

Magnetohydrodynamic Equations Solutions

Abstract

The simplest solution is usually the best solution---Albert Einstein

The system of magnetohydrodynamic (MHD) equations have been solved analytically in this paper. The author applied the technique used in solving the Navier-Stokes equations and applied a new law, the law of definite ratio for MHD. This law states that in MHD, the other terms of the system of equations divide the gravity term in a definite ratio, and each term utilizes gravity to function. The sum of the terms of the ratio is always unity. It is shown that without gravity forces on earth, there would be no magnetohydrodynamics on earth as is known. The equations in the system of equations were added to produce a single equation which was then integrated. Ratios were used to split-up this single equation into sub-equations which were readily integrated, and even, the non-linear sub-equations were readily integrated. Twenty-seven sub-equations were integrated. The linear part of the relation obtained from the integration of the linear part of the equation satisfied the linear part of the equation; and the relation from the integration of the non-linear part satisfied the non-linear part of the equation. The solutions revealed the role of each term in magnetohydrodynamics. In particular, the gravity term is the indispensable term in magnetohydrodynamics. The solutions of the MHD equations were compared with the solutions of the N-S equations, and there were similarities and dissimilarities.

Solutions of the Magnetohydrodynamic Equations

This system consists of four equations and one is to solve for $V_x, V_y, V_z, B_x, B_y, B, P(x)$

$$\left. \begin{array}{l}
 \text{Magnetohydrodynamic Equations} \\
 1. \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad \text{--- continuity equation} \\
 2. \quad \overbrace{\rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z}}^{\text{Navier-Stokes}} = \overbrace{-\frac{\partial p}{\partial x} + \frac{1}{\mu} (\nabla \times B) \times B + \rho g_x}^{\text{Lorentz force}} \\
 3. \quad \rho \frac{\partial B}{\partial t} = \nabla \times (V \times B) + \eta \nabla^2 B \\
 \quad \rho \frac{\partial B}{\partial t} = \nabla \times (V \times B) + \eta \left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2} \right) \\
 \quad (\eta = \text{magnetic diffusivity}) \\
 4. \quad \nabla \cdot B = 0 \\
 \quad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0
 \end{array} \right\}$$

Step 1:

1. If ρ is constant : (for incompressible fluid)

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad \text{--- continuity equation}$$

$$2. \quad \overbrace{\rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z}}^{\text{Navier - Stokes}} = \overbrace{-\frac{\partial p}{\partial x} + \frac{1}{\mu} (\nabla \times B) \times B + \rho g_x}^{\text{Lorentz force}}$$

$$\rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{\mu} (B_z \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) - B_y \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) + \rho g_x)$$

$$\boxed{\rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{\mu} (B_z \frac{\partial B_x}{\partial z} - B_z \frac{\partial B_z}{\partial x} - B_y \frac{\partial B_y}{\partial x} + B_y \frac{\partial B_x}{\partial y}) + \rho g_x}$$

$$3. \quad \rho \frac{\partial B}{\partial t} = \nabla \times (V \times B) + \eta \nabla^2 B$$

$$\rho \frac{\partial B}{\partial t} = \frac{\partial}{\partial y} (V_x B_y - V_y B_x) - \frac{\partial}{\partial z} (V_z B_x - V_x B_z) + \eta \left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2} \right)$$

$$\boxed{\rho \frac{\partial B}{\partial t} = \frac{\partial}{\partial y} V_x B_y - \frac{\partial}{\partial y} V_y B_x - \frac{\partial}{\partial z} V_z B_x + \frac{\partial}{\partial z} V_x B_z + \eta \frac{\partial^2 B_x}{\partial x^2} + \eta \frac{\partial^2 B_x}{\partial y^2} + \eta \frac{\partial^2 B_x}{\partial z^2}}$$

$$4. \quad \nabla \cdot B = 0 \\ \boxed{\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0}$$

Step 2:

After the "vector juggling" one obtains the following system of equations which one will solve.

$$\left\{ \begin{array}{l} 1. \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \\ 2. \rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} + \frac{\partial p}{\partial x} - \frac{1}{\mu} B_z \frac{\partial B_x}{\partial z} + \frac{1}{\mu} B_z \frac{\partial B_z}{\partial x} + \frac{1}{\mu} B_y \frac{\partial B_y}{\partial x} - \frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} = \rho g_x \\ 3. \\ \rho \frac{\partial B_x}{\partial t} - V_x \frac{\partial B_y}{\partial y} - B_y \frac{\partial V_x}{\partial y} + V_y \frac{\partial B_x}{\partial y} + B_x \frac{\partial V_y}{\partial y} + V_z \frac{\partial B_x}{\partial z} + B_x \frac{\partial V_z}{\partial z} - V_x \frac{\partial B_z}{\partial z} - B_z \frac{\partial V_x}{\partial z} - \frac{\eta \partial^2 B_x}{\partial x^2} - \frac{\eta \partial^2 B_x}{\eta \partial y^2} - \frac{\eta \partial^2 B_x}{\eta \partial z^2} = 0 \\ 4. \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \end{array} \right.$$

At a glance, and from the experience gained in solving the Navier-Stokes equations, one can identify equation (2) as the driver equation, since it contains the gravity term, and the gravity term is the subject of the equation. However, since the system of equations is to be solved simultaneously and there is only a single "driver", the gravity term, all the terms in the system of equations will be placed in the driver equation, Equation 2. As suggested by Albert Einstein, Friedrich Nietzsche, and Pablo Picasso, one will think like a child at the next step.

Step 3: Thinking like a ninth grader, one will apply the following axiom:

If $a = b$ and $c = d$, then $a + c = b + d$; and therefore, add the left sides and add the right sides of the above equations. That is, $(1) + (2) + (3) + (4) = \rho g_x$

$$\left\{ \begin{array}{l} \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} + \rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} + \frac{\partial p}{\partial x} - \frac{1}{\mu} B_z \frac{\partial B_x}{\partial z} + \frac{1}{\mu} B_z \frac{\partial B_z}{\partial x} + \frac{1}{\mu} B_y \frac{\partial B_y}{\partial x} - \\ \frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} + \frac{\rho \partial B_x}{\partial t} - V_x \frac{\partial B_y}{\partial y} - B_y \frac{\partial V_x}{\partial y} + V_y \frac{\partial B_x}{\partial y} + B_x \frac{\partial V_y}{\partial y} + V_z \frac{\partial B_x}{\partial z} + B_x \frac{\partial V_z}{\partial z} - V_x \frac{\partial B_z}{\partial z} - B_z \frac{\partial V_x}{\partial z} - \\ \frac{\eta \partial^2 B_x}{\partial x^2} - \frac{\eta \partial^2 B_x}{\eta \partial y^2} - \frac{\eta \partial^2 B_x}{\eta \partial z^2} + \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = \rho g_x \end{array} \right. \quad \text{(Three lines per equation)}$$

