alpha}{\beta}-\frac{1}{\beta}\left(a_0\sqrt{\frac{m^2/_2}{\Omega_{\Lambda 0}}}(1+z)^{-2\nu}-1\right)^{2}\right)+6\right)} \tag{88}$$

In Fig.(5), we plot s as function of z for $m = 0.7, 0.9\ and\ 1$. It's easy to find the relation between r and s.

Using Eq.(85) and Eq.(88), we plot r with s for $m = 0.7, 0.9\ and\ 1$, as shown in Fig.(6), from the plot, we notice that the parameters and show some decreasing behavior for the evolution of our model.

V. *Conclusion:*

We investigated the behavior of deceleration parameter "q" the equation of state parameter "ω_Λ", and the statefinder parameters " $r\ and\ s$" for KK universe enclosed by LGO cutoff. We have evaluated the equation of state parameter. It is found that $EoS\ > -1$ four our model, that assumes that our universe has a phantom model [28, 29] behaved for assumed model, Also by studying the evolution of the deceleration with cosmic time we see that the universe has accelerated expansion behavior.

Finally we study the geometrical parameter s and r the interaction of $s - r$ plane have been considered for different values of m as $m = 0.7,0.9\ and\ 1$. These types of parameters are used to investigate the universe expansion scenarios. Also they are used to get the difference between DE and ΛCDE.

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VI. References:

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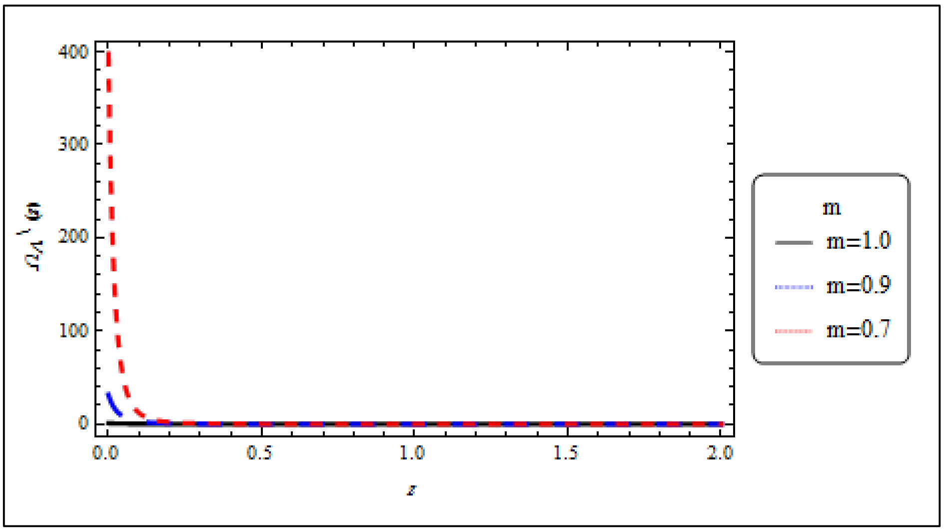

Fig.1 Evaluation of the change in fractal energy density of dark energy (Ω_Λ) according to $\ln(a)$ as a function of redshift parameter (z)

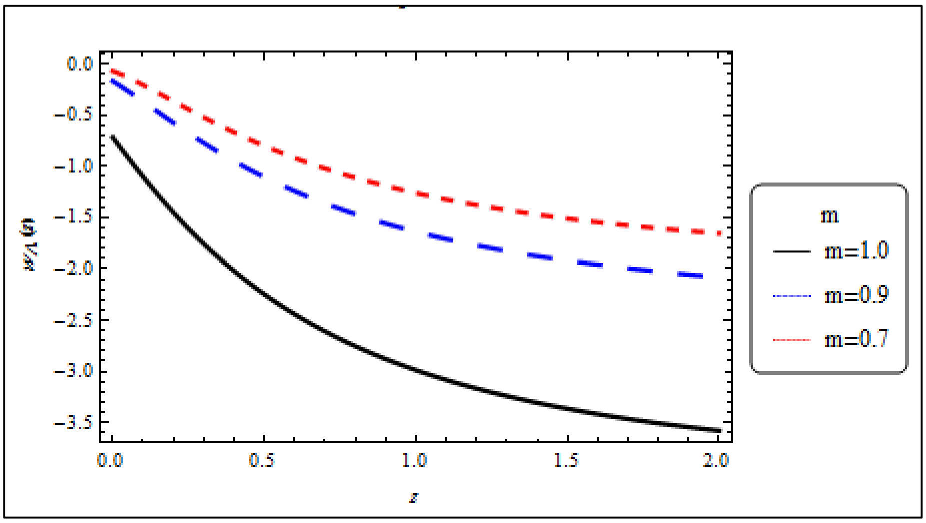

Fig.2 Evaluation of the equation of state (ω_Λ) as a function of redshift parameter (z)

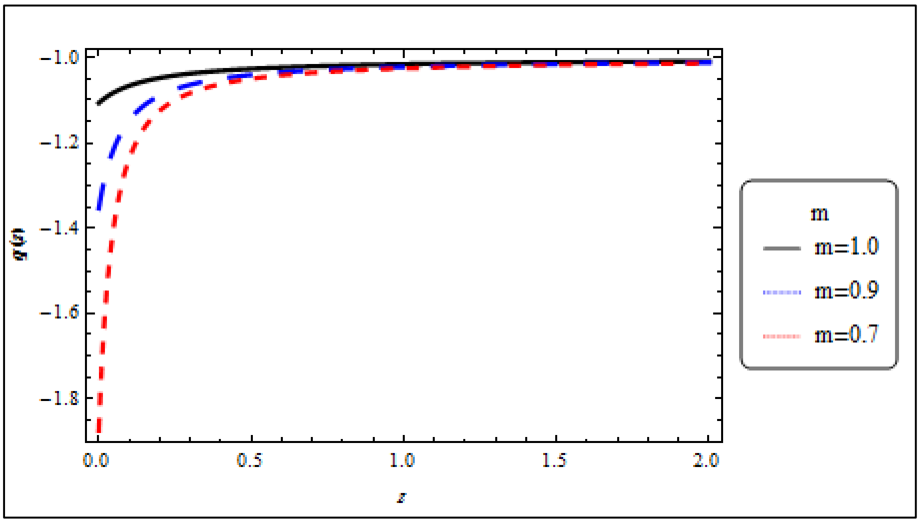

Fig.3 Evaluation of the deceleration parameter (q) as a function of redshift parameter (z)

