The Corpuscular Structure of Matter, the Interaction of Material Particles, and Quantum Phenomena as a Concequence of Selfvariations.

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Abstract

With the term "Law of Selfvariations" we mean an exactly determined increase of the rest mass and electric charge of material particle. In this article we present the basic theoretical investigation of the law of selfvariations. We arrive at the central conclusion that the interaction of material particles, the corpuscular structure of matter, and the quantum phenomena can be justified by the law of Selfvariations. We predict a unified interaction between material particles with a unified mechanism (Unified Selfvariations Interaction, USVI). Every interaction is the result of three clearly distinct terms with clearly distinct consequences in the USVI. We predict a wave equation, whose special cases are the Maxwell equations, the Schrödinger equation, and the related wave equations. We determine a mathematical expression for the total of the conservable physical quantities, and we calculate the current density 4-vector. The corpuscular structure and wave behaviour of matter and their relation emerge clearly, and we give a calculation method for the rest masses of material particles. We prove the «internal symmetry» theorem which justifies the cosmological data. From the study we present, the method for the further investigation of the Selfvariations and their consequences also emerges.

Keywords: Particles and Fields, Quantum Physics, Cosmology.

1. Introduction

The present study is founded on three axioms: The principle of the conservation of the four-vector of momentum, the equation of the Theory of Special Relativity for the rest mass of the material particles, and the law of Selfvariations.

With the term "Law of Selfvariations" we mean an exactly determined increase of the rest mass and electric charge of material particle. It is consistent with the principles of conservation of energy, momentum, angular momentum and electric charge. It is also invariant under the Lorentz-Einstein transformations.

The most immediate consequence of the law of Selfvariations is that the energy, the momentum, the angular momentum, and the electric charge of material particles are distributed in the surrounding spacetime (when the material particle is electrically charged).

In order for the value of the electric charge to increase in absolute value, the electron, in some way, should 'emit' a positive electric charge in the space-time environment. Otherwise, the conservation of the electric charge is violated. Similarly, the increase of the rest mass of the material particle involves the "emission" of negative energy as well as momentum in the space-time surrounding the material particle (spacetime energy-momentum, STEM). The law of Selfvariations describes quantitatively the interaction of material particles with the STEM.

Every material particle interacts both with the STEM emitted by itself due to the selfvariations, and with the STEM originating from other material particles. The material particle and the STEM with which it interacts, comprise a dynamic system which we called "generalized particle". We study this continuous interaction in the present article. For the formulation of the equations the following notation is used:

$W =$ the energy of the material particle

$\mathbf{J} =$ the momentum of the material particle

$m_0 =$ the rest mass of the material particle

$E =$ the energy of the STEM interacting with the material particle

$\mathbf{P} =$ the momentum of the STEM interacting with the material particle

$E_0 =$ the rest energy of the STEM interacting with the material particle

With the above symbolism, the law of Selfvariations for the rest mass is given by equations

$$\begin{aligned}\frac{\partial m_0}{\partial t} &= -\frac{b}{\hbar} E m_0 \\ \nabla m_0 &= \frac{b}{\hbar} \mathbf{P} m_0\end{aligned} \quad . \tag{1.1}$$

in every system of reference $0(\mathrm{t}, x, y, z)$. , $\hbar$ is Planck's constant, b constant, $b \neq 0, b \in \mathbb{C}$ and

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}.$$

The the findings resulting from the law of Selfvariations will be referred to as "the Theory of Selfvariations" (TSV). Initially, we present the TSV in inertial frames of reference.

2. The basic study of the internal structure of the generalized particle

We consider a material particle with rest mass $m_0 \neq 0$. That is, we consider a generalized particle. The rest mass m_0 and the rest energy E_0 given by equations (2.1) and (2.2) respectively according to special relativity [1-4]

$$m_0^2 c^4 = W^2 - c^2 \mathbf{J}^2 \tag{2.1}$$

$$E_0{}^2 = E^2 - c^2 \mathbf{P}^2 \quad . \tag{2.2}$$

We now denote the four-vectors

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ict \\ x \\ y \\ z \end{bmatrix} \tag{2.3}$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{iw}{c} \\ J_x \\ J_y \\ J_z \end{bmatrix} \tag{2.4}$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_x \\ P_y \\ P_z \end{bmatrix} \tag{2.5}$$

where c **is** the vacuum velocity of light snd i is the imaginary unit, $i^2 = -1$.