Step 4: Writing all the linear terms first

$$\left\{ \begin{array}{l} \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} + \rho \frac{\partial V_x}{\partial t} + \frac{\partial p}{\partial x} + \frac{\rho \partial B_x}{\partial t} - \frac{\eta \partial^2 B_x}{\partial x^2} - \frac{\eta \partial^2 B_x}{\eta \partial y^2} - \frac{\eta \partial^2 B_x}{\eta \partial z^2} + \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\ + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} - \frac{1}{\mu} B_z \frac{\partial B_x}{\partial z} + \frac{1}{\mu} B_z \frac{\partial B_z}{\partial x} + \frac{1}{\mu} B_y \frac{\partial B_y}{\partial x} - \frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} - V_x \frac{\partial B_y}{\partial y} - B_y \frac{\partial V_x}{\partial y} \\ + V_y \frac{\partial B_x}{\partial y} + B_x \frac{\partial V_y}{\partial y} + V_z \frac{\partial B_x}{\partial z} + B_x \frac{\partial V_z}{\partial z} - V_x \frac{\partial B_z}{\partial z} - B_z \frac{\partial V_x}{\partial z} = \rho g_x \end{array} \right. \quad \text{(Three lines per equation)}$$

(Since all the terms are now in the same driver equation, let ρg_x "drive them" simultaneously.)

Step 5: Solve the above 28-term equation using the ratio method. (27 ratio terms)

The ratio terms to be used are respectively the following: (Sum of the ratio terms = 1)

$\beta_1, \beta_2, \beta_3, a, b, c, d, f, m, q, r, s, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$

1. $\frac{\partial V_x}{\partial x} = \beta_1 \rho g_x$ $\frac{dV_x}{dx} = \beta_1 \rho g_x$ $V_x = \beta_1 \rho g_x x + C_{16}$	2. $\frac{\partial V_y}{\partial y} = \beta_2 \rho g_x$ $\frac{dV_y}{dy} = \beta_2 \rho g_x$ $V_y = \beta_2 \rho g_x y + C_{17}$	3. $\frac{\partial V_z}{\partial z} = \beta_3 \rho g_x$ $\frac{dV_z}{dz} = \beta_3 \rho g_x$ $V_z = \beta_3 \rho g_x z + C_{18}$	4. $\rho \frac{\partial V_x}{\partial t} = a \rho g_x$ $\frac{\partial V_x}{\partial t} = a g_x$ $V_x = a g_x t + C_{19}$
--	--	--	---

5. $\frac{\partial p}{\partial x} = b\rho g_x$ $\frac{dp}{dx} = b\rho g_x$ $P(x) = b\rho g_x x + C$	6. $\rho \frac{\partial B_x}{\partial t} = c\rho g_x$ $\frac{\partial B_x}{\partial t} = c g_x$ $\frac{dB_x}{dt} = c g_x$ $B_x = c g_x t + C_{1b}$	7. $-\eta \frac{\partial^2 B_x}{\partial x^2} = d\rho g_x$ $\frac{d^2 B_x}{dx^2} = -\frac{d\rho g_x}{\eta}$ $\frac{dB_x}{dx} = -\frac{d\rho g_x x}{\eta} + C_2$ $B_x = -\frac{d\rho g_x x^2}{2\eta} + C_2 x + C_3$
--	--	--

8. $-\eta \frac{\partial^2 B_x}{\partial y^2} = f\rho g_x$ $\frac{d^2 B_x}{dy^2} = -\frac{f\rho g_x}{\eta}$ $\frac{dB_x}{dy} = -\frac{f\rho g_x y}{\eta} + C_4$ $B_x = -\frac{f\rho g_x y^2}{2\eta} + C_4 y + C_5$	9. $-\eta \frac{\partial^2 B_x}{\partial z^2} = m\rho g_x$ $\frac{d^2 B_x}{dz^2} = -\frac{m\rho g_x}{\eta}$ $\frac{dB_x}{dz} = -\frac{m\rho g_x z}{\eta} + C_6$ $B_x = -\frac{m\rho g_x z^2}{2\eta} + C_6 x + C_7$	10. $\frac{\partial B_x}{\partial x} = q\rho g_x$ $\frac{dB_x}{dx} = q\rho g_x$ $B_x = q\rho g_x x + C_{19}$
--	--	---

11. $\frac{\partial B_y}{\partial y} = r\rho g_x$ $\frac{dB_y}{dy} = r\rho g_x$ $B_y = r\rho g_x y + C_{20}$	12. $\frac{\partial B_z}{\partial z} = s\rho g_x$ $\frac{dB_z}{dz} = s\rho g_x$ $B_z = s\rho g_x z + C_{21}$	13. $\rho V_x \frac{\partial V_x}{\partial x} = \omega_1 \rho g_x$ $V_x \frac{dV_x}{dx} = \omega_1 g_x$ $V_x dV_x = \omega_1 g_x dx$ $\frac{V_x^2}{2} = \omega_1 g_x x$ $V_x^2 = 2\omega_1 g_x x$ $V_x = \pm \sqrt{2\omega_1 g_x x} + C_2$	14. $\rho V_y \frac{\partial V_x}{\partial y} = \omega_2 \rho g_x$ $V_y dV_x = \omega_2 g_x dy$ $V_y V_x = \omega_2 g_x y + \psi_y(V_y)$ $V_x = \frac{\omega_2 g_x y}{V_y} + \frac{\psi_y(V_y)}{V_y}$ $V_y \neq 0$
---	---	--	---

15. $\rho V_z \frac{\partial V_x}{\partial z} = \omega_3 \rho g_x$ $V_z \frac{dV_x}{dz} = \omega_3 g_x$ $V_z dV_x = \omega_3 g_x dz$ $V_z V_x = \omega_3 g_x z + \psi_z(V_z)$ $V_x = \frac{\omega_3 g_x z}{V_z} + \frac{\psi_z(V_z)}{V_z}$ $V_z \neq 0$	16. $B_z \frac{\partial B_x}{\partial z} = -\omega_4 \mu \rho g_x$ $B_z dB_x = -\omega_4 \mu \rho g_x dz$ $B_z B_x = -\omega_4 \mu \rho g_x z + \psi_z(B_z)$ $B_x = -\frac{\omega_4 \mu \rho g_x z}{B_z} + \frac{\psi_z(B_z)}{B_z}$ $B_z \neq 0$	17. $B_z \frac{\partial B_z}{\partial x} = \omega_5 \mu \rho g_x$ $B_z \frac{dB_z}{dx} = \omega_5 \mu \rho g_x$ $B_z dB_z = \omega_5 \mu \rho g_x dx$ $\frac{B_z^2}{2} = \omega_5 \mu \rho g_x x$ $B_z^2 = 2\omega_5 \mu \rho g_x x$ $B_z = \pm \sqrt{2\omega_5 \mu \rho g_x x} + C$
---	---	--

