An Alternative Approach to Modelling Elementary Particles

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Abstract: We suggest that electrons, positrons and neutrinos are the fundamental building-blocks of the universe. Based on the concept of self-mass, we have constructed models and derived relations between the mass and charge and the mass and radius for each of these particles. This approach constrains the strengths of the electrostatic and gravitational fields at very short distances. We have also developed models for the proton and neutron in which they are composed of these fundamental constituents and their short-distance interactions. With these models we are able to reproduce the internal charge distribution of the proton and the neutron and derive many results that are in good agreement with measurements, including: (i) relations between the proton and neutron masses and the electron mass; (ii) relations between the proton and neutron masses and their radii; (iii) expressions for the magnetic moments of the proton and neutron; (iv) size estimates of the electron, muon and tau; (v) an explanation of the apparent universal matter-antimatter imbalance.

Introduction

In this paper we summarise and supplement the results and predictions of work reported in four earlier papers [1-4] where we introduced a semi-classical model-dependent approach to calculate the properties of several of the elementary particles, including electrons, positrons, protons and neutrons.

The electron, positron and neutrino are all fundamental point-like particles, each composed of an exact balance of electrostatic and gravitational self-energy. The particle mass is its intrinsic self-mass and this implies a very large gravitational parameter at small distances. Protons and neutrons are small, composite particles containing various combinations of the three fundamental constituents. For example, the proton is assumed to be composed of two positrons and an electron in a structure not unlike that of a simple atom. The neutron is similarly assumed to be composed of two positrons, two electrons and a neutrino. Proton and neutron masses are the relativistic effective masses of their constituents. Because protons and neutrons are composite objects containing three or more independent points of mass, our approach is necessarily model-dependent.

We refer to this approach, collectively, as the e-model.

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We have based our proton and neutron models on Bohr's description of the hydrogen atom. This has enabled us to make testable predictions and to derive expressions for a number of observable physical quantities that provide results that are in good agreement with measurement.

In our model there are only two fundamental force-producing fields. These are gravitation and electrostatics and for both we use the classical $1/r^2$ relationships of Newton and Coulomb. In the case of gravity, we assume that the mass of a fundamental constituent particle is its relativistic mass γm ($= E/c^2$) and that the gravitational parameter is very large at short distances inside the elementary particles (where m is the constituent particle rest-mass, E its energy and γ ($= 1/\sqrt{1-v^2/c^2}$) is the relativistic factor, with v the particle speed and c the speed of light *in vacuo*.)

Another assumption is that there are only four conserved quantities. These are energy, linear momentum, electric charge and angular momentum.

Using this approach we are able to derive simple expressions that connect the properties of several particles, including electrons, protons and neutrons. The charge of the proton is, by construction, exactly equal in magnitude to the charge of the electron. We note that charge is always accompanied by mass. In our approach, neutral particles have either zero rest-mass or are composite. The neutrino is described by the same model as the electron and positron, but with zero charge and therefore zero rest-mass.

Our approach gives masses, magnetic moments and internal charge distributions that are all in remarkably good agreement with measurements, and all models predict that the gravitational parameter becomes much larger at distances below ~ 10^{-14} m. It is conceivable that very sensitive experiments would be able to investigate this prediction.

Electron and Positron

It has long tempted physicists to interpret the mass of an electron as its self-mass. Unfortunately, assuming electrostatic self-energy alone, the self-mass is much larger than the measured electron mass for any reasonable value of the electron radius. What is often overlooked is the gravitational self-energy term because it is generally assumed to be too small to have an effect.

In the presence of both gravitation and electrostatics, a sphere of radius R, charge Q, mass m and "bare" mass m_0 has self-energy that may be written [1, 5]:

$$E - E_0 = (m - m_0)c^2 = \frac{Gm^2}{R} - \frac{kQ^2}{R},$$

where G and k ($=1/4\pi\varepsilon_0$) are gravitational and electrostatic parameters. This expression has a finite value when R is very small, even with $R \sim 0$, if:

$$Gm^2 = kQ^2.$$

This equation has three solutions, with $Q = +e$, $-e$ and 0, where e is the fundamental unit of charge given by 1.602 x 10^{-19} C [6].

