

Ilija Barukčić - Causation And The Law Of Independence.

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# CAUSATION AND THE LAW OF INDEPENDENCE.

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## Abstract

**Titans** like Bertrand **Russell** or Karl **Pearson** warned us to keep our mathematical and statistical hands off causality and at the end David Hume too. **Hume's** scepticism has dominated discussion of causality in both analytic philosophy and statistical analysis for a long time. But more and more researchers are working hard on this field and trying to get rid of this positions. In so far, much of the recent philosophical or mathematical writing on causation ( Ellery Eells (1991), Daniel Hausman (1998), Pearl (2000), Peter Spirtes, Clark Glymour and Richard Scheines (2000), ... ) either addresses to Bayes networks, to the counterfactual approach to causality developed in detail by David Lewis, to Reichenbach's Principle of the Common Cause or to the Causal Markov Condition. None of this approaches to causation investigated the relationship between causation and the law of

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independence to a necessary extent. Nonetheless, the relationship between causation and the law of independence, one of the fundamental concepts in probability theory, is very important. May an effect occur in the absence of a cause? May an effect fail to occur in the presence of a cause? In so far, what does constitute the causal relation? On the other hand, if it is unclear what does constitute the causal relation, maybe we can answer the question, *what does not constitute the causal relation*. So far, a cause as such can not be independent from its effect and vice versa, if there is a deterministic causal relationship. This publication will prove, that the law of independence defines causation to some extent **ex negativo**.

## Introduction

Attempts to analyse the relationship between cause and effect in terms of probability theory are based on the fact that causes can raise (Patrick Suppes (1970) ) or lower ( Germund Hesslow (1976) ) the probabilities of their effects. Probabilistic theories of causation offer a potential advantage over regularity theories (especially John Stuart Mill (1843), John Mackie (1974) ). It is a remarkable fact that probabilistic approaches to causation are compatible with indeterminism.

## Methods

According to David Hume, causes are followed by their effects. But, there are a number of well-known difficulties with this position. Before preceding to the formal proof of the relationship between causation and the law of independence it can be helpful to shaken our faith in the position, that the asymmetry of causation ( Hausman (1998) ) is based on the temporal asymmetry between cause and

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effect. For example, the day is followed by the night. But it is widely accepted that the day is therefore not the cause of the night (post hoc, ergo propter hoc). If the cause as such happens only before the effect, this rules out that the cause can happen after its effect. Thus, if causes precede their effects in time then it seems plausible, however, that there is no causation at all. The definition of causation in terms of temporal asymmetry has a number of disadvantages. Hence, there is no valid proof, that the asymmetry between cause and effect is based on the temporal asymmetry, it is not proved yet, that the cause must precede the effect in time. Causal direction is not identical with temporal direction.

## Results

Causal investigation of the world around us using the tools of probability theory is often based on random variables. For a variety of reasons this is our starting point too. It is common to distinguish “the cause” as such and “a cause” ( Mill (1843) ). The first difficulty is to define, what is the cause, what is the effect. It is a remarkable fact that there are various, usually imprecise definitions of cause (f. e. Aristotle's doctrine of the four causes) and effect. In order to avoid certain major errors of definition, let us just talk about the cause or about the effect.

*Theorem. The determination of the effect by the cause and vice versa.*

Let us perform a thought experiment. Let  $C_t$  denote the cause, a random variable at the (space) time  $t$ . Let  $E(C_t)$  denote the expectation value of the cause at the (space) time  $t$ . Let  $E(C_t) \neq 0$ . Let  $E_t$  denote the effect, a random variable at the (space) time  $t$ . Let  $E(E_t)$  denote the expectation value of the effect at the ( space ) time  $t$ . Let  $E(C_t, E_t)$  denote the expectation value of cause and effect at the

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(space) time  $t$ . Let  $\sigma( C_t , E_t )$  or  $\text{Cov}( C_t , E_t )$  denote the covariance of cause and effect at the (space) time  $t$ . Then, according to the law of independence, one of the fundamental concepts in probability theory, the cause has nothing to do with the effect, or the effect is not determined by the cause, or cause and effect are absolutely independent from each other or there is no causal relationship if

$$\sigma( C_t , E_t ) = \text{Cov}( C_t , E_t ) = E( C_t , E_t ) - ( E( C_t ) * E( E_t ) ) = 0.$$