18. $B_y \frac{\partial B_y}{\partial x} = \omega_6 \mu \rho g_x$ $B_y \frac{dB_y}{dx} = \omega_6 \mu \rho g_x$ $B_y dB_y = \omega_6 \mu \rho g_x dx$ $\frac{B_y^2}{2} = \omega_6 \mu \rho g_x x$ $B_y^2 = 2\omega_6 \mu \rho g_x x$ $B_y = \pm \sqrt{2\omega_6 \mu \rho g_x x + C}$	19. $-\frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} = \lambda_1 \rho g_x$ $B_y \frac{dB_x}{dy} = -\lambda_1 \mu \rho g_x$ $B_y dB_x = -\lambda_1 \mu \rho g_x dy$ $B_y B_x = -\lambda_1 \mu \rho g_x y + \psi_y(B_y)$ $B_x = -\frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\psi_y(B_y)}{B_y}$ $B_y \neq 0$	20 $-V_x \frac{\partial B_y}{\partial y} = \lambda_2 \rho g_x$ $V_x \frac{dB_y}{dy} = -\lambda_2 \rho g_x$ $V_x dB_y = -\lambda_2 \rho g_x dy$ $V_x B_y = -\lambda_2 \rho g_x y + \psi_x(V_x)$ $B_y = \frac{-\lambda_2 \rho g_x y}{V_x} + \frac{\psi_x(V_x)}{V_x}$ $V_x \neq 0$
---	---	--

21. $-B_y \frac{\partial V_x}{\partial y} = \lambda_3 \rho g_x$ $B_y \frac{dV_x}{dy} = -\lambda_3 \rho g_x$ $B_y dV_x = -\lambda_3 \rho g_x dy$ $B_y V_x = -\lambda_3 \rho g_x y + \psi_y(B_y)$ $V_x = -\frac{\lambda_3 \rho g_x y}{B_y} + \frac{\psi_y(B_y)}{B_y}$ $B_y \neq 0$	22. $V_y \frac{\partial B_x}{\partial y} = \lambda_4 \rho g_x$ $V_y \frac{dB_x}{dy} = \lambda_4 \rho g_x$ $V_y dB_x = \lambda_4 \rho g_x dy$ $V_y B_x = \lambda_4 \rho g_x y + \psi_y(V_y)$ $B_x = \frac{\lambda_4 \rho g_x y}{V_y} + \frac{\psi_y(V_y)}{V_y}$ $V_y \neq 0$	23. $B_x \frac{\partial V_y}{\partial y} = \lambda_5 \rho g_x$ $B_x \frac{dV_y}{dy} = \lambda_5 \rho g_x$ $B_x dV_y = \lambda_5 \rho g_x dy$ $B_x V_y = \lambda_5 \rho g_x y + \psi_x(B_x)$ $V_y = \frac{\lambda_5 \rho g_x y}{B_x} + \frac{\psi_x(B_x)}{B_x}$ $B_x \neq 0$
---	--	--

24. $V_z \frac{\partial B_x}{\partial z} = \lambda_6 \rho g_x$ $V_z \frac{dB_x}{dz} = \lambda_6 \rho g_x$ $V_z dB_x = \lambda_6 \rho g_x dz$ $V_z B_x = \lambda_6 \rho g_x z + \psi_z(V_z)$ $B_x = \frac{\lambda_6 \rho g_x z}{V_z} + \frac{\psi_z(V_z)}{V_z}$ $V_z \neq 0$	25. $B_x \frac{\partial V_z}{\partial z} = \lambda_7 \rho g_x$ $B_x \frac{dV_z}{dz} = \lambda_7 \rho g_x$ $B_x dV_z = \lambda_7 \rho g_x dz$ $B_x V_z = \lambda_7 \rho g_x z + \psi_x(B_x)$ $V_z = \frac{\lambda_7 \rho g_x z}{B_x} + \frac{\psi_x(B_x)}{B_x}$ $B_x \neq 0$	26 $-V_x \frac{\partial B_z}{\partial z} = \lambda_8 \rho g_x$ $V_x \frac{dB_z}{dz} = -\lambda_8 \rho g_x$ $V_x dB_z = -\lambda_8 \rho g_x dz$ $V_x B_z = -\lambda_8 \rho g_x z + \psi_x(V_x)$ $B_z = -\frac{\lambda_8 \rho g_x z}{V_x} + \frac{\psi_x(V_x)}{V_x}$ $V_x \neq 0$
--	--	--

27. $-B_z \frac{\partial V_x}{\partial z} = \lambda_9 \rho g_x$ $B_z \frac{dV_x}{dz} = -\lambda_9 \rho g_x$ $B_z dV_x = -\lambda_9 \rho g_x dz$ $B_z V_x = -\lambda_9 \rho g_x z + \psi_z(B_z)$ $V_x = -\frac{\lambda_9 \rho g_x z}{B_z} + \frac{\psi_z(B_z)}{B_z}$ $B_z \neq 0$

Step 6: One collects the integrals of the sub-equations, above, for $V_x, V_y, V_z, B_x, B_y, B_z, P(x)$

$V_x(x,y,z,t) = \text{(sum of integrals from sub - equations \#1, \#4,\#13,\#14,\#15,\#21,\#27)}$ $\beta_1 \rho g_x x + a g_x t \pm \sqrt{2\omega_1 g_x x} + \frac{\omega_2 g_x y}{V_y} - \frac{\lambda_3 \rho g_x y}{B_y} + \frac{\omega_3 g_x z}{V_z} - \frac{\lambda_9 \rho g_x z}{B_z} + \underbrace{\frac{\psi_z(V_z)}{V_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(B_z)}{B_z}}_{\text{arbitrary functions}} + C_1;$
$\text{(integral from sub-equation \#5)}$ $P(x) = b \rho g_x x + C_2$
$\text{(sum of integrals from sub-equations \#2,\#23)}$ $V_y(y) = \beta_2 \rho g_x y + \frac{\lambda_5 \rho g_x y}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_3$
$\text{(sum of integrals from sub-equations \#3, \#25)}$ $V_z(z) = \beta_3 \rho g_x z + \frac{\lambda_7 \rho g_x z}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_4$
$\text{(sum of integrals from sub - equations \#6, \#7, \#8, \#9, \#10, \#16,\#19, \#22, \#24)}$ $B_x(x,y,z,t) =$ $B_x = -\frac{\rho g_x}{2\eta} (dx^2 + fy^2 + mz^2) + q \rho g_x x + C_2 x + C_4 y + C_6 z + c g_x t - \frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\lambda_4 \rho g_x y}{V_y} - \frac{\omega_4 \mu \rho g_x z}{B_z} +$ $\frac{\lambda_6 \rho g_x z}{V_z} + \underbrace{\frac{\psi_z(B_z)}{B_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_7$
$\text{(sum of integrals from sub-equations \#11,\#18,\#20)}$ $B_y = r \rho g_x y \pm \sqrt{2\omega_6 \mu \rho g_x x} - \frac{\lambda_2 \rho g_x y}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_8$
$\text{(sum of integrals from sub-equations \#12,\#17,\#26)}$ $B_z = s \rho g_x z \pm \sqrt{2\omega_5 \mu \rho g_x x} - \frac{\lambda_8 \rho g_x z}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_{21}$

