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CAUSATION AND THE LAW OF INDEPENDENCE.

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Abstract

Titans like Bertrand **Russell** or Karl **Pearson** warned us to keep our mathematical and statistical hands off causality and at the end David Hume too. **Hume**'s scepticism has dominated discussion of causality in both analytic philosophy and statistical analysis for a long time. But more and more researchers are working hard on this field and trying to get rid of this positions. In so far, much of the recent philosophical or mathematical writing on causation (Ellery Eells (1991), Daniel Hausman (1998), Pearl (2000), Peter Spirtes, Clark Glymour and Richard Scheines (2000), ...) either addresses to Bayes networks, to the counterfactual approach to causality developed in detail by David Lewis, to Reichenbach's Principle of the Common Cause or to the Causal Markov Condition. None of this approaches to causation investigated the relationship between causation and the law of

independence to a necessary extent. Nonetheless, the relationship between causation and the law of independence, one of the fundamental concepts in probability theory, is very important. May an effect occur in the absence of a cause? May an effect fail to occur in the presence of a cause? In so far, what does constitute the causal relation? On the other hand, if it is unclear what does constitute the causal relation, maybe we can answer the question, *what does not constitute the causal relation*. So far, a cause as such can not be independent from its effect and vice versa, if there is a deterministic causal relationship. This publication will prove, that the law of independence defines causation to some extent **ex negativo**.

Introduction

Attempts to analyse the relationship between cause and effect in terms of probability theory are based on the fact that causes can raise (Patrick Suppes (1970)) or lower (Germund Hesslow (1976)) the probabilities of their effects. Probabilistic theories of causation offer a potential advantage over regularity theories (especially John Stuart Mill (1843), John Mackie (1974)). It is a remarkable fact that probabilistic approaches to causation are compatible with indeterminism.

Methods

According to David Hume, causes are followed by their effects. But, there are a number of well-known difficulties with this position. Before preceding to the formal proof of the relationship between causation and the law of independence it can be helpful to shaken our faith in the position, that the asymmetry of causation (Hausman (1998)) is based on the temporal asymmetry between cause and effect. For example, the day is followed by the night. But it is widely accepted that the day is therefore not the cause of the night (post hoc, ergo propter hoc). If the cause as such happens only before the effect, this rules out that the cause can happen after its effect. Thus, if causes precede their effects in time then it seems plausible, however, that there is no causation at all. The definition of causation in terms of temporal asymmetry has a number of disadvantages. Hence, there is no valid proof, that the asymmetry between cause and effect is based on the temporal asymmetry, it is not proved yet, that the cause must precede the effect in time. Causal direction is not identical with temporal direction.

Results

Causal investigation of the world around us using the tools of probability theory is often based on random variables. For a variety of reasons this is our starting point too. It is common to distinguish "the cause" as such and "a cause" (Mill (1843)). The first difficulty is to define, what is the cause, what is the effect. It is a remarkable fact that there are various, usually imprecise definitions of cause (f. e. Aristotle's doctrine of the four causes) and effect. In order to avoid certain major errors of definition, let us just talk about the cause or about the effect.

Theorem. The determination of the effect by the cause and vice versa.
Let us perform a thought experiment. Let C_t denote the cause, a random variable at the (space) time t. Let $E(C_t)$ denote the expectation value of the cause at the (space) time t. Let $E(C_t) \neq 0$. Let E_t denote the effect, a random variable at the (space) time t. Let $E(E_t)$ denote the expectation value of the effect at the (space) time t. Let $E(C_t , E_t)$ denote the expectation value of cause and effect at the

(space) time t. Let $\sigma(C_t , E_t)$ or $Cov(C_t , E_t)$ denote the covariance of cause and effect at the (space) time t. Then, according to the law of independence, one of the fundamental concepts in probability theory, the cause has nothing to do with the effect, or the effect is not determined by the cause, or cause and effect are absolutely independent from each other or there is no causal relationship if

$$\sigma(C_t , E_t) = Cov(C_t , E_t) = E(C_t , E_t) - (E(C_t) * E(E_t)) = 0.$$

Proof.

