Notable observation on a property of Carmichael numbers

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I conjecture that for any Carmichael number C is true one of the following two statements: (i) there exist at least one prime q, q lesser than Sqr (C), such that p = (C - q)/(q - 1) is prime, power of prime or semiprime m*n, n > m, with the property that n - m + 1 is prime or power of prime or n + m - 1 is prime or power of prime; (ii) there exist at least one prime q, q lesser than Sqr (C), such that p = (C - q)/((q - 1)*2^n) is prime or power of prime. In two previous papers I made similar assumptions on the squares of primes of the form 10k + 1 respectively 10k + 9 and I always believed that Fermat pseudoprimes behave in several times like squares of primes.

Conjecture:

For any Carmichael number C is true one of the following two statements:

(i) there exist at least one prime q, q lesser than Sqr (C), such that p = (C - q)/(q - 1) is prime, power of prime or semiprime m*n, n > m, with the property that n - m + 1 is prime or power of prime or n + m - 1 is prime or power of prime;

(ii) there exist at least one prime q, q lesser than Sqr (C), such that p = (C - q)/((q - 1)*2^n) is prime or power of prime.

Verifying the conjecture:
(for the first ten Carmichael numbers)

C = 561 and (C - 5)/4 = 139, prime; also (C - 11)/10 = 5*11, semiprime such that 11 - 5 + 1 = 7, prime; also (C - 17)/(16*2) = 17, prime;

C = 1105 and (C - 7)/6 = 3*61, semiprime such that 61 - 3 + 1 = 59, prime; also (C - 13)/12 = 7*13, semiprime such that 13 - 7 + 1 = 7, prime and 13 + 7 - 1 = 19, prime; also (C - 17)/(16*2^2) = 17, prime;

C = 1729 and (C - 5)/4 = 431, prime; also (C - 17)/16 = 107, prime; also (C - 37)/36 = 47, prime;

C = 2465 and (C - 29)/28 = 3*29, semiprime such that 29 - 3 + 1 = 27 = 3^3, power of prime and 29 + 3 - 1 = 31, prime;
\[ C = 2821 \text{ and } (C - 7)/6 = 7 \times 67, \text{ semiprime such that } 67 - 7 + 1 = 61, \text{ prime and } 67 + 7 - 1 = 73, \text{ prime; also } (C - 11)/10 = 281, \text{ prime; also } (C - 31)/30 = 3 \times 31, \text{ semiprime such that } 31 - 3 + 1 = 29, \text{ prime;}
\]
\[ C = 6601 \text{ and } (C - 5)/4 = 17 \times 97, \text{ semiprime such that } 97 - 17 + 1 = 81 = 3^4, \text{ power of prime and } 97 + 17 - 1 = 113, \text{ prime; also } (C - 7)/6 = 7 \times 157, \text{ semiprime such that } 157 - 7 + 1 = 151, \text{ prime and } 157 + 7 - 1 = 163, \text{ prime; also } (C - 11)/10 = 659, \text{ prime; also } (C - 23)/22 = 13 \times 23, \text{ semiprime such that } 23 - 13 + 1 = 11, \text{ prime; also } (C - 31)/30 = 3 \times 73, \text{ semiprime such that } 73 - 3 + 1 = 71, \text{ prime; also } (C - 61)/60 = 109, \text{ prime;}
\]
\[ C = 8911 \text{ and } (C - 23)/(22 \times 2^2) = 101, \text{ prime; also } (C - 31)/(30 \times 2^3) = 37, \text{ prime; also } (C - 67)/(66 \times 2) = 67, \text{ prime;}
\]
\[ C = 10585 \text{ and } (C - 7)/6 = 41 \times 43, \text{ semiprime such that } 43 - 41 + 1 = 3, \text{ prime and } 43 + 41 - 1 = 83, \text{ prime; also } (C - 13)/12 = 881, \text{ prime; also } (C - 19)/18 = 587, \text{ prime; also } (C - 29)/28 = 13 \times 29, \text{ semiprime such that } 29 - 13 + 1 = 17, \text{ prime and } 13 + 29 - 1 = 41, \text{ prime; also } (C - 37)/36 = 293, \text{ prime; also } (C - 43)/42 = 251, \text{ prime; also } (C - 73)/(73 \times 2) = 73, \text{ prime;}
\]
\[ C = 5841 \text{ and } (C - 5)/4 = 37 \times 107, \text{ semiprime such that } 107 - 37 + 1 = 71, \text{ prime; also } (C - 11)/10 = 1583, \text{ prime; also } (C - 13)/12 = 1319, \text{ prime; also } (C - 61)/60 = 263, \text{ prime; also } (C - 67)/66 = 239, \text{ prime; also } (C - 73)/72 = 3 \times 73, \text{ semiprime such that } 73 - 3 + 1 = 71, \text{ prime; also } (C - 89)/88 = 179, \text{ prime; also } (C - 97)/(96 \times 2^2) = 41, \text{ prime;}
\]
\[ C = 29341 \text{ and } (C - 7)/6 = 4889, \text{ prime; also } (C - 31)/30 = 977, \text{ prime; also } (C - 61)/(60 \times 2^3) = 61, \text{ prime.} \]