Step 7: Find the test derivatives for the linear part

1.	2.	3.	4.	5.	6.
$\frac{\partial V_x}{\partial x} = (\beta_1 \rho g_x)$	$\frac{\partial V_y}{\partial y} = (\beta_2 \rho g_x)$	$\frac{\partial V_z}{\partial z} = (\beta_3 \rho g_x)$	$\frac{\partial V_x}{\partial t} = (a g_x)$	$\frac{\partial p}{\partial x} = (b \rho g_x)$	$\frac{dB_x}{dt} = (c g_x)$

7.	8.	9.	10.	11.	12.
$\frac{\partial^2 B_x}{\partial x^2} = -\frac{d \rho g_x}{\eta}$	$\frac{\partial^2 B_x}{\partial y^2} = -\frac{f \rho g_x}{\eta}$	$\frac{\partial^2 B_x}{\partial z^2} = -\frac{m \rho g_x}{\eta}$	$\frac{\partial B_x}{\partial x} = q \rho g_x$	$\frac{\partial B_y}{\partial y} = r \rho g_x$	$\frac{\partial B_z}{\partial z} = s \rho g_x$

Test derivatives for the nonlinear part

13.	14.	15.	16.	17.
$\frac{\partial V_x}{\partial x} = \frac{\omega_1 g_x}{V_x}$	$\frac{\partial V_x}{\partial y} = \frac{\omega_2 g_x}{V_y}$	$\frac{\partial V_x}{\partial z} = \frac{\omega_3 g_x}{V_z}$	$\frac{\partial B_x}{\partial z} = -\frac{\omega_4 \mu \rho g_x}{B_z}$	$\frac{\partial B_z}{\partial x} = \frac{\omega_5 \mu \rho g_x}{B_z}$

18.	19.	20.	21.	22.
$\frac{\partial B_y}{\partial x} = \frac{\omega_6 \mu \rho g_x}{B_y}$	$\frac{\partial B_x}{\partial y} = -\frac{\lambda_1 \mu \rho g_x}{B_y}$	$\frac{\partial B_y}{\partial y} = -\frac{\lambda_2 \rho g_x}{V_x}$	$\frac{\partial V_x}{\partial y} = -\frac{\lambda_3 \rho g_x}{B_y}$	$\frac{\partial B_x}{\partial y} = \frac{\lambda_4 \rho g_x}{V_y}$

23.	24.	25.	26.	27.
$\frac{\partial V_y}{\partial y} = \frac{\lambda_5 \rho g_x}{B_x}$	$\frac{\partial B_x}{\partial z} = \frac{\lambda_6 \rho g_x}{V_z}$	$\frac{\partial V_z}{\partial z} = \frac{\lambda_7 \rho g_x}{B_x}$	$\frac{\partial B_z}{\partial z} = -\frac{\lambda_8 \rho g_x}{V_x}$	$\frac{\partial V_x}{\partial z} = -\frac{\lambda_9 \rho g_x}{B_z}$

Step 8: Substitute the above test derivatives respectively in the following 28-term equation

$$\left\{ \begin{aligned} & \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} + \rho \frac{\partial V_x}{\partial t} + \frac{\partial p}{\partial x} + \frac{\rho \partial B_x}{\partial t} - \frac{\eta \partial^2 B_x}{\partial x^2} - \frac{\eta \partial^2 B_x}{\eta \partial y^2} - \frac{\eta \partial^2 B_x}{\eta \partial z^2} + \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\ & + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} + \rho V_z \frac{\partial V_x}{\partial z} - \frac{1}{\mu} B_z \frac{\partial B_x}{\partial z} + \frac{1}{\mu} B_z \frac{\partial B_z}{\partial x} + \frac{1}{\mu} B_y \frac{\partial B_y}{\partial x} - \frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} - V_x \frac{\partial B_y}{\partial y} - B_y \frac{\partial V_x}{\partial y} \\ & + V_y \frac{\partial B_x}{\partial y} + B_x \frac{\partial V_y}{\partial y} + V_z \frac{\partial B_x}{\partial z} + B_x \frac{\partial V_z}{\partial z} - V_x \frac{\partial B_z}{\partial z} - B_z \frac{\partial V_x}{\partial z} = \rho g_x \end{aligned} \right. \quad \text{(Three lines per equation)}$$

$$\left\{ \begin{aligned} & (\beta_1 \rho g_x) + (\beta_2 \rho g_x) + (\beta_3 \rho g_x) + \rho(a g_x) + (b \rho g_x) + \rho(c g_x) - \eta \left(-\frac{d \rho g_x}{\eta}\right) - \eta \left(-\frac{f \rho g_x}{\eta}\right) - \eta \left(-\frac{m \rho g_x}{\eta}\right) + \\ & (q \rho g_x) + (r \rho g_x) + (s \rho g_x) + \rho V_x \left(\frac{\omega_1 g_x}{V_x}\right) + \rho V_y \left(\frac{\omega_2 g_x}{V_y}\right) + \rho V_z \left(\frac{\omega_3 g_x}{V_z}\right) - \frac{1}{\mu} B_z \left(-\frac{\omega_4 \mu \rho g_x}{B_z}\right) + \\ & \frac{1}{\mu} B_z \left(\frac{\omega_5 \mu \rho g_x}{B_z}\right) + \frac{1}{\mu} B_y \left(\frac{\omega_6 \mu \rho g_x}{B_y}\right) - \frac{1}{\mu} B_y \left(-\frac{\lambda_1 \mu \rho g_x}{B_y}\right) - V_x \left(-\frac{\lambda_2 \rho g_x}{V_x}\right) - B_y \left(-\frac{\lambda_3 \rho g_x}{B_y}\right) + V_y \left(\frac{\lambda_4 \rho g_x}{V_y}\right) + \\ & B_x \left(\frac{\lambda_5 \rho g_x}{B_x}\right) + V_z \left(\frac{\lambda_6 \rho g_x}{V_z}\right) + B_x \left(\frac{\lambda_7 \rho g_x}{B_x}\right) - V_x \left(-\frac{\lambda_8 \rho g_x}{V_x}\right) - B_z \left(-\frac{\lambda_9 \rho g_x}{B_z}\right) = \rho g_x \end{aligned} \right. \quad \text{(Four lines per equation)}$$

