Four conjectures on the numbers of the form (p+270)*n-n+1 where p and p+270 primes

Abstract. In this paper we conjecture that there exist an infinity of primes, respectively squares of primes, respectively semiprimes with a certain property, respectively Poulet numbers of the form (p + 270)*n – n + 1, for any p prime greater than or equal to 7, if p + 270 is also a prime number.

Conjecture 1:

There exist an infinity of primes of the form (p + 270)*n – n + 1, for any p prime greater than or equal to 7, if p + 270 is also a prime number.

Examples:
(for p = 7, 11, 13, 23, 37 up to n = 15)

: for p = 7, 277 also prime and are obtained, for n = 3, 5, 6, 7, 11, 12 (...) the primes 829, 1381, 1657, 1933, 3037, 3313 (...)

: for p = 11, 281 also prime and are obtained, for n = 9, 10, 12, 15 (...) the primes 2521, 2801, 3361, 4201 (...)

: for p = 13, 283 also prime and are obtained, for n = 4, 6, 9, 15 (...) the primes 1129, 1693, 2539, 4231 (...)

: for p = 23, 293 also prime and are obtained, for n = 3, 6, 13 (...) the primes 877, 1753, 3797 (...)

: for p = 37, 307 also prime and are obtained, for n = 2, 3, 5, 7, 10, 12, 15 (...) the primes 613, 919, 1531, 2143, 3061, 3673, 4591 (...)

Conjecture 2:

There exist an infinity of squares of primes of the form (p + 270)*n – n + 1, for any p prime greater than or equal to 7, if p + 270 is also a prime number.

Examples:
(for p = 7, 11, 13, 23, 37 up to n = 15)

: for p = 7, 277 also prime and are obtained, for n = 8 (...) the squares of primes 2209 (= 47^2) (...)

: for p = 11, 281 also prime and are obtained, for n = 3, 6 (...) the squares of primes 841 (= 29^2), 1681 (= 41^2) (...)

Conjecture 3:

There exist an infinity of semiprimes q1*q2 of the form q1*q2 = (p + 270)*n – n + 1 having the property that q2 – q1 + 1 is prime, for any p prime greater than or equal to 7, if p + 270 is also a prime number.

Examples:
(for p = 7, 11, 13, 23, 37 up to n = 15)

: for p = 7, 277 also prime and are obtained, for n = 1, 10, 13, 14, 15 (...) the semiprimes 553 (= 7*79 and 79 – 7 + 1 = 73, prime), 2761 (= 11*251 and 251 – 11 + 1 = 241, prime), 3589 (= 37*97 and 97 – 37 + 1 = 61, prime), 3865 (= 5*773 and 773 – 5 + 1 = 769, prime), 4141 (= 41*101 and 101 – 41 + 1 = 61, prime (...)

: for p = 11, 281 also prime and are obtained, for n = 4, 7 (...) the semiprimes 1121 (= 19*59 and 59 – 19 + 1 = 41, prime), 1961 (= 37*53 and 53 – 37 + 1 = 17, prime) (...)

: for p = 13, 283 also prime and are obtained, for n = 5, 14 (...) the semiprimes 1411 (= 17*83 and 83 – 17 + 1 = 67, prime), 3949 (= 11*359 and 359 – 11 + 1 = 349, prime (...)

: for p = 23, 293 also prime and are obtained, for n = 9 (...) the semiprimes 2629 (= 11*239 and 239 – 11 + 1 = 229, prime) (...)

: for p = 37, 307 also prime and are obtained, for n = 6, 13, 14 (...) the semiprimes 1837 (= 11*167 and 167 – 11 + 1 = 157, prime), 3979 (= 23*173 and 173 – 23 + 1 = 151, prime), 4285 (= 5*857 and 857 – 5 + 1 = 853, prime (...)

Conjecture 4:

There exist an infinity of Poulet numbers of the form (p + 270)*n – n + 1, for any p prime greater than or equal to 7, if p + 270 is also a prime number.

Examples:
(for p = 7, 11, 13, 23, 37 up to n = 15)

: for p = 7, 277 also prime and are obtained, for n = 4 (...) the Poulet numbers 1105 (...)
: for p = 11, 281 also prime and are obtained, for n = 2 (...) the Poulet numbers 561 (...)
: for p = 13, 283 also prime and are obtained, for n = 10 (...) the Poulet numbers 2821 (...)