Mathematical Principle about the Mass Change of Moving Object along with the Velocity

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Abstract: British physicist Paul Dirac, using Albert Einstein's special theory of relativity, and Erwin Schrödinger's theory, clever reasoning obtained the behavior of electrons and positrons, and predicted antiparticles presence. In the standard model Higgs field at the center of the thought lies in: even if is in a state of lowest energy, space is not empty. Through particle in the space will be more or less with the Higgs field, the effect to make particles in motion produced a "sticky" characteristics, namely mass, as a result of the Higgs field is not carrying a net electric charge or colors, photon and gluons don't interact with it, so there is still no mass, for the mass of the neutrinos. Based on these use quaternion, matrix method and linear combination of relationship, combined with Albert Einstein's special theory of relativity, and Louis de Broglie's thought, it is concluded that there are the three kinds of changes in the mass of moving objects, the relativistic state, the phase state, the relativistic state and phase state superimposed "superposition state", and through the use of de Broglie theory of phase harmony theorem, deduced the mass of moving object, regardless is in the relativity state, or in the phase state, the same wavelength of matter waves, according to their different frequency propagation, their momentum is equal, and predicted that the mass of a moving object, in the relativistic state and the phase state "superposition state", the same wavelength of matter waves, according to a different frequency propagation, momentum is equal, and research the relationship between mass and energy of moving objects, obtained the evolution models of particles and anti-particles, and the mass model of elementary particles.

Keywords: Relativistic mass, Phase mass, Dimensionless number W, The mass and energy field, Quaternion form of time and spatial relationships, Evolution Model of particles and antiparticles, Mass model of elementary particles.

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Introduction

Since the famous physicist Albert Einstein created the special theory of relativity, the theory of the famous French physicist Louis de Broglie, using the special theory of relativity and the German physicist Max Planck's theory, the creation of the material wave theory, followed by Austrian physicist Erwin Schrödinger, based on the wave theory of matter, wave mechanics was established, then the British physicist Paul Dirac, combined with special relativity and the Schrodinger equation, built can describe the physical behavior of fermions Dirac equation, accurately describe the behavior of electrons and positrons, and predicted the existence of antimatter.

Later, the British physicist Peter Higgs and other theoretical physicists proposed the Higgs mechanism, and predicted a spin-0 boson: Higgs boson. A few years later, Gerald Guralnik, Carl Hagen, and Tom Kibble, will integrate these concepts become a more realistic theory, which is the predecessor of the standard model.

Among the standard model, the Higgs field lead to spontaneous symmetry breaking, and giving mass specification propagator and fermions, Higgs field is the Higgs field quantum excitation, which is obtained by self-interaction mass It is considered the source of the mass of elementary particles generated.

In Einstein's time, the Austrian physicist Wolfgang E.Paul, neutrinos mention hypothesis, then the United States physicist Frederick Reines and Clyde Cowan, the use of β decay product of
nuclear reactors produce antineutrinos, observed neutrino-induced reaction. Later the Italy particle physicist Bruno Pontecorv, proposed the concept of neutrino oscillations, until modern times, Japan physicist Takaaki Kajita and Canada physicist Arthur B.Mcdonald found the phenomenon of neutrino oscillations, suggesting that indicates that neutrinos is to have mass.

1. On the Relationship between Mass and Energy of the Three Formulas

According to Einstein’s special theory of relativity, a rest mass $m_0$ of the object, if the object with velocity $v = \beta c$ relative to an observer at constant velocity, and for simplicity, our observer is stationary, by the known results of relativistic mechanics, the observer will measure the mass of the body as $m_1$,

$$m_1 = m_0 \sqrt{1 - \beta^2} = \frac{m_0}{\sqrt{1 - \beta^2}} = \frac{m_0}{\sqrt{1 - \beta^2}} = m_0(1 - \beta^2)^{1/2} \quad (1)$$

In order to facilitate the discussion later, (1) is the expression of Einstein’s special theory of relativity, then (1) is called $m_1$ in Einstein’s special theory of relativity mass called relativistic mass, (1) can be called the relativistic mass formula, in order to facilitate the discussions, the (1) is called the first mass formula.

In the second article, I used two methods to derive a formula:

$$m_2 = \frac{m_0}{(1 - \beta^2)^{3/2}} = m_0(1 - \beta^2)^{3/2} \quad (2)$$

(2) type can from de Broglie theory, proved is derived “phase wave group velocity equals the velocity of moving object” theorem, by using of principle is derived out, so (2) type in the of $m_2$ will called de Broglie meaning phase mass, referred to phase mass, (2) type can called phase mass formula, to convenient discussion, put (2) type called second mass formula.

In the special theory of relativity, Einstein’s formula of first quality, by power series expansion of the Taylor series, namely:

$$m_1 = m_0 \left(1 - \beta^2\right)^{\frac{1}{2}} \approx m_0 \left(1 + \frac{1}{2} \beta^2 + \frac{3}{8} \beta^4 + \cdots\right)$$

the second mass formula, or by power series expansion of the Taylor series, namely:

$$m_1 = m_0 \left(1 - \beta^2\right)^{\frac{3}{2}} \approx m_0 \left(1 + \frac{3}{2} \beta^2 + \frac{15}{8} \beta^4 + \cdots\right).$$

Above first mass formula and second mass formula of expand type, first items are $m_0$, from second items began, to remaining all items, are not same , $\beta = 0$ ( namely: $v = 0$ ),

$$m_1 = m_2 = m_0$$ is \( \beta \neq 0 \) ( namely: $v \neq 0$ ), $m_1 \neq m_2$ so fully describe obtained a conclusion, objects of mass has such of nature: movement, objects of mass with speed will occurred different of changes, that: movement, objects of mass with speed will occurred, relativity changes and phase changes. In other words, objects moving, mass in gas, namely: object motion, mass in relativistic state or phase state. So when motion mass would likely be in the relativistic state and phase state "superposition", when this is motion, a change in mass, then:

$$m_3 = \frac{m_1 + m_2}{2} = m_0 \left(1 + \beta^2 + \frac{9}{8} \beta^4 + \cdots\right) \quad (3)$$

(3) it is called $m_3$ in the third mass, (3) is called the third mass formula. Physical meaning of the third mass formula is: relativistic mass and phase mass, described the movements, changes in the mass and speed of an object, then the relativistic mass and phase mass of linear superposition, also describes the movements, changes in the mass and speed of an object.