$$\left\{ \begin{aligned} & \beta_1 \rho g_x + \beta_2 \rho g_x + \beta_3 \rho g_x + a \rho g_x + b \rho g_x + c \rho g_x + d \rho g_x + f \rho g_x + m \rho g_x + q \rho g_x + r \rho g_x + s \rho g_x + \omega_1 \rho g_x \\ & + \omega_3 \rho g_x + \omega_5 \rho g_x + \omega_6 \rho g_x + \lambda_1 \mu \rho g_x + \lambda_2 \rho g_x + \lambda_3 \rho g_x + \lambda_4 \rho g_x + \lambda_5 \rho g_x + \omega_2 \rho g_x + \omega_3 \rho g_x \\ & + \lambda_6 \rho g_x + \lambda_7 \rho g_x + \lambda_8 \rho g_x + \lambda_9 \rho g_x = \rho g_x \end{aligned} \right. \quad \text{(Three lines per equation)}$$

$$\left\{ \begin{array}{l} \beta_1 g_x + \beta_2 g_x + \beta_3 g_x + a g_x + b g_x + c g_x + d g_x + f g_x + m g_x + q g_x + r g_x + s g_x + \omega_1 g_x + \omega_3 g_x + \omega_5 g_x \\ + \omega_6 g_x + \lambda_1 g_x + \lambda_2 g_x + \lambda_3 g_x + \lambda_4 g_x + \lambda_5 g_x + \omega_2 g_x + \omega_3 g_x + \lambda_6 g_x + \lambda_7 g_x + \lambda_8 g_x + \lambda_9 g_x = g_x \end{array} \right. \quad (2 \text{ lines})$$

$$\left\{ \begin{array}{l} g_x (\beta_1 + \beta_2 + \beta_3 + a + b + c + d + f + m + q + r + s + \omega_1 + \omega_3 + \omega_5 + \lambda_3 + \lambda_4 + \lambda_5 + \omega_2 + \omega_3 + \lambda_6 + \lambda_7 \\ + \omega_6 + \lambda_1 + \lambda_2 + \lambda_8 + \lambda_9) = g_x \end{array} \right. \quad (\text{Two lines per equation})$$

$$g_x(1) = g_x \quad (\text{Sum of the ratio terms} = 1)$$

$$g_x = g_x \quad \text{Yes}$$

Since an identity is obtained, the solutions to the 28-term equation are as follows

$V_x(x, y, z, t) = \quad (\text{sum of integrals from sub-equations \#1, \#4, \#13, \#14, \#15, \#21, \#27})$ $\beta_1 \rho g_x x + a g_x t \pm \sqrt{2\omega_1 g_x x} + \frac{\omega_2 g_x y}{V_y} - \frac{\lambda_3 \rho g_x y}{B_y} + \frac{\omega_3 g_x z}{V_z} - \frac{\lambda_9 \rho g_x z}{B_z} + \underbrace{\frac{\psi_z(V_z)}{V_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(B_z)}{B_z}}_{\text{arbitrary functions}} + C_1;$
$(\text{integral from sub-equation \#5})$ $P(x) = b \rho g_x x + C_2$
$(\text{sum of integrals from sub-equations \#2, \#23})$ $V_y = \beta_2 \rho g_x y + \frac{\lambda_5 \rho g_x y}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_3$
$(\text{sum of integrals from sub-equations \#3, \#25})$ $V_z = \beta_3 \rho g_x z + \frac{\lambda_7 \rho g_x z}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_4$
$(\text{sum of integrals from sub-equations \#6, \#7, \#8, \#9, \#10, \#16, \#19, \#22, \#24})$ $B_x(x, y, z, t) =$ $B_x = -\frac{\rho g_x}{2\eta} (dx^2 + fy^2 + mz^2) + q \rho g_x x + C_2 x + C_4 y + C_6 z + c g_x t - \frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\lambda_4 \rho g_x y}{V_y} - \frac{\omega_4 \mu \rho g_x z}{B_z} +$ $\frac{\lambda_6 \rho g_x z}{V_z} + \underbrace{\frac{\psi_z(B_z)}{B_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_7$
$(\text{sum of integrals from sub-equations \#11, \#18, \#20})$ $B_y = r \rho g_x y \pm \sqrt{2\omega_6 \mu \rho g_x x} - \frac{\lambda_2 \rho g_x y}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_8$
$(\text{sum of integrals from sub-equations \#12, \#17, \#26})$ $B_z = s \rho g_x z \pm \sqrt{2\omega_5 \mu \rho g_x x} - \frac{\lambda_8 \rho g_x z}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_{21}$

Step 9: The linear part of the relation satisfies the linear part of the equation (in Step 8; and the non-linear part of the relation satisfies the non-linear part of the equation. The solutions are above.

Analogy for the Identity Checking Method: If one goes shopping with American dollars and Japanese yens (without any currency conversion) and after shopping, if one wants to check the cost of the items purchased, one would check the cost of the items purchased with dollars against the receipts for the dollars; and one would also check the cost of the items purchased with yens against the receipts for the yens purchase. However, if one converts one currency to the other, one would only have to check the receipts for only a single currency, dollars or yens. This conversion case is similar to the linearized N-S equations, where there was no partitioning in identity checking.

Important insight

One observes above that the most important insight of the above solutions is the indispensability of the gravity term in MHD. Observe that if gravity, g_x , were zero, all the non-constant terms in each solution would be zero. These results can be stated emphatically that without gravity forces on earth, there would be no magnetohydrodynamics on earth as is known. It would not therefore be meaningful to write a system of MHD equations without the gravity term, since there would be no magnetohydrodynamics.

Supporter Equation Contributions (see also viXra:1405.0251)

Note above that there are 28 terms in the driver equation, and 27 supporter equations, Each supporter equation provides useful information about the driver equation. The more of these supporter equations that are integrated, the more the information one obtains about the driver equation. However, without solving a supporter equation, one can sometimes write down some characteristics of the integration relation of the supporter equation by referring to the subjects of the supporter equations of the Navier-Stokes equations. For example, if one uses $(\eta \partial^2 B_x / \partial x^2)$ as the subject of a supporter equation here, the curve for the integration relation obtained would be parabolic, periodic, and decreasingly exponential. Using $\rho(\partial V / \partial t)$ as the subject of the supporter equation, the curve would be periodic and decreasingly exponential. Using $(\partial p / \partial x)$, the curve would be parabolic.