$$+ E_t = + E_t$$

The starting point of our proof is the **identity** of $+E_t = +E_t$ (Hegel 1998, p. 411). E_t is only itself, simple equality with itself, it is only self-related and unrelated to an other, it is distinct from any relation to an other, E_t contains nothing other but only itself. In this way, there does not appear to be any relation to an other, any relation to an other is removed, any relation to an other has vanished. Consequently, E_t is just itself and thus somehow the absence of any other determination. E_t is in its own self only itself and nothing else. In this sense, E_t is identical only with itself, E_t is thus just the 'pure' E_t. Let us consider this in more detail, E_t is not the transition into its opposite, the negative of E_t is not as necessary as the E_t itself, E_t is not confronted by its other. E_t is without any opposition or contradiction, is not against an other, is not opposed to an other, is identical with itself and has passed over into pure equality with itself.

But lastly, although identity and difference are somehow different, identity is not difference, identity is in its own self different. Thus, E_t immediately negates itself. E_t is at the same time in its self-sameness different from itself and thus self-contradictory. Since $E_t = E_t$ it excludes at the same time the other out of itself, it is E_t and it is nothing else, it is at the same time not-E_t, E_t is thus non-being as non-being of its other. In excluding its own other it is excluding itself in its own self. By excluding its other, E_t makes itself into the other of what it excludes from itself, or E_t makes itself into its own opposite, E_t is thus simply the transition of itself into its opposite. E_t is therefore alive only in so far as it contains such a contradiction within itself.

The non-being of its other is at the end the sublation of its other. This non-being is the non-being of itself, a non-being which has its non-being in its own self and not in another, each contains thus a reference to its other. Not-E_t is the pure other of E_t. But at the same time, not-E_t only shows itself in order to vanish, the other of E_t is not. E_t and not-E_t are distinguished and at the same time both are related to one and the same E_t, each is that what it is as distinct from its own other. Identity is thus to some extent at the same time the vanishing of otherness. E_t is itself and its other, E_t has its determinateness not in an other, but in its own self. E_t is thus self-referred and the reference to its other is only a self-reference. On closer examination E_t therefore is, only in so far as its Not-E_t is, E_t has within itself a relation to its other. In other words, E_t is in its own self at the same time different from something else or E_t is something. It is widely accepted that something is different from nothing, thus while $E_t = E_t$ it is at the same time different from nothing or from **non -** E_t **.** From this it is evident, that the other side of the identity $E_t = E_t$ is the fact, that E_t cannot at the same time be E_t and not E_t. In fact, if $E_t = E_t$ then E_t is not at the same time non E_t! What emerges from this consideration is, therefore, even if $E_t = E_t$ it is a self-contained opposition. E_t is only in so far as E_t contains this contradiction within it, E_t is inherently self-contradictory. E_t is thus only as the other of the other. In so far, E_t includes within its own self its own non-being, a relation to something else different from its own self. Thus, E_t is at the same time the unity of identity with difference. E_t is itself and at the same time its other too, E_t is thus contradiction. Difference as such imply contradiction because it unites sides which are, only in so far as they are at the same time not the same. E_t is only in so far as the other of E_t, the non- E_t is. E_t is thus that what it is only through the other, through the non- E_t, through the non-being of itself. Thus we obtain

$$+E_t - E_t = 0.$$

$+E_t$ and $-E_t$ are negatively related to one another and both are indifferent to one another, E_t is separated in the same relation. E_t is itself and its other, it is self-referred, its reference to its other is thus a reference to itself, its non-being is thus only a moment in it. E_t is in its own self the opposite of itself, it has within itself the relation to its other, it is a simple and self-related negativity. Each of them are determined against the other, the other is in and for itself and not as the other of an other. E_t is in its own self the negativity of itself. E_t therefore is, only in so far as its non-being is and vice versa. Non - E_t therefore is, only in so far as its non-being is, both are through the non-being of its other, both as opposites cancel one another in their combination.