If we assume that this is a model of the electron and positron when $Q = \pm e$ and $m = m_e$, we can calculate the value of the ratio k_0/G_0 from the measured value of m_e/e [6]:

$$k_0/G_0 = m_e^2/e^2 = 3.24 \text{ x } 10^{-23} \text{ kg}^2/\text{C}^2,$$

where we refer to the short-range values of these parameters as k_0 and G_0.

This value of the ratio gives, by definition, an electron (and positron) mass exactly equal to its self-mass, m_e = 9.11 x 10^{-31} kg (= 0.511 MeV). The electron and the positron have exactly the same mass and equal and opposite charge. They could be two charge states of the same particle or they could be particle and anti-particle.

We note that this model does not address the origin of the electron and positron spin, $h/4\pi$, where h is the Planck constant.

Neutrino

If we set the charge Q to zero in the above relation, this solution of the model suggests a point-like particle with zero rest-mass and presumably the same spin as the electron. It is natural to identify this particle as the neutrino.

In the e-model the only fundamental particles are the electron, the positron and the neutrino. There is only one kind of neutrino. It is its own antiparticle and it is massless. The particles referred to as mu-neutrino and tau-neutrino in the Standard Model are simply excited states of the neutrino. This picture is not inconsistent with data [7].

Proton

If the electron, positron and neutrino are the fundamental particles then a natural extension of the above is to assume that proton and neutron are composed of these fundamental entities. We know that neither protons nor neutrons are point-like particles. Both have an internal distribution of charge that has been measured (see figs. 1 and 2) and for both particles, the internal charge falls to zero by a radius of approximately 3 - 3.5 fm (1 fm = 10^{-15} m) [8]. In addition, scattering experiments have shown that there are point-like objects (scattering centres) inside both proton and neutron [9].

The motion of the particles inside the proton will surely be complex. However a simple assumption for the proton model that allows calculations is that it is a composite sphere containing three fundamental point-like constituents in an orbital structure not unlike that of a simple atom. We assume that these are one electron and two positrons and we have used the measured charge distribution of the proton to investigate the scale of its internal structure [3, 8]. It is a natural consequence of this model that the charge and spin of the proton are exactly equal in magnitude to the charge and spin of the electron.

We start the process using an iterative method to fit the measured internal charge distribution to a sum of three charge distributions. A best fit (shown as the dashed line on fig. 1) is obtained using Breit-Wigner line shapes with the following parameters:

	Orbital Radius (fm)	Width (fm)
electron	0	1.20 ± 0.05
positron at R_1	0.35 ± 0.02	0.97 ± 0.05
positron at R_2	0.47 ± 0.02	0.93 ± 0.05

For both the positron orbits, the orbital speeds (v_1 and v_2) are approximately equal to c (to better than 1 part in 10^6).

Using this model, the magnetic moment of the proton can be estimated from the sum of two positron current loops plus the mass-scaled magnetic moment of the central electron:

$$\mu_p = \mu_e \frac{m_e}{m_p} + \frac{ecR_1}{2} + \frac{ecR_2}{2}$$

With the orbital radii of the positrons from the table above and orbital speeds assumed equal to c, we obtain the following values for these terms:

		J/T	nuclear magnetons
electron	$\mu_e(m_e/m_p)$	-5.06×10^{-27}	-1.0
positron loop at R_1	$ecR_1/2$	$(8.41 \pm 0.48) \times 10^{-27}$	1.66 ± 0.09
positron loop at R_2	$ecR_2/2$	$(11.29 \pm 0.48) \times 10^{-27}$	2.23 ± 0.09
Total		$(14.6 \pm 0.7) \times 10^{-27}$	2.90 ± 0.14

This gives a result in good agreement with the measured value of 2.793 nuclear magnetons [6].

Finally, the effective mass of the three components gives the proton mass (m_p) in terms of the electron mass (m_e):

$$m_p = m_e(1 + \gamma_1 + \gamma_2),$$

where $\gamma_{1,2}$ are the relativistic factors ($= 1/\sqrt{1 - v_{1,2}^2/c^2}$).