**Comparison of Solutions of Navier-Stokes Equations
and
Solutions of Magnetohydrodynamic Equations**

Navier-Stokes x-direction solution

$V_x(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2) + C_1x + C_3y + C_5z + fg \pm \sqrt{2hgx} + \frac{ngy}{V_y} + \frac{qgz}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$
$P(x) = d\rho g_x x \qquad (V_y \neq 0, V_z \neq 0)$

For magnetohydrodynamic solutions, see previous page

1. V_x for MHD system resembles the V_x for the Euler solution part of N-S solution.
2. $P(x)$ for N-S and MHD equations are the same.
3. V_y and V_z for MHD are different from those of N-S solution.
4. B_x is parabolic and resembles V_x for N-S, except for the absence of the square root function.
5. B_y and B_z resemble the Euler solution part of the N-S solution.

Conclusion

The author proposed and applied a new law to solve the system of magnetohydrodynamic equations. This law states that in magnetohydrodynamics, all the other terms in the system of equations divide the gravity term in a definite ratio, and each term utilizes gravity to function. The experience gained in solving the Navier-Stokes equations guided the author to solve the MHD equations.

It was shown that without gravity forces on earth, there would be no magnetohydrodynamics on earth as is known. The equations in the system of equations were added to produce a single equation which was then integrated. Ratios were used to split-up the single equation, and the resulting sub-equations were readily integrated; and even, the nonlinear sub-equations were readily integrated.

Twenty-seven sub-equations were integrated. The linear part of the relation obtained from the integration of the linear part of the equation satisfied the linear part of the equation; and the relation from the integration of the non-linear part satisfied the non-linear part of the equation. Comparison of the solutions of MHD equations with the solutions of the N-S equations revealed the following:

(a) V_x for MHD system resembles the V_x for the non-linear part of the N-S solution; (b) $P(x)$ for N-S and MHD equations are the same; (c) V_y and V_z for MHD are different from those of N-S solutions; (d) B_x is parabolic and resembles V_x for N-S solution, except for the absence of the square root function; and (e) B_y and B_z resemble the non-linear part of N-S solution.

By solving algebraically and simultaneously for $V_x, V_y, V_z, B_x, B_y, B_z$, the solutions could be expressed in term of x, y, z and t .

In applications, the ratio terms may perhaps be determined using information such as initial and boundary conditions or may have to be determined experimentally. Finally, for any magnetohydrodynamic design, one should always maximize the role of gravity for cost-effectiveness, durability, and dependability. Perhaps, a law for magnetohydrodynamics should read "Sum of everything else equals ρg "; and this would imply the proposed new law that the other terms in the system of equations divide the gravity term in a definite ratio, and each term utilizes gravity to function.

Note: The liquid pressure, P at the bottom of a liquid of depth h units is given by $P = \rho gh$.

From the MHD solutions in this paper, $P(x) = b\rho gx$ from integrating $\frac{dp}{dx} = b\rho g$ where b is ratio term. Each of the other terms in the MHD equation must also be set equal to the product of a ratio term and ρg . This result implies that the approach used in solving the MHD equations is valid.

P.S.

The author spent more time on "vector juggling" than on the integration of the equations, since no complete system without vector notation was available either in textbooks or on-line. The integration took less time because of the experience with the N-S equations. Any error in the vector juggling part, if any, can be integrated within minutes.

INDEX-Power of Ratios

30-60-90 degree triangle 58

45-45-90 degree triangle 57

A

Acceleration 76

Addition

of mixed numbers 146, 147

Angle of depression 56

Angle of elevation 55

Applications of Proportion in Engineering 89

Applications of Ratios in Business 91

Axioms 181

B

Base

finding the base 171

Biology Ratios 90

Boyle's Law 72

Business and economics applications 111

C

Calculations Involving Percent 166

Calculations Involving Percent (%) 169

Capacitance 77

Charles' Law 73

Checking Solutions of Equations 185

Chemical Equations 80

Chemical similarity 89

Circular motion 76

Classification of Equations 110

Closed-loop transfer function 94

CMI Millennium Prize Problem Requirements 138

Combined Gas Law 75

Combustion 93

Common Fractions 156

Comparison

of decimals 158

of Fractions 148

Comparison of Congruency and Similarity of Triangles 48

Complex Decimals 161

converting to a fraction 161

Complex Fractions

division 154

Compound Interest 91

Converting

a Complex decimal to a fraction 161

a decimal to a fraction 159

Coulomb's Law 77

Cross Rates 91

Cross-multiplication 187

D

- decimal digits 157
- decimal fraction 156
- Decimal places 157
 - number of 157
 - of complex decimals 161
- Decimals
 - addition 157
 - changing to percent 166
 - comparison of decimals 158
 - complex decimals 161
 - dividing by a decimal 163
 - dividing by a power of 10 158
 - dividing by a whole number 162
 - multiplication by a decimal 158
 - multiplying by a power of 10 158
 - subtraction 157
- Density 79
- Differential Equations 93
- Dimensional Consistency 96
- Discount Problems 179
- Dividing decimals 158
- Division
 - by a decimal 163
 - complex fractions 154
 - of Fractions 153
 - of Mixed Numbers 154
- Dosage Calculations 83
- Driver Equation 110
- Dynamic similarity 89

E

- Elasticity 76
- Electric Current 77
- Electric field 77
- Engineering Ratios 90
- Equation of a line
 - horizontal line 68
 - vertical line 69
- Equations
 - checking solutions 185
 - cross-multiplication 187
 - of Straight Lines 62
- equivalent decimal fraction 156
- Equivalent decimals 157
- Equivalent Fractions 143, 145
 - forming 143
- Euler Equations of Fluid flow 112, 235

F

- First Degree Equations 181
- Fluid flow design considerations 111
- Food Preparation, 86
- Fraction
 - unit fraction 141
- Fractions 141
 - addition 145
 - arranging in decreasing or increasing order 150
 - comparison 148
 - converting to a decimal 164
 - decimal fractions 156
 - division 153
 - equivalent fractions 143
 - improper fraction 141
 - largest fraction 149
 - like fraction 145
 - multiplication 153
 - ordering 150
 - reducing to lowest terms 142
 - smallest fraction 149
 - subtraction 146
 - terms 141
 - unlike fractions 145
 - writing in words 142
- Friction 76