Three mass formulas, when $\beta = 0$ ( namely: $v = 0$ ) is,

$$m_1 = m_2 = m_0$$ that represents the mass of the body when at rest, rest mass is $m_0$. Using the Taylor series, the first mass formula and the second mass formula to expand, where $\beta^2 < 1$, obviously,
when $\beta$ value is small, the first mass formula and the second mass formula can be taken approximately separately:

$$m_i = m_0 \left(1 + \frac{1}{2} \beta^2 \right), \quad m_z = m_0 \left(1 + \frac{3}{2} \beta^2 \right)$$

Similarly, the third mass formula approximate formula:

$$m_i = m_0 \left(1 + \beta^2 \right)$$

In Einstein’s special theory of relativity, the most important is the concept of mass-energy relation: mass and energy are always associated with each other, namely:

$$E_i = m_i c^2 = m_0 c^2 \left(1 + \beta^2 \right)^{\frac{1}{2}}, \quad E_z = m_z c^2 = m_0 c^2 \left(1 + \beta^2 \right)^{\frac{3}{2}}, \quad E_y = m_y c^2 = m_0 c^2 \left(1 + \beta^2 + \frac{9}{8} \beta^4 + \cdots \right)$$

Equation (7), (8), (9) called the first mass-energy equation, the second mass-energy equation and the third energy formula, corresponds with the approximate relationship:

$$e_i = m_i c^2 \left(1 + \frac{1}{2} \beta^2 \right) = m_0 c^2 + \frac{1}{2} m_0 v_i^2$$

$$e_z = m_z c^2 \left(1 + \frac{3}{2} \beta^2 \right) = m_0 c^2 + \frac{3}{2} m_0 v_z^2$$

$$e_y = m_y c^2 \left(1 + \beta^2 \right) = m_0 c^2 + m_0 v_y^2$$

2, The Second Mass Formula and Momentum Relations Discussion

Second mass formula has been derived in two ways, the first method is by De Broglie’s theory is derived the second mass formula, De Broglie was intended to prove that " phase wave group velocity equals the velocity of moving object " is given, $V = \frac{1}{\beta}$ (This derived from: $V \cdot v = c^2, v = \beta c$, $\hbar$ is the Planck constant, $\mu$ is a frequency: $\mu = \frac{m_0 c^2}{\hbar} \left(1 - \beta^2 \right)^{\frac{1}{2}}$, $V$ and $\mu$ as a function of $\beta$, then: $\frac{dV}{d\beta} = \frac{c^2}{\beta}$, that: $dV = \frac{c^2}{\beta} \cdot d\beta$, $d\left(\frac{\mu}{V}\right) = \frac{m_0 c^2}{\hbar} d\left(\frac{1}{\sqrt{1 - \beta^2}}\right) = \frac{m_0 c^2}{\hbar} \frac{1}{(1 - \beta^2)^{\frac{3}{2}}}$ (13).

The (13) on both sides at the same time multiplied by $hc$ to get: $h c \cdot \frac{d\left(\frac{\mu}{V}\right)}{d\beta} = m_0 c^2 (1 - \beta^2)^{\frac{5}{2}}$, you only need to prove that this left energy-related, then:

$$h c \cdot \frac{d\left(\frac{\mu}{V}\right)}{d\beta} = \frac{m_0 c^2}{1 - \beta^2} d\left(\frac{\mu}{V}\right) = \frac{1}{\beta} (\mu + d\mu)$$

The second method is to introduce velocity $v^* = \frac{l}{\Delta t}$, In Einstein’s theory of relativity, $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$, $v = \frac{l}{\Delta t_0}$, and the momentum $p = \frac{m_0 v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$, located in line with the relativistic case, velocity is $v^* = \frac{l}{\Delta t_0} \sqrt{1 - \left(\frac{v}{c}\right)^2}$, relative to the object itself, $\Delta t_0 = \frac{l}{v}$, then: $v^* = \frac{l}{\Delta t_0} \sqrt{1 - \left(\frac{v}{c}\right)^2}$. In the De Broglie theory, De Broglie proved " phase wave group velocity equals the speed of moving object " is, the velocity of a moving body is energy transfer speeds, their introduction of velocity $v^*$, does not have a physical meaning, is obtained by the momentum $p = \frac{m_0 v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = m_0 v = m_2 v^*$, to reason out the
It has long been recognized that because of the photon mass is zero, so it has two perpendicular to the direction of momentum of transverse polarization, and general mass of vector particle, there are three polarization direction, two of which are transverse polarization, is polarized in the direction of the movement.

From the above physical model, to understand the $p_1, p_2, p_3$, then, $p_i = m_i v$ is the momentum of the direction of motion.

One of the $p_2 = 3m_0 v$, is the direction of the movement of the object of the momentum, the other two, and the direction of motion perpendicular. Two of the $p_3 = 2m_0 v$, both of which are perpendicular to the direction of movement of the object. It is very important to understand the $p_1, p_2, p_3$, which is the birth of some of the important ideas, giving the inspiration.