**Proof.**

$$+ E_t = + E_t$$

The starting point of our proof is the **identity** of  $+E_t = +E_t$  (Hegel 1998, p. 411).  $E_t$  is only itself, simple equality with itself, it is only self-related and unrelated to an other, it is distinct from any relation to an other,  $E_t$  contains nothing other but only itself. In this way, there does not appear to be any relation to an other, any relation to an other is removed, any relation to an other has vanished. Consequently,  $E_t$  is just itself and thus somehow the absence of any other determination.  $E_t$  is in its own self only itself and nothing else. In this sense,  $E_t$  is identical only with itself,  $E_t$  is thus just the 'pure'  $E_t$ . Let us consider this in more detail,  $E_t$  is not the transition into its opposite, the negative of  $E_t$  is not as necessary as the  $E_t$  itself,  $E_t$  is not confronted by its other.  $E_t$  is without any opposition or contradiction, is not against an other, is not opposed to an other, is identical with itself and has passed over into pure equality with itself.

But lastly, although identity and difference are somehow different, identity is not difference, identity is in its own self different. Thus,  $E_t$  immediately negates itself.  $E_t$  is at the same time in its self-sameness

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different from itself and thus self-contradictory. Since  $E_t = E_t$  it excludes at the same time the other out of itself, it is  $E_t$  and it is nothing else, it is at the same time  $\text{not-}E_t$ ,  $E_t$  is thus non-being as non-being of its other. In excluding its own other it is excluding itself in its own self. By excluding its other,  $E_t$  makes itself into the other of what it excludes from itself, or  $E_t$  makes itself into its own opposite,  $E_t$  is thus simply the transition of itself into its opposite.  $E_t$  is therefore alive only in so far as it contains such a contradiction within itself.

The non-being of its other is at the end the sublation of its other. This non-being is the non-being of itself, a non-being which has its non-being in its own self and not in another, each contains thus a reference to its other.  $\text{Not-}E_t$  is the pure other of  $E_t$ . But at the same time,  $\text{not-}E_t$  only shows itself in order to vanish, the other of  $E_t$  is not.  $E_t$  and  $\text{not-}E_t$  are distinguished and at the same time both are related to one and the same  $E_t$ , each is that what it is as distinct from its own other. Identity is thus to some extent at the same time the vanishing of otherness.  $E_t$  is itself and its other,  $E_t$  has its determinateness not in an other, but in its own self.  $E_t$  is thus self-referred and the reference to its other is only a self-reference. On closer examination  $E_t$  therefore is, only in so far as its  $\text{Not-}E_t$  is,  $E_t$  has within itself a relation to its other. In other words,  $E_t$  is in its own self at the same time different from something else or  $E_t$  is something. It is widely accepted that something is different from nothing, thus while  $E_t = E_t$  it is at the same time different from nothing or from **non** -  $E_t$ . From this it is evident, that the other side of the identity  $E_t = E_t$  is the fact, that  $E_t$  cannot at the same time be  $E_t$  and not  $E_t$ . In fact, if  $E_t = E_t$  then  $E_t$  is not at the same time non  $E_t$ ! What emerges from this consideration is, therefore, even if  $E_t = E_t$  it is a self-contained opposition.  $E_t$  is only in so far as  $E_t$  contains this

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contradiction within it,  $E_t$  is inherently self-contradictory.  $E_t$  is thus only as the other of the other. In so far,  $E_t$  includes within its own self its own non-being, a relation to something else different from its own self. Thus,  $E_t$  is at the same time the unity of identity with difference.  $E_t$  is itself and at the same time its other too,  $E_t$  is thus contradiction. Difference as such imply contradiction because it unites sides which are, only in so far as they are at the same time not the same.  $E_t$  is only in so far as the other of  $E_t$ , the non-  $E_t$  is.  $E_t$  is thus that what it is only through the other, through the non-  $E_t$ , through the non-being of itself. Thus we obtain

$$+E_t - E_t = 0.$$

$+E_t$  and  $-E_t$  are negatively related to one another and both are indifferent to one another,  $E_t$  is separated in the same relation.  $E_t$  is itself and its other, it is self-referred, its reference to its other is thus a reference to itself, its non-being is thus only a moment in it.  $E_t$  is in its own self the opposite of itself, it has within itself the relation to its other, it is a simple and self-related negativity. Each of them are determined against the other, the other is in and for itself and not as the other of an other.  $E_t$  is in its own self the negativity of itself.  $E_t$  therefore is, only in so far as its non-being is and vice versa. Non -  $E_t$  therefore is, only in so far as its non-being is, both are through the non-being of its other, both as opposites cancel one another in their combination.