G

- Gas Laws 75
- Gay-Lussac's Law 74
- Geometric similarity 89
- golden number 93
- Golden Ratio 93
- golden rectangle, 93
- golden section 93
- Gram-molecular weight 80
- Graph
 - of a Horizontal Line 68
- Graphs
 - vertical lines 69

H

- Horizontal line
 - equation 68

I

- Illumination 78
- Improper Fraction 141
 - converting to a mixed number 141
- Income and Expenses 92
- Index of refraction 77, 78
- Interest rate 91
- Intravenous flow rate calculation 85

K

- Kinematic Viscosity 77

L

- Law of Definite composition 79
- Law of Multiple Proportions 79
- LCD 145
- Like Fractions 145
- Linear Equations 181
 - solutions 186
- Linear Magnification 78
- Linearization of the Non-Linear Terms 102
- Lines
 - equations 62
- Lowest terms
 - of fractions 142

M

- Mach number 93
- Machine Design 87
- Magnetic Intensity 77
- Magnetohydrodynamic Equations 120, 291
- Mechanical Advantage 76
- Mechanical similarity 89
- Mixed Number 141
- Mixed numbers
 - addition 147
 - converting to a fraction 141
 - division 154
 - in words 142
 - multiplication 154
 - subtraction 151
- Model-Prototype Design 89
- Modulus of elasticity 76
- Mole 80
- Mole fraction 80
- Molecular weight 80
- Molecule 80
- Multiplication
 - mixed numbers 154
 - of a decimal by a decimal or by a whole number 158
 - of Fractions 153
- Multiplying decimals 157

N

- NP problems 202
- Navier-Stokes Equation 97
- Navier-Stokes Equations 95, 247
- Newton's second law 76
- Nursing Math 83
- Nutrition & Recipes 86

O

- Ohm's law 77
- Other Business Ratio Terms 92

P

- P vs NP 202
- Parity Ratio 91
- Percent
 - changing to a decimal 167
 - changing to a fraction 167
- Percent (%) 166
- Percent problems 179, 180
- Percentage 169
- Power 76
- Powers of 10, 156
- Pressure 76
- Price-Earnings (P.E.) Ratio 91
- Proper Fraction 141
- Proportion 39
 - compound proportion 33
 - direct proportion 8
 - inverse proportion 9
 - types of proportion 7, 10
- Proportion Problems
 - methods for solving direct proportion problems 12
 - methods for solving inverse problems 28

R

- Radian-Degree Conversions 50
- Rate 91
- Rate %
 - finding 174
- Ratio 39
 - comparing quantities 3
 - dividing a quantity into parts 5
 - reducing to lowest terms 1
- Ratio scale 91
- Ratio test in calculus 93
- Reciprocals 153
- Resistance 77
- Reynolds number, 93
- Right Triangle Trigonometry 53
- Round-off place 162

S

- Salary Change Problem 179
- Sales Tax Problem 180
- Sales Tax Problems 177
- Science Ratios 90
- signs of the slopes of lines 65
- Similar Triangles 39
 - properties 41
- Simple Interest 91
- Slope
 - of a horizontal Line 60
 - of a Vertical Line 61

Slope of a line
 given two points on the line 60
 horizontal line 65
 sign of 65
 vertical line 65
 smallest fraction 149
 Snell's law 78
 Solutions of 3-D Navier-Stokes Equations 114
 Solutions of 4-D Linearized Navier-Stokes Equations 111
 Solutions of 4-D Navier-Stokes Equations 116
 Solutions of Navier-Stokes Equations
 Method 2 128
 Solutions of the Magnetohydrodynamic Equations 120
 Solving First Degree Equations 183
 Special triangles 57
 Specific Gravity 79
 Streamlines 93
 Subtraction
 decimals 157
 Mixed numbers 151
 Supporter equation 110

T

Tasks in Dosage Calculations 83
 Telescope 78
 Thermal similarity (89
 Traveling salesman 220
 Trigonometric Functions 53
 Trigonometric Ratios 53

U

Unit fraction 141
 Unitary Method 17
 Units Label Method 13
 Units Label or Dimensions Method 83

V

Valence 80
 Vapor density 80
 Variation
 compound variation 33
 direct variation 7
 Variation Method 14
 Velocity 76
 Vertical line
 equation 69
 Void Ratio 93

W

Whole numbers
 dividing by a power of 10 158
 multiplying by a power of 10 157

Some useful conversion factors**Units of length**

1 cm = .3937 in = .0328 ft = .01094 yd
 1 m = 100 cm = 39.3701 in = 3.2808 ft = 1.0936 yd
 1 in. = 2.54 cm = .0833 ft = .0254 m
 1 ft = 12 in. = 30.48 cm = .3048 m = .3333 yd
 1 mile = 1760 yd = 5280 ft = 1.6093 km
 1 yd = 3 ft = 36 in. = .9144 m = 91.44 cm
 1 km = .62137 mile = 1000 m = 100,000 cm

Units of mass

1 kg = 1000 g = 2.2046 lb = 35.274 oz
 1 lb (avdp) = 453.592 g = 16 oz
 1 metric ton = 1000 kg = 10^6 g = 1.1023 ton
 1 ton = 907.1847 kg = 2000 lb = .9072 metric ton
 1 gm = .03527 oz = 1000 mg = .0022046 lb
 1 oz = 28.3495 g = .0625 lb = 16 drams
 1 long ton (British) = 2240 lb

Units of volume

1 liter = 1000 cm³ = 61.0237 in.³ = .26417 gal = 1.0567 qt = .03531 ft³ = 2.113 pt
 1 gal (U.S.) = 4 qt = 3.7854 liter = 8 pt = 231 in.³ = .13368 ft³
 1 qt = 2 pt = .946353 liter = 946.353 cm³ = 57.75 in.³ = .25 gal = .034201 ft³
 1 cord = 128 ft³
 1 pt = .473 liter = .5 qt
 1 ft³ = 1728 in.³
 1 yd³ = 27 ft³ = 46656 in.³

Units of area

1 ft² (sq. ft) = 144 in.² (sq. in.)
 1 yd² (sq. yd) = 9 ft² (sq. ft) = 1296 sq. in.
 1 mile² (sq. mile) = 640 acres
 1 m² = 10⁴ cm²
 1 acre = 4840 yd²