3, The Spatio-temporal Relationship between the Mass and Energy Field

Quaternions are Ireland mathematician Hamilton, invented the mathematical concept, it is simple to Super-complex. Plurals are formed by real with the imaginary unit $i, j, k$, and they have the following relationship: $i^2 = j^2 = k^2 = -1, i^0 = j^0 = k^0 = 1$. Each Quaternion is a linear combination of $1, i, j, k$, the Quaternion, generally expressed as $a + bi + cj + dk$, where $a, b, c, d$ is a real number.

For $i, j, k$, itself of geometry meaning, can understanding for a rotating, which, $\hat{i}$ rotating, said X axis and Y axis intersect surface in the, X axis are to, to Y axis are to of rotating, $\hat{j}$ rotating, said Z axis and X axis intersect surface in the, Z axis are to, to X axis are to of rotating, $\hat{k}$ rotating, said Y axis and Z axis intersect surface in the,
Y axis are to, to Z axis are to of rotating \(-i, -j, -k\) respectively representative \(i, j, k\) rotating of reverse rotating.

Discussed a relationship between the mass and energy, to study the mass-energy field, now introducing Quaternions, but at this point the use of Quaternions, is no longer a simple Hamiltonian Quaternions, but to promote a Quaternion, then the time-space relationship mass-energy field is set to: \(O = a_o + a_i + a_j + a_k\).

\(i = j = k = \sqrt{-1}\). In Quaternions, the \(i, j, k\) understood as a kind of rotation, in the mass-energy field of space and time relationship, \(i, j, k\) understood as a kind of transformation, \(\sqrt{-1}\) is called mass-energy field conversion factor of space and time, remember as \(\alpha\). Type \(a_o, a_i, a_j, a_k\) as the product of mass factor and space velocity, to as \(\gamma\) mass factor, time and space transform velocity respectively for \(u_o, u_1, u_2, u_3\), and \(|\hbar| = \sqrt{u_{1}^{2} + u_{2}^{2} + u_{3}^{2}}\) called part \(O\) vector transformation relationship between time and space velocity model.

In the About the Mass Change of Moving Object along with the Velocity article the second mass formula is derived, the relationship between relativistic mass \(m_1\) and phase mass \(m_2\): \(m_2 : W = \frac{m_1}{m_2} = \frac{1}{1 - \beta^2}\), the dimensionless number says, is the significance of the theory of between relativistic mass \(m_1\) and phase mass \(m_2\), close to a degree of the rest mass \(m_0\), also said that the third mass of \(m_3\) near the rest mass \(m_0\), because when the dimensionless number \(W = 1\) (namely \(\beta = 0, v = 0\)), \(m_1 = m_2 = m_3 = m_0\). In using the third mass formula, Taylor series expansion approximation to calculate, when the dimensionless number \(W\) is close to 1, the smaller the \(\beta\), the smaller the \(v\), the smaller is the error of the approximate calculation results. Dimensionless number \(W\), the measure error of approximate calculation, so for mass-energy field, the dimensionless number

\[ W = \frac{1}{1 - \alpha^2 \beta^2} = \frac{1}{1 - (\alpha \beta)^2}, \quad \beta = \frac{|\hbar|}{c}. \]

Whether it is the first mass formula, or the second to third mass formula, which factors \((1 - \beta^2)\) is a very important part, it will be three mass formula, and three mass corresponding to the formula all transformation between closely related. Therefore, in the mass-energy relation of time and space, mass factor \(\gamma = \pm \frac{1}{\sqrt{1 - (\alpha \beta)^2}} = \pm (1 - \alpha^2 \beta^2)^{-\frac{1}{2}},\) in which the \(\beta = \frac{|\hbar|}{c}, \quad W = \gamma^2\). In special relativity, the first mass-energy formula \(E_i = m_o c^2 = m_o c^2 (1 + \beta^2)^{\frac{1}{2}},\) derived \(E_i^2 = p^2 c^2 + m_0^2 c^4, \quad p = m_o v = m_o \sqrt{(1 - \beta^2)^{\frac{1}{2}}}\). In the mass-energy relation of time and space, \(E_i^2 = p^2 c^2 + m_0^2 c^4, \quad p = m_o v = m_0 \sqrt{(1 - \alpha^2 \beta^2)^{\frac{1}{2}}}, \beta = \frac{|\hbar|}{c},\) which is \(E_i^2 = p^2 c^2 + m_0^2 c^4\) open quadratic equal to \(E_i^2 = \pm \sqrt{p^2 c^2 + m_0^2 c^4}\) and \(E_i = \mp \sqrt{p^2 c^2 + m_0^2 c^4}, \quad p = m_o v = m_o \sqrt{(1 - \alpha^2 \beta^2)^{\frac{1}{2}}}, \beta = \frac{|\hbar|}{c}.

Mass factor \(\gamma\) by \(\gamma = E_i / m_o c^2\) obtain, therefore, mass factor \(\gamma\) takes a positive sign or a negative sign, and is not the only factor \((1 - \beta^2)^{\frac{1}{2}}\).

In the About the Mass Change of Moving Object along with the Velocity article, the second mass formula was derived, the most important of which is \(m_o v = m_o v^* = m_0 \sqrt{(1 - \beta^2)^{\frac{1}{2}}},\) the relationship among the summary and discussion, the use of the phase of DE Broglie harmonious theorem can be derived, it said that the mass of the moving object, whether in the theory of relativity, or is in the phase state, the
momentum is \( m_0 \nu (1 - \beta^2)^{1/2} \). DE Broglie proved, phase wave group velocity equals the speed of moving object, that is, the mass of the moving object, whether in the theory of relativity, or phase, all in the same motion velocity \( V \), so in a conclusion, the mass of the moving object, whether in the theory of relativity, or phase, the momentum is \( m_0 \nu (1 - \beta^2)^{1/2} \), kinetic energy is \( E_i = \frac{1}{2} m_0 \nu^2 (1 - \beta^2)^{1/2} \).