The relativistic Bohr quantum conditions [10] for the two orbital electrons give:

$$\gamma_1 m_e c R_1 = \hbar \text{ and } \gamma_2 m_e c R_2 = \hbar .$$

Therefore:

$$m_p = m_e + \frac{\hbar}{cR_1} + \frac{\hbar}{cR_2},$$

where R_1 and R_2 are the radii from the above table and $\hbar = h/2\pi$.

This gives m_p/m_e = 1926 ± 100 and therefore m_p = (1.75 ± 0.09) x 10^{-27} kg = 984 ± 50 MeV, in good agreement with the measured value of m_p = 938.3 MeV [6]. Slightly different values of R_1 and R_2 (well within their experimental uncertainty) give the exact proton mass.

If quarks are the assumed constituents rather than electrons and positrons, this approach works almost as well for the m_p calculation; but it leaves no momentum for the gluons. In addition, using quarks gives poor agreement with the measured magnetic moment.

Neutron

The experimental uncertainties on the neutron internal charge distribution (fig. 2) are quite large, especially around the charge peaks at R_1 ~ 0.3 fm and R_2 ~ 0.9 fm where they are approximately 15 to 20% [4, 8]. In addition, the internal structure of the neutron is perhaps more complex than that of the proton. So we adopt a slightly different approach to investigate the scale of the neutron internal structure.

The neutron charge distribution is obtained by using and comparing proton data and deuteron data. This means that we are modeling a stabilized neutron and not a free neutron, which might have a somewhat different internal arrangement.

Like the proton, the neutron is also a composite particle. In order to allow simple calculations, we assume that the neutron contains two electrons, two positrons and a neutrino in an orbital structure similar to the proton structure. It is therefore a natural consequence of this model that the neutron spin is exactly equal to the proton (and electron and neutrino) spin and the neutron charge is exactly zero.

Again we start the process using an iterative procedure to fit the measured internal charge distribution to a sum of electron and positron charge distributions. There are several good fits with a positron at R_1 in the range ~ 0.28–0.32 fm and an electron at R_2 in the range ~ 0.7–1.1 fm. For both these orbits, the orbital speed is again approximately equal to c (to better than 1 part in 10^5). All fits require one of the electrons and one of the positrons at R = 0 fm.

Using this model, the magnetic moment of the neutron can be estimated from the sum of the two current loops. The mass-scaled magnetic moments of the central electron and positron cancel. The contribution from the neutrino is assumed to be negligible, so:

$$\mu_n = \frac{ecR_1}{2} - \frac{ecR_2}{2}.$$

The effective mass of the 5 constituents gives the neutron mass (m_n) in terms of the electron mass (m_e):

$$m_n = m_e(2 + \gamma_1 + \gamma_2) + \frac{E_\nu}{c^2} = 2m_e + \frac{\hbar}{cR_1} + \frac{\hbar}{cR_2} + \frac{E_\nu}{c^2},$$

where we have again used the relativistic Bohr quantum conditions and R_1 and R_2 are the positron and electron radii. If we assume that the neutrino energy $E_\nu = 0$ and use the measured values for m_n and μ_n [6] these two equations can be solved for R_1 and R_2 giving R_1 near to 0.3 fm and R_2 near to 0.7 fm. It is perhaps worth emphasizing that these values for R_1 and R_2 give values for the neutron mass and magnetic moment in <u>exact</u> agreement with measurement [6]. However, they do not give a satisfactory fit to the neutron internal charge distribution and they leave no energy for the neutrino.

If we fix R_1 = 0.3 fm and vary R_2 and the four widths, we obtain several acceptable fits with various values for the neutrino energy. The fitted charge distribution shown in fig. 2 (dashed curve) represents a compromise using four Breit-Wigner line shapes with the following parameters:

	Radius (fm)	Width (fm)	m.m. term	J/T	n.m.
e^+	0	1.3	$\mu_e(m_e/m_n)$	5.05 x 10^{-27}	1.0
e^-	0	1.3	$-\mu_e(m_e/m_n)$	-5.05 x 10^{-27}	-1.0
e^+ at R_1	0.3	0.4	$ecR_1/2$	7.2 x 10^{-27}	1.4
e^- at R_2	0.75	1.3	$-ecR_2/2$	-18.0 x 10^{-27}	-3.5

In this Table, the second and third columns refer to the fit to the neutron internal charge distribution shown in fig. 2 and the last three columns refer to the magnetic moment calculation (m.m. means magnetic moment and n.m. means nuclear magneton). Experimental uncertainties are typically ~ 5 to 10%.

