Conjecture on the primes of the form \((q+n)2^n + 1\) where \(q\) odd prime

Abstract. In this paper I first conjecture that for any non-null positive integer \(n\) there exist an infinity of primes \(p\) such that the number \(q = (p - 1)/2^n - n\) is also prime and than I conjecture that for any odd prime \(q\) there exist an infinity of positive integers \(n\) such that the number \(p = (q + n)*2^n + 1\) is prime.

Conjecture:

For any non-null positive integer \(n\) there exist an infinity of primes \(p\) such that the number \(q = (p - 1)/2^n - n\) is also prime.

Examples:
(for \(n = 1\))

: for \(p = 13\), \((13 - 1)/2^1 - 1 = 5\), prime;
: for \(p = 17\), \((17 - 1)/2^1 - 1 = 7\), prime;
: for \(p = 29\), \((29 - 1)/2^1 - 1 = 13\), prime;
: for \(p = 37\), \((37 - 1)/2^1 - 1 = 17\), prime;
: for \(p = 41\), \((41 - 1)/2^1 - 1 = 19\), prime;
: for \(p = 61\), \((61 - 1)/2^1 - 1 = 29\), prime;

[...]
: for \(p = 104537\), \((104537 - 1)/2^1 - 1 = 52267\), prime;
: for \(p = 104729\), \((104729 - 1)/2^1 - 1 = 52363\), prime.

Examples:
(for \(n = 2\))

: for \(p = 29\), \((29 - 1)/2^2 - 2 = 5\), prime;
: for \(p = 37\), \((37 - 1)/2^2 - 2 = 7\), prime;
: for \(p = 53\), \((53 - 1)/2^2 - 2 = 11\), prime;
: for \(p = 61\), \((61 - 1)/2^2 - 2 = 13\), prime;

[...]
: for \(p = 104693\), \((104693 - 1)/2^2 - 2 = 26171\), prime.
: for \(p = 104717\), \((104717 - 1)/2^2 - 2 = 26177\), prime.

Examples:
(for \(n = 3\))

: for \(p = 113\), \((113 - 1)/2^3 - 3 = 11\), prime;
: for \(p = 192\), \((192 - 1)/2^3 - 3 = 23\), prime;
: for \(p = 257\), \((256 - 1)/2^3 - 3 = 29\), prime;
: for \(p = 353\), \((353 - 1)/2^3 - 3 = 41\), prime.
Examples:
(for n = 4)

: for p = 113, \((113 - 1)/2^4 - 4 = 3\), prime;
: for p = 337, \((337 - 1)/2^4 - 4 = 17\), prime;
: for p = 433, \((433 - 1)/2^4 - 4 = 23\), prime.

Examples:
(for n = 5)

: for p = 577, \((577 - 1)/2^5 - 5 = 13\), prime.

Examples:
(for n = 6)

: for p = 577, \((577 - 1)/2^6 - 6 = 3\), prime;
: for p = 104513, \((10451 - 1)/2^6 - 6 = 1627\), prime.

Conjecture:

For any odd prime q there exist an infinity of positive integers n such that the number \(p = (q + n)2^n + 1\) is prime.

: for q = 3, the least n for which p is prime is n = 4, because \((3 + 4)2^4 + 1 = 113\), prime;
: for q = 5, the least n for which p is prime is n = 1, because \((5 + 1)2^1 + 1 = 13\), prime;
: for q = 7, the least n for which p is prime is n = 1, because \((7 + 1)2^1 + 1 = 17\), prime;
: for q = 11, the least n for which p is prime is n = 2, because \((11 + 2)2^2 + 1 = 53\), prime;
: for q = 13, the least n for which p is prime is n = 1, because \((13 + 1)2^1 + 1 = 29\), prime;
: for q = 17, the least n for which p is prime is n = 1, because \((17 + 1)2^1 + 1 = 37\), prime;
: for q = 19, the least n for which p is prime is n = 1, because \((19 + 1)2^1 + 1 = 41\), prime;
: for q = 23, the least n for which p is prime is n = 2, because \((23 + 2)2^2 + 1 = 101\), prime;
: for q = 29, the least n for which p is prime is n = 1, because \((29 + 1)2^1 + 1 = 61\), prime;
: for q = 31, the least n for which p is prime is n = 5, because \((31 + 5)2^5 + 1 = 1153\), prime [note the interesting fact that for n = 4 is obtained \((31 + 4)2^4 + 1 = 561\), the first absolute Fermat pseudoprime].

Taking seven larger consecutive primes were obtained:
for $q = 104693$, the least $n$ for which $p$ is prime is $n = 8$, because $(104693 + 8) \times 2^8 + 1 = 26803457$, prime;

for $q = 104701$, the least $n$ for which $p$ is prime is $n = 2$, because $(104701 + 2) \times 2^2 + 1 = 418813$, prime;

for $q = 104707$, the least $n$ for which $p$ is prime is $n = 2$, because $(104707 + 2) \times 2^2 + 1 = 418837$, prime;

for $q = 104711$, the least $n$ for which $p$ is prime is $n = 4$, because $(104711 + 4) \times 2^4 + 1 = 1675441$, prime;

for $q = 104717$, the least $n$ for which $p$ is prime is $n = 7$, because $(104717 + 7) \times 2^7 + 1 = 13404673$, prime;

for $q = 104723$, the least $n$ for which $p$ is prime is $n = 1$, because $(104723 + 1) \times 2^1 + 1 = 209449$, prime;

for $q = 104729$, the least $n$ for which $p$ is prime is $n = 8$, because $(104729 + 8) \times 2^8 + 1 = 26812673$, prime;

Note the relative small value of $n$ for which the first prime is found!