Using this notation, equations (1.1), (2.1) and (2.2) are written in the form of equations (2.6), (2.7) and (2.8)

$$\frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k m_0, k = 0,1,2,3 \tag{2.6}$$

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 + m_0{}^2 c^2 = 0 \tag{2.7}$$

$$P_0^2 + P_1^2 + P_2^2 + P_3^2 + \frac{E_0^2}{c^2} = 0. \tag{2.8}$$

After differentiating equation (2.7) with respect to $x_k, k = 0,1,2,3$ we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} + m_0 c^2 \frac{\partial m_0}{\partial x_k} = 0$$

and with equation (2.6) we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} + \frac{b}{\hbar} P_k m_0^2 c^2 = 0$$

and with equation (2.7) we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar} P_k \left(J_0^2 + J_1^2 + J_2^2 + J_3^2 \right) = 0$$

$$J_0\left(\frac{\partial J_0}{\partial x_k}-\frac{b}{\hbar}P_k J_0\right)+J_1\left(\frac{\partial J_1}{\partial x_k}-\frac{b}{\hbar}P_k J_1\right)$$
$$+J_2\left(\frac{\partial J_2}{\partial x_k}-\frac{b}{\hbar}P_k J_2\right)+J_3\left(\frac{\partial J_3}{\partial x_k}-\frac{b}{\hbar}P_k J_3\right)=0, k=0,1,2,3 \quad . \tag{2.9}$$

We now symbolize

$$\frac{\partial J_i}{\partial x_k}-\frac{b}{\hbar}P_k J_i=\lambda_{ki},\ k,i=0,1,2,3. \tag{2.10}$$

With this notation, equation (2.9) can be written in the form

$$J_0\lambda_{k0}+J_1\lambda_{k1}+J_2\lambda_{k2}+J_3\lambda_{k3}=0,\ k=0,1,2,3. \tag{2.11}$$

We now need the 4×4 matrix T as given by equation

$$T=\begin{bmatrix}\lambda_{00} & \lambda_{01} & \lambda_{02} & \lambda_{03}\\ \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13}\\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23}\\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33}\end{bmatrix}. \tag{2.12}$$

With this notation, equation (2.11) can be written in the form

$$TJ=0. \tag{2.13}$$

We now prove the following theorem:

Theorem 2.1′′For $m_0\neq 0$, and for every $k,i=0,1,2,3$ equation (2.14) holds

$$\frac{\partial P_i}{\partial x_k}=\frac{\partial P_k}{\partial x_i},k\neq i,k,i=0,1,2,3\,'' \tag{2.14}$$

Proof. Indeed, by differentiating equation (2.6) with respect to $x_i,\ i=0,1,2,3$ we get

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right)=\frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

and using the identity

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right)=\frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right)$$

we get

$$\frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right)=\frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

and with equation (2.6) we have

$$\frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_i m_0\right)=\frac{b}{\hbar}\frac{\partial}{\partial x_i}\left(P_k m_0\right)$$

$$P_i\frac{\partial m_0}{\partial x_k}+m_0\frac{\partial P_i}{\partial x_k}=P_k\frac{\partial m_0}{\partial x_i}+m_0\frac{\partial P_k}{\partial x_i}$$

and with equation (2.6) we have

$$P_i\frac{b}{\hbar}P_k m_0+m_0\frac{\partial P_i}{\partial x_k}=P_k\frac{b}{\hbar}P_i m_0+m_0\frac{\partial P_k}{\partial x_i}$$

$$m_0\left(\frac{\partial P_i}{\partial x_k}-\frac{\partial P_k}{\partial x_i}\right)=0$$

and since $m_0\neq 0$, we obtain equation (2.14). □

We now prove the following theorem:

Theorem 2.2 ´´For every $k,i,v=0,1,2,3$ the following equation holds

$$\frac{\partial\lambda_{ki}}{\partial x_v}-\frac{b}{\hbar}P_v\lambda_{ki}=\frac{\partial\lambda_{vi}}{\partial x_k}-\frac{b}{\hbar}P_k\lambda_{vi}\,.´´ \qquad (2.15)$$

Proof. Indeed, by differentiating equation (2.10) with respect to x_v, $v=0,1,2,3$ we get

$$\frac{\partial\lambda_{ki}}{\partial x_v}=\frac{\partial}{\partial x_v}\left(\frac{\partial J_i}{\partial x_k}\right)-\frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

and with identity

$$\frac{\partial}{\partial x_v}\left(\frac{\partial J_i}{\partial x_k}\right)=\frac{\partial}{\partial x_k}\left(\frac{\partial J_i}{\partial x_v}\right)$$

we get

$$\frac{\partial\lambda_{ki}}{\partial x_v}=\frac{\partial}{\partial x_k}\left(\frac{\partial J_i}{\partial x_v}\right)-\frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

and with equation (2.10) we have

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_v J_i + \lambda_{vi}\right) - \frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}\frac{\partial}{\partial x_k}\left(P_v J_i\right) - \frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k} + \frac{b}{\hbar}J_i\frac{\partial P_v}{\partial x_k} - \frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v} - \frac{b}{\hbar}J_i\frac{\partial P_k}{\partial x_v}$$

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v} + \frac{b}{\hbar}J_i\left(\frac{\partial P_v}{\partial x_k} - \frac{\partial P_k}{\partial x_v}\right)$$

and with equation (2.14) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v}$$

and with equation (2.10) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\left(\frac{b}{\hbar}P_k J_i + \lambda_{ki}\right) - \frac{b}{\hbar}P_k\left(\frac{b}{\hbar}P_v J_i + \lambda_{vi}\right)$$

and we finally have

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\lambda_{ki} - \frac{b}{\hbar}P_k\lambda_{vi} .$$

which is equation (2.15). □

3. Physical quantities λ_{ki}, $k,i = 0,1,2,3$ and the conservation principles of energy and momentum

The physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ are related to the conservation of energy and momentum of the generalized particle. This investigation we will present in this section. We prove the following theorem:

Theorem 3.1 ′′If the generalized particle conserves its momentum along the axes $x_i, i = 0,1,2,3$, that is

$$J_i + P_i = c_i = \text{constant} . \tag{3.1}$$

then the following equation holds

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{\hbar}\left(c_k P_i - c_i P_k\right) \tag{3.2}$$

for every $k, i = 0,1,2,3,\ k \neq i.$ ´´

Proof. Combining equations (2.14) and (3.1) we obtain

$$\frac{\partial}{\partial x_k}\left(c_i - J_i\right) = \frac{\partial}{\partial x_i}\left(c_k - J_k\right)$$

$$\frac{\partial J_i}{\partial x_k} = \frac{\partial J_k}{\partial x_i}$$

and with equation (2.10) we get

$$\frac{b}{\hbar} P_k J_i + \lambda_{ki} = \frac{b}{\hbar} P_i J_k + \lambda_{ik}$$

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right)$$

which is equation (3.2). The rest of equations (3.2) are derived taking into account equation (3.1). (3.2). Equation (3.2) holds for $k \neq i,\ k, \mathrm{i} = 0,1,2,3$, since equation (2.14), from which equation (3.2) results is an identity $k = i$ and gives no information in this case.□

We now prove the following theorem:

Theorem 3.2. TSV theorem for the symmetry of indices:

´´If the generalized particle conserves its momentum along the axes x_i and x_k with $k \neq i$, the following equivalences hold

1. $\lambda_{ik} = \lambda_{ki} \Leftrightarrow J_k P_i = J_i P_k \Leftrightarrow c_i J_k = c_k J_i \Leftrightarrow c_k P_i = c_i P_k$. (3.3)