Further, the identity of  $E_t = E_t$  is an identity over time. Time as such involves in a very general way something like an alteration.  $E_t$  undergoes alteration, it goes outside itself. In general, any alteration of  $E_t$ , the effect, raises subtle problems. How can the effect remain the same and yet change? If  $E_t$  changes, must there be a cause for this

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change or is an uncaused change possible? Is it extremely implausible to deny caused change? Thus, if  $E_t = E_t$  and if  $E_t$  changes too, then  $E_t$  must at the same time at least be non-identical to itself. In so far,  $E_t$  must include a difference within itself or to say it more mathematically, there must be an expectation value of  $E_t$ . According to Kolmogorov it holds true that **"If  $x$  and  $y$  are equivalent then  $E(x) = E(y)$ ."** (Kolmogorov 1956, p. 39). Thus we get the next equation.

$$E(E_t) = E(E_t).$$

If  $E_t = E_t$  then  $E(E_t) = E(E_t)$ . This does not mean that it must hold true that  $E_t = E(E_t)$ ! If it is only that  $E_t = E_t$ , how can an advance to something different be made? Let us suppose, that  $E_t$  is not alone. In other words, it is true that

$$E(E_t) * 1 = E(E_t).$$

Let  **$E(C_t) = E(C_t)$** . Let  $E(C_t) \neq 0$ , thus  **$E(C_t)/E(C_t) = 1$** . It is  $E(E_t) = E(E_t)$  and  $E(C_t) = E(C_t)$  but both are not one. The self-identity of both is thus the indifference of each towards the other which is distinguished from it. In the same relation, both are rigidly held as separated. Both have a separate existence and are without any relation to an other. In this case, a cause has no relation to an effect, nothing changes by the cause, effect  $E_t$  is like it is, thus we obtain

$$E(E_t) * (E(C_t) / E(C_t)) = E(E_t)$$

or

$$E(E_t) * E(C_t) = E(E_t) * E(C_t)$$

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Each of both stands isolated from each other, is separated from each other, each is only on its own. By this separation of one from the other, both are related not to one another, each is valid on its own and without any respect to an other. In so far, according to Kolmogorov, it is " $E(\mathbf{X} \mathbf{Y}) = \dots = E(\mathbf{X} E(\mathbf{Y})) = E(\mathbf{X}) * E(\mathbf{Y})$ " (Kolmogorov 1956, p. 60). Thus we obtain

$$E(C_t, E_t) = E(C_t) * E(E_t)$$

or

$$E(C_t, E_t) - E(C_t) * E(E_t) = 0$$

However, in general,

if there is

**no causal relationship**

between random variables,

then it is true that

$$\sigma(C_t, E_t) = \text{Cov}(C_t, E_t) = E(C_t, E_t) - (E(C_t) * E(E_t)) = 0.$$

**Q. e. d.**



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## Conclusions

If the cause is isolated from the effect and vice versa, if both are separated from each other, if each is only itself and without any relation to an other, if cause and effect are absolutely independent from each other or if the cause and the effect are **not determined** by each other (Barukčić 2006, p. 44 ), then it is true that

$$\sigma(C_t, E_t) = \text{Cov}(C_t, E_t) = E(C_t, E_t) - (E(C_t) * E(E_t)) = 0.$$

If  $\sigma(C_t, E_t) \neq 0$ , this does not proof at once that there is a causal relationships between investigated random variables. We can only state that there is no causal relationship, if  $\sigma(C_t, E_t) = 0$ . In so far, the law of independence, one of the fundamental laws in nature, statistics and probability theory is valid for the relationship between cause and effect. If the effect at the same time is absolutely independent from the cause and vice versa, if the cause at the same time has nothing to do with the effect, if there isn't a constant and deterministic relation between cause and effect, then it holds true that

$$\sigma(C_t, E_t) = \text{Cov}(C_t, E_t) = E(C_t, E_t) - (E(C_t) * E(E_t)) = 0.$$

Under this circumstances a cause can not produces an effect. Independence is thus the other of causation and at the same time an absolutely necessary part of causation ( Barukcic 2006, p. 44 ) too, independence defines causation to some extent **ex negativo**.

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