Ignoring the neutrino term, these values for R_1 and R_2 give m_n = (1.64 ± 0.15) x 10^{-27} kg = 922 ± 90 MeV. This is already in good agreement with the measured value of 939.6 MeV [6]. With a 17.6 MeV neutrino, the formula gives the <u>exact</u> neutron mass. The Bohr quantum condition for the neutrino orbit gives $E_\nu R_\nu = \hbar c$ so the radius of the neutrino orbit is ~ 11 fm.

The magnetic moment of the neutron is the sum of the four terms in the last two columns. This gives μ_n = -(10.8 ± 1.0) x 10^{-27} J/T = -(2.1 ± 0.2) nuclear magnetons, in reasonable agreement with the measured value of -1.91 nuclear magnetons [6].

As in the proton case, it is not clear what is the significance, if any, of Breit-Wigners providing a better fit (i.e. same fit parameters but smaller χ^2) than Gaussians.

If quarks are assumed to be the charged constituents of the neutron rather than electrons and positrons, this method gives poor agreement with the values for the neutron mass and magnetic moment, and it again leaves no momentum for the gluons.

These simple models of the proton and the neutron give good agreement with experimental observations. Both models suggest that it might be possible to produce single protons and neutrons in e^+e^- collisions below the proton-antiproton threshold. This should be tested experimentally.

The Gravitational Parameter

The electron e-model is based on a balanced combination of electrostatic and gravitational self-energy. In order for the model to give the correct electron mass, the ratio of gravitational to electrostatic parameter has to be approximately 10^{43} times greater than its macroscopic value! This can only be interpreted in terms of a much larger gravitational parameter with, perhaps, a smaller electrostatic parameter.

In order to estimate the value of the gravitational parameter (G_0) using the proton and neutron e-models, we need an equation-of-motion in each case.

Since the motion of constituents inside the proton and neutron is likely to be complex, it is difficult to establish an exact equation-of-motion. For the proton case, we have tried several alternatives [1-3]. They all give values of G_0 in the range 10^{28} to 10^{30} Nm^2/kg^2. The neutron is more complicated, but alternatives also give $G_0 \sim 10^{29}$ Nm^2/kg^2 [4].

Here we adopt a simpler, more general approach using a chargeless, orbiting test mass to assess the magnitude of G_0 and R in different cases. It is similar to the method used in astrophysics to determine the mass of an unknown object. For example, the technique is used to determine the mass of the black-hole candidate at the centre of our galaxy. If m is the mass of the particle (proton or neutron, say) and M_{test} the mass of the test particle in an orbit of radius R and orbital speed v, the equation-of-motion of the test particle is given by:

$$\frac{M_{test}v^2}{R} = \frac{G_0 M_{test} m}{R^2}.$$

The minimum value of R is given when $v = c$ and we interpret this as the radius within which all of the particle charge resides:

$$R = \frac{G_0 m}{c^2} \quad \text{or} \quad G_0 = \frac{Rc^2}{m}.$$

This result is independent of the orbiting test particle mass.

If we assume that this value of R gives an estimate of the radius of the particle, this method can be used to determine G_0 if R is known. For example, for proton and neutron, with $R \sim 3.5$ fm, G_0 is $\sim 2.0 \times 10^{29}$ Nm^2/kg^2 for both particles. This is in good agreement with earlier methods using various alternative equations-of-motion [1-4].

Other Nuclei

The idea of the previous section can be used to investigate the size of any elementary particle. If an estimate of the particle radius has been determined experimentally, then this technique allows us to determine the value of G_0. In particular, we can calculate the value of G_0 inside each nucleus.