Symbols for units

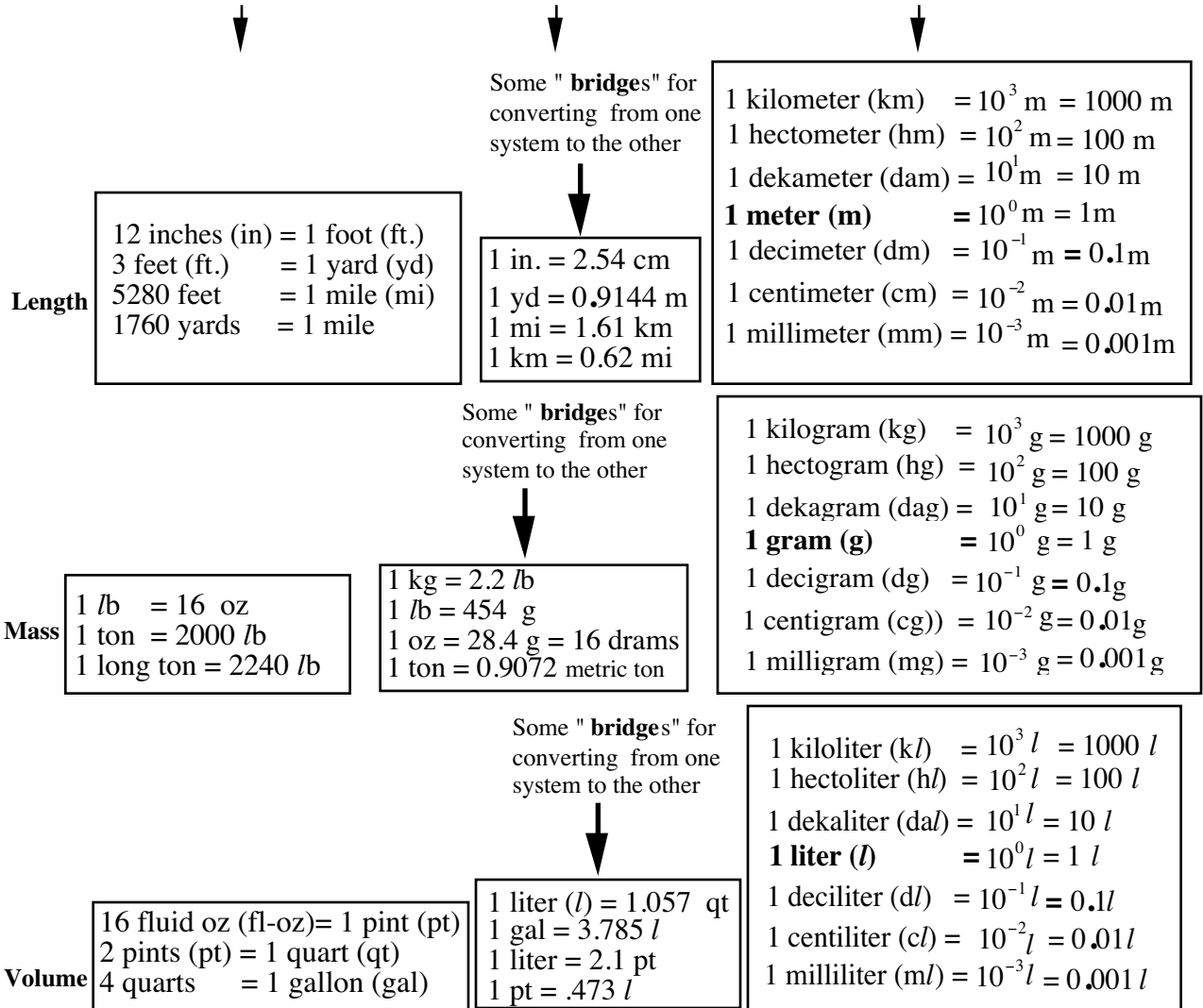
cm = centimeter	gal = gallon	g = gram
m = meter	qt = quart	kg = kilogram
in. = inch	pt = pint	lb = pound
ft = foot	oz = ounce	mg = milligram
yd = yard		
km = kilometer		
mi = mile		

Some prefixes (International System)

Prefix	Power
tera	10 ¹²
giga	10 ⁹
mega	10 ⁶
kilo	10 ³
hecto	10 ²
deka	10 ¹
deci	10 ⁻¹
centi	10 ⁻²
milli	10 ⁻³
micro	10 ⁻⁶
nano	10 ⁻⁹
pico	10 ⁻¹²

Conversion Factors for Measurements

American System (British System) Interconversion (Factors) Metric System



Must remember the following (metric system):

100 cm = 1 m
 1000 m = 1 km

1000 mg = 1 g
 1000 g = 1 kg

1000 ml = 1 l
 1 ml = 1 cc = 1 cm³
 1000 cc = 1 l

Mnemonic device (metric system)

k - ilo - 10^3
 h - ecto - 10^2
 d - eka - 10^1
 d - eci - 10^{-1}
 c - enti - 10^{-2}
 m - illi - 10^{-3}

Say the following aloud:

Step 1: First go down vertically as kei-eitch-dii-dii-see-em, then Step 2
 Step 2: Kilo-hecto-deka-deci-centi-milli, and then note how the powers decrease vertically downwards.
 Examples: 1 Kilometer = 10^3 meter; 1 milligram = 10^{-3} gram;
 1 centimeter = 10^{-2} meter = $\frac{1}{100}$ meter ---> 100 centimeters = 1 meter.

Mathematical Modeling

Some Reciprocal Relationships

- 1. Arithmetic** If A working alone can do a piece of work in time t_A ; B working alone can do the same work in time t_B ; C working alone can do the same work in time t_C , and if A, B, and C working together, can do the same work in time t_{ABC} , then

$$\boxed{\frac{1}{t_{ABC}} = \frac{1}{t_A} + \frac{1}{t_B} + \frac{1}{t_C}}$$

That is, the reciprocal of the working-together time equals the sum of the reciprocals of working-alone times (individual times).

- 2. Geometry:** For any triangle, the reciprocal of the inradius (R) equals the sum of the reciprocals of the exradii (r_1, r_2 , and r_3).

$$\text{Thus } \boxed{\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

- 3. Physics** (Electricity) For electrical resistances in parallel (in an electric circuit), the reciprocal of the combined resistance, R , equals the sum of the reciprocals of the separate resistances, r_1, r_2 , and r_3 .

$$\text{Thus } \boxed{\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

- 4. Physics** (Optics)

For two thin lenses in contact, the reciprocal of the combined focal length, F , equals the sum of the reciprocals of the separate focal lengths, f_1 and f_2 .

$$\text{Thus } \boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}}$$

- 5. Physics** (Optics) For spherical mirrors and thin lenses, the reciprocal of the focal length F equals the sum of the reciprocals of the object distance, d_o and the image distance d_i .

$$\text{Thus } \boxed{\frac{1}{F} = \frac{1}{d_o} + \frac{1}{d_i}}$$

- 6. Physics** (Mechanics). If two bubbles of radii r_1, r_2 , coalesce into a double bubble, the radius,

R , of the partition is given by

$$\boxed{\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}}$$