The mass of the moving object in the theory of relativity state or phase, momentum is \( m_0 \nu (1 - \beta^2)^{1/2} \), then just mentioned in the About the Mass Change of Moving Object along with the Velocity last " \( p_1 = m_0 \nu \) is an object moves in the direction of momentum, and \( p_2 = 3m_0 \nu \) one of them, is the object movement direction of the momentum." is consistent, then the " \( p_2 = 3m_0 \nu \) the rest of the two, and vertical direction, \( p_3 = 2m_0 \nu \) of the two, are perpendicular to the movement direction of the object" is also true, because only \( p_2 = 3m_0 \nu \) of the remaining two, and vertical direction, to make the mass of the moving object, is in the phase state, the kinetic energy is \( E_i = \frac{1}{2} m_0 \nu^2 (1 - \beta^2)^{1/2} \), this also means that \( p_2 = 3m_0 \nu \) of the remaining two perpendicular to the direction of motion, the kinetic energy of a moving object in the phase state, without any contribution, so \( p_3 = 2m_0 \nu \) is perpendicular to the movement direction of the object, the system had no contribution, which makes the mass of the moving object, in the theory of relativity of state and phase superposition "superposition", became the state is not necessary to exist, but that these conditions are established: \( \beta \) is very small, \( V \) is far less than the velocity of light.

Normally, a dimensionless number \( W = \frac{1}{1 - \beta^2} \), when velocity \( V \) infinitely close to the speed of light, the dimensionless number \( W \) tends to infinity. The dimensionless number \( W \), the significance of relativistic mass \( m_1 \) and phase mass \( m_2 \), infinitely far away from the rest mass \( m_0 \), the mass of the moving object with velocity becomes infinite. When \( v = c \), the mass energy relations, the conversion of mass into energy, and at this time only in the form of the energy of the mass times the speed of light squared, for kinetic energy is \( E_i = 0 \), in other words, when \( v = c \), the kinetic energy \( E_i \) disappeared due to the reason of mass into energy. So, related to the kinetic energy of \( p_1 = m_0 \nu \), with the loss of kinetic energy, can look at this moment, only \( p_2 = 3m_0 \nu \) the rest of the two perpendicular to the direction of motion, and \( p_1 = 2m_0 \nu \), this would mean that the theory of relativity of state and phase superposition "superposition", in this case, there is the value and significance, then the third formula approximation of the mass:

\[
e_i = m_0 c^2 + m_0 \nu^2
\]

and can be used to solve many problems. But, when \( v = c \), whether it is dimensionless number \( W \), is the first mass formula and the second mass formula, both the denominator is zero, the relationship is known, the mass to any formula related to the mass, will be meaningless.

Using quaternions mathematical concepts, in order to solve these problems, this article analyzes the difficulties of. In mass-energy field,

\[
O = o_1 + o_2 + o_3 = \mu_1 + \mu_2 + j + \mu_3 k
\]

is relationship between time and space, in which \( i = j = k = \alpha = \sqrt{-1} \). The nature of the quaternion, the related matrix, said:

\[
P = \begin{pmatrix}
\alpha_0 - \alpha_1 i & -\alpha_0 + \alpha_1 j \\
\alpha_1 + \alpha_2 j & \alpha_0 + \alpha_2 i
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\mu_0 - \mu_1 \alpha & -\mu_0 + \mu_1 \alpha \\
\mu_1 + \mu_2 \alpha & \mu_0 + \mu_2 \alpha
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\mu_0 - \mu_1 \alpha & -\mu_0 + \mu_1 \alpha \\
\mu_1 + \mu_2 \alpha & \mu_0 + \mu_2 \alpha
\end{pmatrix}
\]
Evolution of a physical system, the initial state and final state, it is obvious that in mass-energy field, for an object system evolution of the mass and energy, the initial state is 
\[ m_0c^2, \quad O = o_0 + o_1 + o_2 + o_3 = \gamma u_0 + \gamma u_1 + \gamma u_2 + \gamma u_3. \]
the scalar part \( O_0 \) relationship between time and space, said it is the object's state at the beginning of \( m_0c^2 \), in the process of transformation of time and space changes, so, in the mass-energy field, in time and space transformation, relationship between time and space velocity \( U_{0j} \) equals the velocity of light, therefore, the relationship between time and space is 
\[ O = o_0 + o_1 + o_2 + o_3 = \gamma u_0 + \gamma u_1 + \gamma u_2 + \gamma u_3, \]
in which \( i = j = k = \alpha = \sqrt{-1} \), is:
\[
P = \begin{pmatrix}
  o_i - o_j & o_i + o_j \\
  o_i - o_j & o_i + o_j \\
  o_i - o_j & o_i + o_j \\
  o_i - o_j & o_i + o_j 
\end{pmatrix}
\]

So, for the final state of system evolution, using the third formula approximation of the mass-energy, but not the original \( e_i = m_i c^2 + m_i \), it's about mass-energy formula of relationship between time and space in the field:
\[
Q = \varphi(m)c^2 + \varphi(m)U^2,
\]
among them, the dimensionless number \( W = \frac{1}{1-(\alpha \beta)^2} \), mass factor \( \gamma = \pm \frac{1}{\sqrt{1-(\alpha \beta)^2}} \), for the final state of evolution:
\[
Q = \varphi(m)c^2 + \varphi(m)U^2 = \frac{1}{2}m_0c^2 + \frac{1}{2}m_0\alpha^2 \gamma^2 \gamma^2 = -m_0c^2.
\]