From the previous section $G_0 = Rc^2/m$, and since R is proportional to $A^{1/3}$ and m is proportional to A (where A is the total number of protons and neutrons in the nucleus), the value of G_0 must slowly decrease as the radius of the particle increases. For example, the value of G_0 inside the uranium nucleus should be a factor $\sim$ 40 less than inside the proton.

We know that $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 for $R > 0.01$ m [6]. With a value for the gravitational parameter almost forty orders of magnitude greater inside the proton, it is conceivable that it is still larger than the macroscopic value on the scale of the atom. This could be investigated experimentally.

Muon and Tau

In the e-model, the muon and the tau are excited states of the electron. Their associated neutrinos are excited states of the fundamental neutrino noted above. It is interesting that this is not inconsistent with the results of the experiment that "discovered" the muon neutrino [7].

Again the test particle idea can be used. If we assume that the value of G_0 obtained for proton and neutron is also a good estimate for the muon (with a slightly smaller value for the tau), we obtain $R \sim 0.4$ fm for the muon and $R \sim 5$ fm for the tau.

This method can also be used to estimate the radius of the electron and the neutrino. Assuming the proton or neutron value of G_0 we obtain $R \sim 2 \times 10^{-18}$ m for the electron and $R \sim 0$ for the neutrino.

It is conceivable that these predicted values of *R* for electron, muon and tau could be tested experimentally. In fact the current controversy regarding the difference in the radius of the proton when measured using muons or electrons [6, 11, 12] might be evidence that the muon has a significantly larger radius than the electron.

The magnetic moment of the muon has been measured very precisely [6] and precise QED calculations have also been made [13]. The two differ slightly by an amount = 3.3 x 10^{-33} J/T = 3.6 x 10^{-10} Bohr magnetons. This is consistent with upper limit measurements of the neutrino magnetic moment [6].

Matter and Antimatter

If the proton is composed of an electron and two positrons, then the particle we refer to as the antiproton is two electrons and a positron, and proton and antiproton have exactly the same mass and exactly equal and opposite charge. If the positron is the antiparticle of the electron then the proton is composed of more antimatter than matter. The hydrogen atom, consisting of a proton and an electron, has an equal amount of matter and antimatter.

On the other hand, perhaps there is no antimatter. Perhaps electron and positron are two charge states of the same particle. The so-called antiproton is simply a negatively charged proton. This approach would necessitate the existence of two distinct types of neutron. They are both composed of two electrons, two positrons and a neutrino, but they have different internal structure. One of them is what we normally call the antineutron.

Conclusion, Discussion and Predictions

The Standard Model of particle physics has become complex and cumbersome. It is, at best, an incomplete and unwieldy description of the sub-atomic environment. We don't intend to review the well-known weaknesses of the Standard Model here; instead, we have stepped back from it and asked ourselves: "What is it we know for sure and how can these facts be used to build a model of the universe without the multiple parameters and arbitrary features of the Standard Model?"

In the e-model described here, the fundamental particles are electrons, positrons and neutrinos. With the assumption that these are the constituents of all other particles, we are able to derive simple relationships for protons and neutrons and calculate physical properties that are in good agreement with experimental data.

The electron and positron are point-like (radius ~ 0) fundamental particles whose masses come from the combination of electrostatic and gravitational self-energies. The self-mass calculation suggests an identity that relates the mass, the charge and the strengths of the gravitational and electrostatic forces. The neutrino is a similar, point-like object with zero charge and therefore zero mass (and zero radius).

The proton is an atom-like structure with two positrons in orbit around an electron. The centripetal force is provided by both electrostatics and gravitation (with gravitation dominating). The neutron is a similar object composed of two electrons, two positrons and a neutrino. In both cases the known internal charge distributions are reproduced in a natural and unforced manner. The proton and neutron masses are given by the effective masses of their constituents. Semi-classical calculations provide numerical estimates of the masses and magnetic moments that compare very well with measured values. In both cases the gravitational field strength at short distances (less than ~ 10^{-14} m) is predicted to be ~ 3 x 10^{39} times greater than the measured, macroscopic value. Folding this into the electron model implies that the electrostatic field strength at very short distances (less than ~ 10^{-18} m) is ~ 1500 times weaker than the macroscopic value.