2. $\lambda_{ik} = -\lambda_{ki} \Leftrightarrow \lambda_{ki} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{2\hbar}\left(c_k P_i - c_i P_k\right)$. (3.4)

$k, i = 0,1,2,3,\ k \neq i$.´´

Proof. The theorem is an immediate consequence of equation 3.2. □

We now consider the four-vector C, as given by equation

$$C = J + P = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}. \qquad (3.5)$$

When the generalized particle conserves its momentum along every axis, then the four-vector C is constant. Also, we denote M_0 the total rest mass of the generalized particle, as given by equation

$$C^T C = c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2 \mathrm{c}^2. \qquad (3.6)$$

where C^T is the transposed of the column vector C.

For reasons that will become apparent later in our study, we give the following definitions: We name the symmetry $\lambda_{ik} = \lambda_{ki},\ k \neq i,\ k,i = 0,1,2,3$ internal symmetry, and the symmetry $\lambda_{ik} = -\lambda_{ki},\ k \neq i,\ k,i = 0,1,2,3$ external symmetry. We now prove the following theorem:

Theorem 3.3. Internal Symmetry Theorem:

′′ If the generalized particle conserves its momentum in every axis, the following hold:

1. $\lambda_{ik} = \lambda_{ki}$ for every $k,i = 0,1,2,3 \quad \Leftrightarrow \ J$, P and C are parallel

$$\Leftrightarrow P = \Phi J \text{ where } \Phi \in \mathbb{C},\ \Phi \neq 0. \qquad (3.7)$$

2. For $\Phi = -1$ the following equation holds:

$$E_0 = \pm m_0 c^2 \qquad (3.8)$$

3. For $\Phi \neq -1$ the following equations hold:

$$\Phi = K \exp\left[-\frac{b}{\hbar}\left(c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3\right)\right] \qquad (3.9)$$

$$m_0 = \pm \frac{M_0}{1+\Phi} \qquad (3.10)$$

$$E_0 = \pm \frac{M_0 c^2 \Phi}{1+\Phi} \qquad (3.11)$$

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3 \qquad (3.12)$$

$$P_i = \frac{\Phi c_i}{1+\Phi}, \mathrm{i} = 0,1,2,3 \qquad (3.13)$$

where K is a dimensionless constant physical quantity.

4. We have $\lambda_{ik} = \lambda_{ki}$ for every $k, \mathrm{i} = 0,1,2,3$

$$\Leftrightarrow \qquad (3.14)$$

$\lambda_{ki} = 0$ for every $k, \mathrm{i} = 0,1,2,3$. ′′

Proof. Equivalence (3.7) results immediately from equivalence (3.3). For $\Phi = 0$ from the last of equivalence (3.7) we obtain $P = 0,$ which is impossible, since in this case the Selfvariations of the rest mass $m_0 \neq 0$, do not exist, as seen from equation (2.6). Therefore, $\Phi \neq 0.$ For $\Phi = -1$ from the last of equivalence (3.7) we obtain $P = -J$ and from equations (2.7) and (2.8) we obtain

$$E_0^2 = m_0^2 c^4$$

which is equation (3.8).

For $\Phi \neq -1$ from the last of equivalence (3.7) we obtain $P_i = \Phi J_i$ for every $i = 0,1,2,3$ and with equation (3.1) $J_i + P_i = c_i$ we initially obtain equations (3.12) and (3.13). Then, combining equations (2.7) and (3.12) we get

$$m_0^2 c^2 + \frac{1}{(\Phi+1)^2}\left(c_0^2 + c_1^2 + c_2^2 + c_3^2\right) = 0$$

and with equation (3.6) we obtain equation

$$m_0^2 c^2 - \frac{M_0^2 c^2}{(\Phi+1)^2} = 0 \qquad (3.15)$$

and we finally have

$$m_0 = \pm \frac{M_0}{1+\Phi}$$

which is equation (3.10). Similarly, combining equations (2.8) and (3.13) we obtain equation (3.11). We now prove that function Φ is given by equation (3.9).