Through on spatio-temporal relationship \( O \) and system evolution of eventually state \( Q \) of research told us, early state for \( +m_0c^2 \) of has mass elementary particles, can from positive energy state \( +m_0c^2 \) evolution for negative energy state \( -m_0c^2 \), then for the reverse transformation, early state for \( -m_0c^2 \) of elementary particles, spatio-temporal relationship \( O \) vector partial transformation velocity respectively for the speed of light, that
\[
u_i = u_2 = u_3 = c, \quad |v| = \sqrt{u_1^2 + u_2^2 + u_3^2} = \sqrt{3}c, \quad \text{so} \quad \beta = \frac{|v|}{c} = \sqrt{3}, \quad \text{spho-temporal transform factor} \quad \alpha = \sqrt{-1}, \quad \text{the dimensionless number} \quad W = \frac{1}{1-(\alpha \beta)^2} = 0.25 \quad \text{the mass factor} \quad \gamma = \frac{1}{\sqrt{1-(\alpha \beta)^2}} = 1/2,
\]
therefore, the relationship between time and space is
\[
O = o_i + o_2 + o_3 = \gamma u_0 + \gamma u_1 + \gamma u_2 + \gamma u_3 = \begin{pmatrix}
  1/2 - c/2 - c/2 - c/2 \\
  1/2 + c/2 + c/2 + c/2 \\
  1/2 - c/2 - c/2 - c/2 \\
  1/2 + c/2 + c/2 + c/2 
\end{pmatrix}
\]
i = j = k = \sqrt{-1} \alpha \varphi(m) = \gamma m_0 = \frac{1}{2} m_0, \quad U = \alpha |v|,
So, for mass-energy field of spatio-temporal relationship \( O \) and system evolution of eventually state \( Q \), allowed to have the mass of fundamental particles have positive energy and negative energy, has continuous of positive energy state, began \(+m_c^2\)
(equivalent to the dimensionless number \( W = 1 \), namely \( \beta = 0, p = 0 \)), as a mirror reflection, has continuous of negative energy state, began \(-m_c^2\) (also in dimensionless number \( W = 1 \), that \( \beta = 0, p = 0 \) of conditions ), that is to say, there are a positive energy \(+m_c^2\) mass fundamental particles and a negative energy \(-m_c^2\) mass fundamental particles, mutual for particles and antiparticles. This is obvious, spatio-temporal relationship \( O \) and system evolution of eventually state \( Q \), these two mathematical expressions, unlike Paul Dirac’s theory, limited to electronics, but universal.

Thus, in the mass-energy field, each evolution of particles and antiparticles, spatio-temporal relationship \( O \) vector partial transformation velocity respectively for the speed of light, namely
\[
u_i = u_i = c, \quad |\nu| = \sqrt{u_i^2 + u_j^2 + u_k^2} = \sqrt{3}c, \quad \beta = \frac{|\nu|}{c} = \sqrt{3}, \quad \text{sptatio-temporal transformation factor } \alpha = \sqrt{-1}, \text{ dimensionless number}\]
\[
W = \frac{1}{1 - (\alpha \beta)} = 0.25, \quad \text{mass factors } \gamma = \pm \frac{1}{\sqrt{1 - (\alpha \beta)^2}} = \pm \frac{1}{2}, \quad \text{and spatio-temporal relation}\]
\[
O = a_0 + a_1 + a_2 + a_3 = \gamma \nu + m_0 \beta + m_j \beta + m_k \beta = \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} i + \frac{1}{2} c j + \frac{1}{2} c k \\
\frac{1}{2} c - \frac{1}{2} i - \frac{1}{2} c j - \frac{1}{2} c k
\end{pmatrix},
\]
i = j = k = \sqrt{-1}, \quad \varphi(m) = m_0 = \pm \frac{1}{2} m_0, \quad U = \alpha |\nu|,
\[
|\nu| = \sqrt{u_i^2 + u_j^2 + u_k^2} = \sqrt{3}c, \quad \text{for the evolution of the final state of the system } Q = \varphi(m)c^2 + \varphi(m)U^2 = \left[ -m_c^2 + m_c^2 \right]. \text{ Here the relation between its corresponding matrix:}
\]
\[
P = \begin{pmatrix}
\gamma - a_0, & \gamma a_1 - a_2, & \gamma a_3 - a_3 \\
\gamma a_0 + a_2, & \gamma a_1 + a_1, & \gamma a_3 + a_3 \\
\gamma a_0 + a_3, & \gamma a_1 - a_1, & \gamma a_2 - a_2
\end{pmatrix}
\]
\[
\pm \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} i + \frac{1}{2} c j + \frac{1}{2} c k \\
\frac{1}{2} c - \frac{1}{2} i - \frac{1}{2} c j - \frac{1}{2} c k
\end{pmatrix}
\]
, the values \( \det(P) = c^2 \), \( m_i \cdot \det(P) = m_i c^2 \) characterization is that each particle and antiparticle, whether positive energy fundamental particles, or negative energy fundamental particles, which itself contains energy \( m_c^2 \), then the particles and antiparticles annihilate, \( m_c^2 \) is twice times the energy generated by, rather than cancel each other out for 0. Therefore, evolutionary models of particles and antiparticles, dimensionless number \( W = 0.25 \), in the mass-energy field, by the spatio-temporal relationship \( O \), the matrix \( P \) and the evolution of the final state of the system \( Q \) to defined.