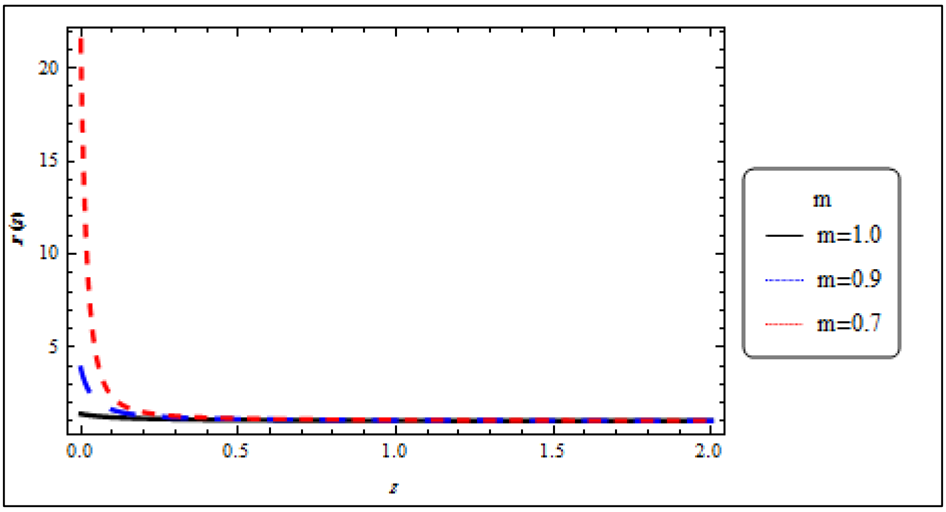

Fig.4 Evaluation of the statefinder parameter (r) as a function of redshift parameter (z)

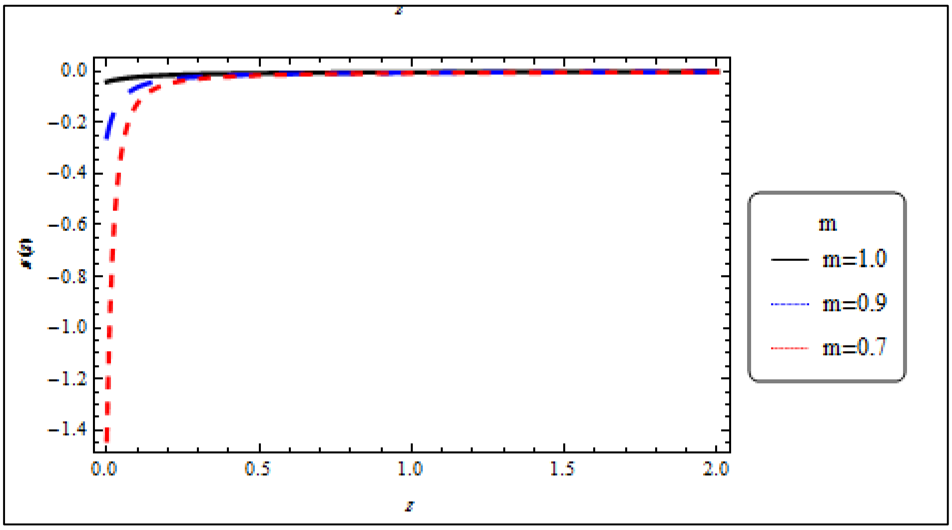

Fig.5 Evaluation of the statefinder parameter (s) as a function of redshift parameter (z)

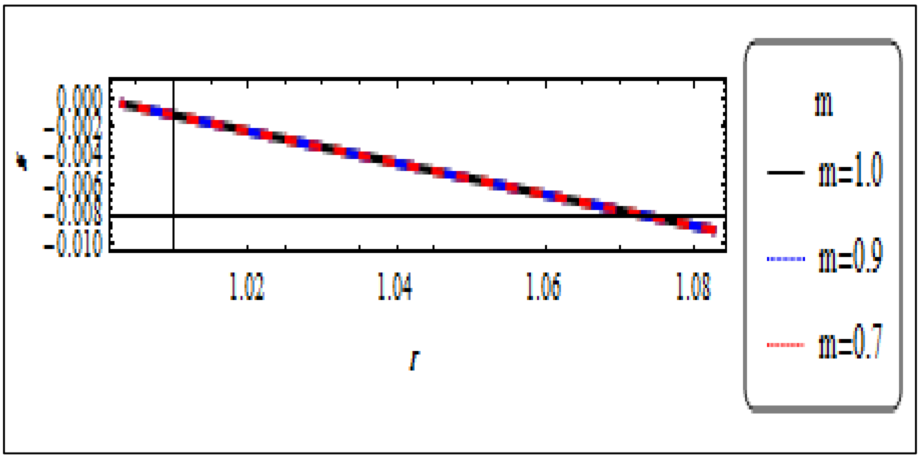

Fig.6 Evaluation of the statefinder parameter (s) as a function of statefinder parameter (r)