Further, the identity of $E_t = E_t$ is an identity over time. Time as such involves in a very general way something like an alteration. E_t undergoes alteration, it goes outside itself. In general, any alteration of E_t, the effect, raises subtle problems. How can the effect remain the same and yet change? If E_t changes, must there be a cause for this change or is an uncaused change possible? Is it extremely implausible to deny caused change? Thus, if $E_t = E_t$ and if E_t changes too, then E_t must at the same time at least be non-identical to itself. In so far, E_t must include a difference within itself or to say it more mathematically, there must be an expectation value of E_t. According to Kolmogorov it holds true that **"If *x* and *y* are equivalent then *E(x) = E(y)* ."** (Kolmogorov 1956, p. 39). Thus we get the next equation.

$$\mathbf{E(E_t) = E (E_t).}$$

If $E_t = E_t$ then $E(E_t) = E(E_t)$. This does not mean that it must hold true that $E_t = E(E_t)$! If it is only that $E_t = E_t$, how can an advance to something different be made? Let us suppose, that E_t is not alone. In other words, it is true that

$$E(E_t) \mathbf{* 1} = E (E_t).$$

Let $\mathbf{E(C_t) = E (C_t).}$ Let $E(C_t) \neq 0$, thus $\mathbf{E(C_t)/E (C_t)} = \mathbf{1}$. It is $E(E_t) = E(E_t)$ and $E(C_t) = E(C_t)$ but both are not one. The self-identity of both is thus the indifference of each towards the other which is distinguished from it. In the same relation, both are rigidly held as separated. Both have a separate existence and are without any relation to an other. In this case, a cause has no relation to an effect, nothing changes by the cause, effect E_t is like it is, thus we obtain

$$E(E_t) \mathbf{* (E(C_t) / E(C_t))} = E (E_t)$$

or

$$E(E_t) * E(C_t) = E (E_t) * E(C_t)$$

Each of both stands isolated from each other, is separated from each other, each is only on its own. By this separation of one from the other, both are related not to one another, each is valid on its own and without any respect to an other. In so far, according to Kolmogorov, it is " **E(X Y)** = ... = E(X E(Y)) **= E(X) * E(Y)** " (Kolmogorov 1956, p. 60). Thus we obtain

$$E(C_t , E_t) = E (C_t) * E(E_t)$$

or

$$E(C_t , E_t) - E (C_t) * E(E_t) = 0$$

However, in general,

if there is

no causal relationship

between random variables,

then it is true that

$$\boldsymbol{\sigma(C_t , E_t) = Cov(C_t , E_t) = E(C_t , E_t) - (E (C_t) * E(E_t)) = 0.}$$

Q. e. d.

Conclusions

If the cause is isolated from the effect and vice versa, if both are separated from each other, if each is only itself and without any relation to an other, if cause and effect are absolutely independent from each other or if the cause and the effect are **not determined** by each other (Barukčić 2006, p. 44), then it is true that

$$\sigma(C_t , E_t) = \mathrm{Cov}(C_t , E_t) = E(C_t ,E_t) - (E (C_t) * E(E_t)) = 0.$$

If $\sigma(C_t , E_t) \neq 0$, this does not proof at once that there is a causal relationships between investigated random variables. We can only state that there is no causal relationship, if $\sigma(C_t , E_t) = 0$. In so far, the law of independence, one of the fundamental laws in nature, statistics and probability theory is valid for the relationship between cause and effect. If the effect at the same time is absolutely independent from the cause and vice versa, if the cause at the same time has nothing to do with the effect, if there isn't a constant and deterministic relation between cause and effect, then it holds true that

$$\boldsymbol{\sigma(C_t , E_t) = \mathrm{Cov}(C_t , E_t) = E(C_t , E_t) - (E (C_t) * E(E_t)) = 0.}$$

Under this circumstances a cause can not produces an effect. Independence is thus the other of causation and at the same time an absolutely necessary part of causation (Barukcic 2006, p. 44) too, independence defines causation to some extent **ex negativo.**

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