Because of mass factor \( \gamma \) and spatio-temporal covariance transform velocity \( u_i, u_j, u_k, u_s \), implied in spatial and temporal transformation of mass-energy field, velocity vector of any objects can’t travel faster than the speed of light, but magnitude of velocity may be greater than the speed of light. In the mass-energy field, the velocity’s magnitude of spatio-temporal relationship \( O \) vector part transform is \( |\nu| = \sqrt{u_i^2 + u_j^2 + u_k^2} \), so be associated with \( |\nu| \), and spatio-temporal relationship \( O \) equivalent of form
\[
O_H = \gamma c + \gamma |\nu| = \gamma c + \gamma |\nu| \alpha, \quad \text{which } i = \alpha = \sqrt{-1}, \text{ is related to } |\nu|
\]
and matrix \( P \) equivalent of form \( P_H = \begin{pmatrix}
\gamma c & \gamma |\nu| \\
\gamma |\nu| \alpha & \gamma \alpha
\end{pmatrix} \), so, the evolution model of particles and antiparticles, dimensionless number \( W = 0.25 \), in the mass-energy field, also can by
\[
O_{H} = \gamma + \beta |q| = \gamma + \beta |p| = \left\{ \begin{array}{l}
\frac{1}{2} c^2 - \frac{1}{2} \sqrt{3} c \cdot \sqrt{-1} \\
\frac{1}{2} c^2 + \frac{1}{2} \sqrt{3} c \cdot \sqrt{-1}
\end{array} \right\
\]
\[
P_{H} = \left( \begin{array}{c}
\gamma \\
\beta |q| \\
\gamma |p|
\end{array} \right) = \left( \begin{array}{c}
\frac{1}{2} c^2 + \frac{1}{2} \sqrt{3} c \cdot \sqrt{-1} \\
\frac{1}{2} c^2 - \frac{1}{2} \sqrt{3} c \cdot \sqrt{-1}
\end{array} \right),
\]
and
\[
[\text{system evolution of eventually state } Q \text{ to determine}].
\]

In the mass-energy field, for \( +m_{0}c^{2} \) energy elementary particles, relativity mass \( m_{0} = m_{0} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{1}{2}} \), phase mass \( m_{2} = m_{0} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{3}{2}} \), and corresponds to \( E_{1} = m_{0}c^{2} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{1}{2}} \), \( E_{2} = m_{0}c^{2} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{3}{2}} \), so on \( E_{1} \) and \( E_{2} \) of approximate, dimensionless number \( W = \frac{1}{1 - (a\beta)^{2}} \), is on \( E_{1} \) of approximate:

\[
e_{1} = m_{0}c^{2} + \frac{1}{2} m_{0}U^{2}, \quad \text{on } E_{1} \text{ of approximate: } e_{2} = m_{0}c^{2} + \frac{3}{2} m_{0}U^{2},
\]
which spatio-temporal transform factor \( \alpha = \sqrt{-1} \), \( \beta = \frac{H}{c} \),

\[
U = \alpha |q|,
\]
and on relativity state and phase state superimposed

"superposition state" is:

\[
e_{3} = m_{0}c^{2} + m_{0}U^{2}.
\]
So, for \(-m_{0}c^{2}\) energy elementary particles, relativity mass \( m_{1} = -m_{0} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{1}{2}} \), phase mass \( m_{2} = -m_{0} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{3}{2}} \), and corresponds to

\[
E_{1} = -m_{0}c^{2} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{1}{2}}, \quad E_{2} = -m_{0}c^{2} \left( 1 - \alpha^{2} \beta^{2} \right)^{\frac{3}{2}}, \text{ so on } E_{1} \text{ and } E_{2} \text{ of approximate, dimensionless number } W = \frac{1}{1 - (a\beta)^{2}} \text{, is on } E_{1} \text{ of approximate:}
\]

\[
e_{1} = -m_{0}c^{2} - \frac{1}{2} m_{0}U^{2}, \quad \text{on } E_{1} \text{ of approximate: } e_{2} = -m_{0}c^{2} - \frac{3}{2} m_{0}U^{2},
\]
in which spatio-temporal transform factor

\[
\alpha = \sqrt{-1}, \quad \beta = \frac{H}{c}, \quad U = \alpha |p|, \quad \text{and on relativity state and phase state superimposed }\]

"superposition state" is:

\[
e_{3} = -m_{0}c^{2} - m_{0}U^{2}.
\]

Analysis \( +m_{0}c^{2} \) energy elementary particles and \(-m_{0}c^{2}\) energy elementary particles, mass-energy fields, in relativistic state and phase state, the use of \( O_{H} = \gamma + \beta |q| \) and \( P_{H} = \left( \begin{array}{c}
\gamma \\
\beta |q| \\
\gamma |p|
\end{array} \right) \) in connection with \( |q| \), dimensionless number \( W = \frac{1}{1 - (a\beta)^{2}} \), spatio-temporal transform factor \( \alpha = \sqrt{-1} \), \( \beta = \frac{H}{c} \), mass factor

\[
\gamma = \pm \frac{1}{\sqrt{1 - (a\beta)^{2}}}, \quad U = \alpha |q|,
\]
When \( |q| = c \), the dimensionless number

\[
W = 0.5, \quad \beta = \frac{H}{c} = 1, \quad \text{mass factor } \gamma = \pm \frac{\sqrt{2}}{2}, \quad U = \alpha |q| = \frac{\sqrt{2}}{c},
\]

\[
O_{H} = \gamma + \beta |q| = \pm \frac{\sqrt{2}}{2} c \pm \frac{\sqrt{2}}{2} c \sqrt{-1}, \quad \text{so, } |q| = c, \quad \text{and}
\]

\[
P_{H} = \left( \begin{array}{c}
\gamma \\
\beta |q| \\
\gamma |p|
\end{array} \right) = \left( \begin{array}{c}
\pm \frac{\sqrt{2}}{2} c \\
\pm \frac{\sqrt{2}}{2} c \sqrt{-1}
\end{array} \right), \quad \text{the values}
\]
\[
det(P_{H}) = c^{2}, \quad \text{therefore } m_{0} \cdot det(P_{H}) = m_{0}c^{2}.
\]
For relativity state and phase state, are:

\[
e_{1} = \left\{ \begin{array}{l}
+ m_{0}c^{2} + \frac{1}{2} m_{0}U^{2} \\
- m_{0}c^{2} - \frac{1}{2} m_{0}U^{2}
\end{array} \right\} = \left\{ \begin{array}{l}
+ \frac{1}{2} m_{0}c^{2} \\
- \frac{1}{2} m_{0}c^{2}
\end{array} \right\}, \quad \text{time}
\]

\[
e_{2} = \left\{ \begin{array}{l}
+ m_{0}c^{2} + \frac{3}{2} m_{0}U^{2} \\
- m_{0}c^{2} - \frac{3}{2} m_{0}U^{2}
\end{array} \right\} = \left\{ \begin{array}{l}
+ \frac{3}{2} m_{0}c^{2} \\
- \frac{3}{2} m_{0}c^{2}
\end{array} \right\}.
\]