We have based our calculations on simple assumptions that can be justified experimentally:

- There are only two fundamental fields: gravitation and electromagnetism.

- There are only four conserved quantities: energy, linear momentum, electric charge and angular momentum.

- There are only three fundamental particles: electron, positron and neutrino. All three are point-like particles. All other elementary particles are composite objects made of combinations of electrons, positrons and neutrinos bound by a combination of gravitation and electrostatics. For example, the proton is composed of an electron and two positrons. The neutron is composed of two electrons, two positrons and a neutrino

We emphasize that, because they have never been directly observed in an experiment, there are neither quarks nor gluons in our models. For similar reasons there are no strong, weak or Higgs fields and there are no *ad hoc* quantum numbers (such as baryon number, lepton number, isospin, strangeness, charm, top, bottom, etc.)

Apart from conservation of energy (preventing the electrons and positrons from moving into smaller orbits), it is not clear what mechanism prevents electron-positron annihilation inside the proton and the neutron, but it is presumably similar to the mechanism that prevents electrons from collapsing into the nucleus of an atom.

The masses and charges of the electron, positron, neutrino, proton and neutron are intrinsic properties of the particles. In addition, the observation that the proton charge is exactly equal and opposite to the electron charge is not a mysterious coincidence. It is a natural consequence of the proton model.

Another natural consequence of the proton model is that there is no mysterious matter-antimatter imbalance in the universe. If at some point in time there was an equal number of electrons and positrons in the universe then this fundamental balance must still be present in the universe today. Protons and antiprotons will be formed whenever there is a high-density state of electrons and positrons and when this occurs there will inevitably be a proton-antiproton imbalance. However, when one takes into account all the particles then there is no matter-antimatter imbalance. In this *scenario*, all atoms contain an equal amount of matter and antimatter. An alternative interpretation of our models is that there is no antimatter. Positrons are simply positively charged electrons and antiprotons are simply negatively charged protons. Even so, in this paper we continue to use the word antiproton rather than negative proton.

Finally we note that our model makes several predictions that might conceivably be experimentally accessible. These include:

- The gravitational parameter G has a new value G_0 that is predicted to be very large (~ 40 orders of magnitude larger than the macroscopic value of G) for distances R less than ~ 10^{-14} m [14]. The electrostatic field strength is predicted to be ~ 1500 times weaker below R ~ 10^{-18} m.

- The proton is predicted to have three charged internal scattering centres; one is at rest, two are relativistic. The neutron is predicted to have four; two at rest and two relativistic.

- Protons and antiprotons are composed of electrons and positrons. It is possible that a well-designed experiment would be able to demonstrate the production of single protons and antiprotons. Operating an electron-positron collider at an energy below the proton-antiproton threshold would be capable of providing unambiguous signatures of these reactions.

- The electron radius is predicted to be ~ 2 x 10^{-18} m, the muon radius is predicted to be ~ 0.4 fm, the tau radius ~ 5 fm.

- The neutrino has zero mass, zero charge, zero radius and non-zero spin. However, we cannot exclude the possibility of a neutrino with very small mass, charge, radius and magnetic moment.