And in mass-energy field, relativity state and phase state superimposed "superposition state":

\[
e_{3} = \pm m_{0}c^{2} \pm m_{0}U^{2} = 0.
\]
So, in mass-energy field, the mass model of elementary particles, dimensionless number \( W = 0.5 \), by the spatio-temporal relationship \( O_{H} \), the matrix \( P_{H} \), \( e_{1} \) and \( e_{2} \) to determine.

Through on evolution model of particle-antiparticles and mass model of elementary particles, mathematics relationship of established, so, in mass-energy field, exists such a surface, this surface in spatio-temporal is between positive energy sea and negative energy sea, when energy value is \( m_{0}c^{2} \), in this surface, has counterclockwise of role makes energy trend counterclockwise for
\[ \frac{1}{2} m_c^2, \text{ has clockwise of role makes energy trend clockwise for} \]
\[ -\frac{1}{2} m_c^2, \text{ this two species different role of cooperation with} \]
direction is a polarization freedom. Annihilate by the positive and negative energies known, the formed system is unstable, so if there are two polarization degree of freedom, which makes it stable and does not affect the system, the two polarization degree of freedom perpendicular to the direction of cooperation. So, there is a such system, for \( m_c \) energy, in counterclockwise role and clockwise role, formed stable of system, cooperation with direction vertical of two polarization freedom, in the positive energy sea, as a mirror reflection, for \( m_c \) energy, in counterclockwise role and clockwise role, formed stable of system, cooperation with direction vertical of two polarization freedom, in the negative energy sea, is this two \( m_c \) energy composition of system, mutual for particles and antiparticles.

So, three polarization freedom, and which a polarization freedom contains two roles, makes energy into for has mass of vector particles, two polarization freedom and contains two role of polarization freedom vertical, and this two polarization freedom of location, direction and angle, is to determine this has mass vector particle’s nature, part factors. Another polarization freedom, by counterclockwise role and clockwise role of cooperation with to out, is to determine this has mass vector particle’s mass, part factors( the weak interaction, strong interaction, and so on, will affect particle’s nature and mass). So also can be said to be four degrees of vector particle energy into mass, but massless particles, there are only two polarization degree of freedom. Equal to a \( \gamma \) photon energy, for example, when no roles of clockwise and counterclockwise, is equal to the energy of a \( \gamma \) photon, has two polarization degree of freedom, shown as a \( \gamma \) photon, roles of clockwise and counterclockwise when equal to the energy of a \( \gamma \) photon, have four polarization degree of freedom, characterized by an electron or a positron, etc. This also showed that, \( \gamma \) photon in the positive energy sea or in the negative energy sea, no counterclockwise role and clockwise role, it is always the form of \( \gamma \) photon.

5, Summary and Discussion

Louis Victor de Broglie thought that his truly great achievement is the phase harmonious theorem rather than two minimum principle of equivalence, harmonious theorem is of universal significance, it applies not only to the classical approximation, but also the entire wave mechanics, and contains all of the features.

The phase harmonious theorem: in association with the moving object, relative to the stationary observer’s frequency is equal to
\[
\mu = \frac{1}{f} m_c c^2 \sqrt{1 - \beta^2} \]
changes periodically, for stationary observation always following a wave with phase, the frequency of the wave is
\[
\mu = \frac{1}{h} m_c c^2 \sqrt{1 - \beta^2}, \]
it’s direction and velocity \( V = \frac{c}{\beta} \) motion of an object moving in same direction.

Using phase harmonious theorem and DE Broglie Formula
\[
p = \frac{h \mu}{V} = \frac{h}{\lambda} \text{ (} \lambda \text{ for the wavelength), combined with relativity mass} \]
\[
m_1 = m_c (1 - \beta^2)^{1/2} \text{ and phase mass } m_2 = m_c (1 - \beta^2)^{3/2}, \text{ to discuss.} \]

Known, wave velocity is expressed as the product of the wavelength and frequency, momentum is expressed as the product of mass and velocity. In de Broglie theory, relative to the same observer, movement objects of energy is equal to \( m_c c^2 \sqrt{1 - \beta^2} \),
corresponding of frequency for \( \mu = \frac{1}{h} m_c c^2 \sqrt{1 - \beta^2}, \) so for frequency is equal to \( \mu = \frac{1}{h} m_c c^2 \sqrt{1 - \beta^2}, \) the momentum is
\[
P = m\lambda \mu = m_c (1 - \beta^2)^{1/2} \left( \frac{1}{p} \right) \left( \frac{1}{h} m_c c^2 \sqrt{1 - \beta^2} \right) = \frac{m_c^2 c^2}{p (1 - \beta^2)}, \text{ for} \]

\[ \text{for} \]
\[ \]
frequency is equal to \( \mu_i = \frac{1}{\hbar} m_i c^2 \sqrt{1 - \beta^2} \), the momentum is

\[
P_i = m_i \lambda \mu_i = m_i (1 - \beta) \frac{1}{2} \left( \frac{1}{\beta} \right)^3 = \frac{m_i c^2}{p(1 - \beta^2)},
\]

apparently

\[
P = P_i = \frac{m_i c^2}{p(1 - \beta^2)} = \frac{m_i c^2}{v \sqrt{1 - \beta^2}} = m V,
\]

while also description has movement objects of mass, in relativistic state or phase state, the momentum is \( m_i (1 - \beta) \frac{1}{2} \), proved has third article “The spatio-temporal relationship between the mass and energy Field” described content.

If the mass of moving object, in relativistic state and phase state superimposed "superposition state", then for the third mass

\[
m_3 = \frac{m_1 + m_2}{2} = \frac{m_1 (1 - \beta^2) + m_2}{2} = \frac{m_1 (1 - \frac{1}{2} \beta^2)}{(1 - \beta^2)^2},
\]

momentum

\[
P_3 = P = m V = m_3 \lambda \mu_3 = \frac{m_3 c^2}{v \sqrt{1 - \beta^2}} = \mu_3,
\]

so

\[
\mu_3 = \frac{P_3}{m_3 \lambda} = \frac{m_3 c^2}{p(1 - \beta^2)} = \frac{m_3 c^2 (1 - \beta^2)^3}{p(1 - \beta^2)^2}.
\]

Cutting-edge research has shown that Neutrino Oscillations due to three types of neutrino mass eigenstates are mixed in with the weak interaction eigenstate. Eigenstates of an electron neutrino has three kinds of mass ingredients, spread some distance into electron neutrinos, muon neutrinos, tau neutrino superposition. Generation and detection of neutrinos is through the weak interaction, while the mass is determined by the mass eigenstates. Due to mixed, produced weak interaction eigenstates are not the mass eigenstates, but three kinds of superposition of eigenstates of mass. Three mass eigenstate in matter waves of different frequencies transmitted, so on the different distance observation neutrinos, will show a weak interaction eigenstates of the different components. When weak interactions to detect neutrinos, will see a different neutrino. So the nature of neutrinos, may is the neutrino in relativistic state, phase state, relativistic state and phase state superimposed "superposition state", according to different frequency

\[
\mu_i = \frac{1}{\hbar} m_i c^2 \sqrt{1 - \beta^2},
\]

transmission.

Through discussions of the mathematical method of relationship between mass and energy, come to universal, evolution model of particles and antiparticles, and mass model of elementary particles, using Quaternion, matrix and linear combinatorial algorithms, of which the most important is the dimensionless number \( W \).

When the dimensionless number \( W = 1 \), the mass factors \( \gamma = \pm 1 \), \( \beta = 0 \), the vector part of spatio-temporal relationship \( O \) is 0, only the scalar part \( O_0 = m_i \cdot \text{det}(P) = m_i c^2 \), the velocity of an object is 0, the mass not relativistic state changes, or phase changes, or relativistic state and phase state superimposed "superposition state" changes, obtained by linear \( m_1 = m_2 = m_3 = m_0 \).

When the dimensionless number \( 0.5 < W < 1 \), the spatio-temporal relationship \( O \) vector part transform modules of velocity is less than the speed of light, \( m_i \cdot \text{det}(P) = m_i c^2 \), objects of mass with velocity, occurred relativistic state changes and phase state changes, then relativistic state and phase state superimposed "superposition state" changes is little effect, can ignored this changes, linear relationship combination study:

\[
m_1 = m_0 (1 - \beta^3)^{\frac{1}{2}},
\]

\[
m_2 = m_0 (1 - \beta^3)^{\frac{1}{2}},
\]

\[
E_i = \pm m_i c^2 = \pm m_0 c^2 (1 + \beta^3)^{\frac{1}{2}},
\]

\[
E_2 = \pm m_2 c^2 = \pm m_0 c^2 (1 + \beta^3)^{\frac{1}{2}}.
\]
When the dimensionless number $W = 0.5$, the mass factor
$$\gamma = \pm \sqrt{\frac{3}{2}}, \beta = 1,$$
the spatio-temporal relationship $O$ vector part transform modules of velocity is equal to the speed of light,
momentum $P = m_0 c^2$, in mass-energy fields, relativistic and phase changes in objects, by the relativistic state and phase state superimposed "superposition state" linear relations of
$$e_1 = \pm m_0 c^2 \pm m U^2 = 0,$$ this changes can be ignored, linear relationship combination study:
$$e_1 = \begin{cases} + m_0 c^2 + \frac{1}{2} m U^2, & \beta = 1 \quad \Rightarrow \quad - \frac{1}{2} m c^2, \\ - m_0 c^2 - \frac{1}{2} m U^2, & \beta = 1 \quad \Rightarrow \quad \frac{1}{2} m c^2, \end{cases}$$
$$e_2 = \begin{cases} + m_0 c^2 + \frac{3}{2} m U^2, & \beta = 3 \quad \Rightarrow \quad - \frac{1}{2} m c^2, \\ - m_0 c^2 - \frac{3}{2} m U^2, & \beta = 3 \quad \Rightarrow \quad \frac{1}{2} m c^2. \end{cases}$$

When the dimensionless number $0.25 < W < 0.5, 1 < \beta < \sqrt{3}$, the spatio-temporal relationship $O$ vector part transform modules of velocity is greater than the speed of light, in mass-energy fields, the evolution of the system is very complex, containing various combinations of particles change, consider linear combinations of relationships in many ways.

When the dimensionless number $W = 0.25$, the mass factor
$$\gamma = \pm \frac{1}{2}, \beta = \sqrt{3},$$
the spatio-temporal relationship $O$ vector part transform modules of velocity is greater than the speed of light, in mass-energy fields, this is the evolution of particles and antiparticles, considering linear relationships:
$$Q = \varphi(m)c^2 + \varphi(m)U^2, \varphi(m) = \gamma m_0.$$