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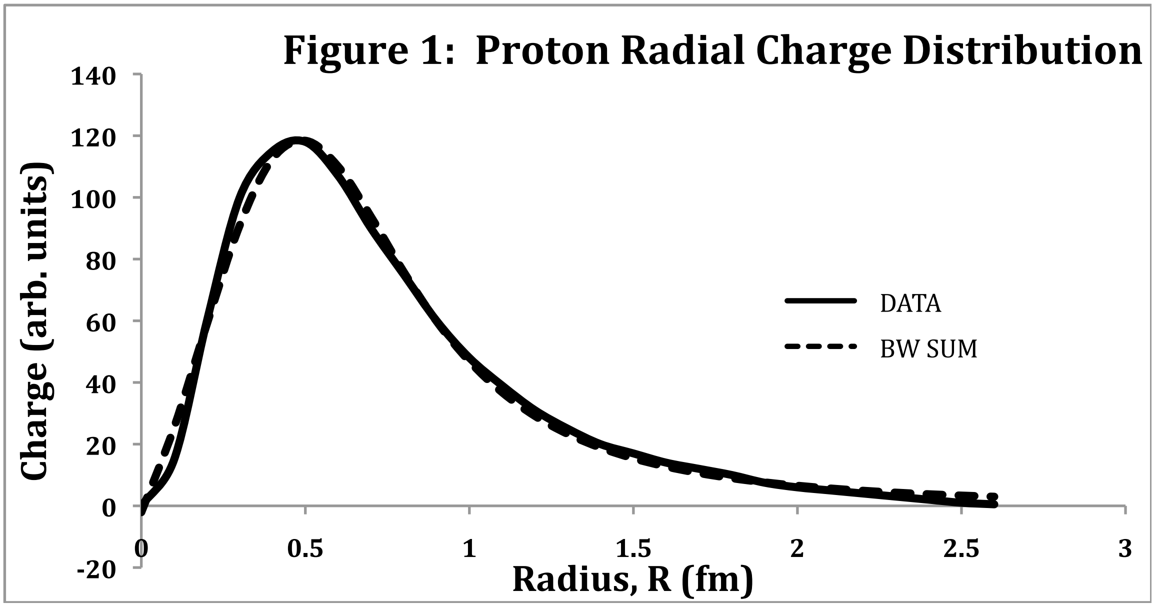

Figure 1

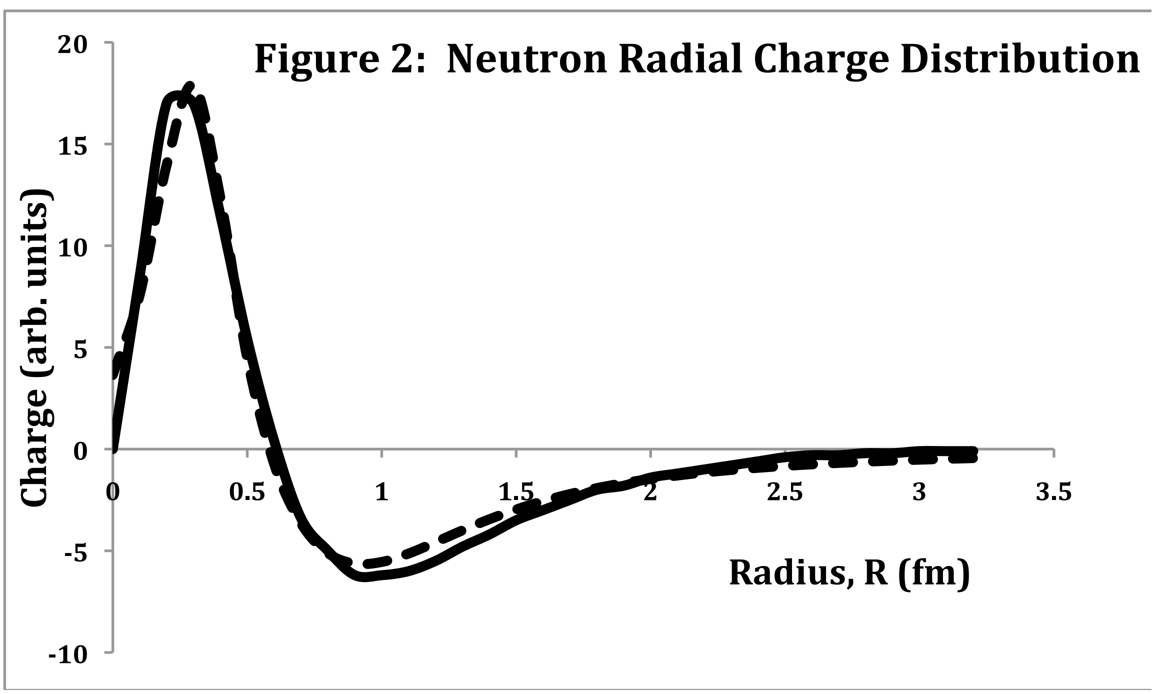

Figure 2

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