Differentiating equation (3.15) with respect to x_v, $v = 0,1,2,3$ and considering equation (2.6) we obtain

$$\frac{2b}{\hbar}P_v m_0^2 c^2 + \frac{2M_0^2 c^2}{(\Phi+1)^3}\frac{\partial\Phi}{\partial x_v} = 0$$

and with equation (3.15) we have

$$\frac{b}{\hbar}P_v \frac{M_0^2 c^2}{(\Phi+1)^2} + \frac{M_0^2 c^2}{(\Phi+1)^3}\frac{\partial\Phi}{\partial x_v} = 0$$

$$\frac{\partial\Phi}{\partial x_v} = -\frac{b}{\hbar}P_v(\Phi+1)$$

and with equation (3.13) for $i = v$ we arrive at equation

$$\frac{\partial\Phi}{\partial x_v} = -\frac{b}{\hbar}c_v\Phi, \quad v = 0,1,2,3. \qquad (3.16)$$

By integration of equation (3.16) we obtain

$$\Phi = K\exp\left[-\frac{b}{\hbar}\left(c_0X_0 + c_1X_1 + c_2X_2 + c_3X_3\right)\right]$$

where K is the integration constant, which is equation (3.9).

Combining equations (2.10), (3.12) and (3.13) for $k = 0,1,2,3$ we obtain

$$\lambda_{ki} = \frac{\partial}{\partial x_k}\left(\frac{c_i}{1+\Phi}\right) - \frac{b}{\hbar}\frac{\Phi c_k}{1+\Phi}\frac{c_i}{1+\Phi}$$

$$\lambda_{ki} = -\frac{c_i}{(1+\Phi)^2}\frac{\partial\Phi}{x_k} - \frac{b}{\hbar}\frac{\Phi c_k c_i}{(1+\Phi)^2}$$

and with equation (3.16) for $v = k$ we obtain

$$\lambda_{ki} = \frac{c_i}{(1+\Phi)^2}\frac{b}{\hbar}c_k\Phi - \frac{b}{\hbar}\frac{\Phi c_k c_i}{(1+\Phi)^2}$$

$$\lambda_{ki} = 0. \ \square$$

According to the previous theorem, internal symmetry is equivalent to the parallelism of the four-vectors J, P. Starting from this conclusion we can determine the physical content of the internal symmetry.

In an isotropic space the spontaneous emission of generalized photons by the material particle is isotropic. Due to the linearity of the Lorentz-Einstein transformations, this isotropic emission has as a consequence the parallelism of the four-vectors J, P ([5] par.

5.3). Thus, the theorem of internal symmetry 3.3 holds for the spontaneous emission of generalized photons by the material particle due to Selfvariations .

In the following paragraphs, we will make clear that the internal symmetry refers to a spontaneous internal increase of the rest mass and the electrical charge of the material particles, independent of any external causes. The consequences of this increase is the cosmological data, as we'll see in Paragraph 11. Also, the internal symmetry is associated with Heisenberg's uncertainty principle.

We start the investigation of the external symmetry with the proof of the following theorem:

Theorem 3.4. First theorem of the TSV for the external symmetry: ′′If the generalized particle conserves its momentum along every axis, and the symmetry $\lambda_{ik} = -\lambda_{ki}$ holds for every $\mathrm{k} \neq \mathrm{i}, k, i = 0,1,2,3$, then:

1. $$\begin{aligned} &c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = 0 \\ &c_i J_{vk} + c_k J_{iv} + c_v J_{ki} = 0 \\ &c_i P_{vk} + c_k P_{iv} + c_v P_{ki} = 0 \end{aligned} \qquad (3.17)$$

for every $i \neq v, v \neq k, k \neq i,\ \ k, i, v = 0,1,2,3$.

2. $$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \lambda_{ki} - \frac{bc_v}{2\hbar}\lambda_{ki} = -\frac{b}{\hbar} J_v \lambda_{ki} + \frac{bc_v}{2\hbar}\lambda_{ki} \qquad (3.18)$$

for every $k \neq i, k, i, v = 0,1,2,3$.

3. $$\lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21} = 0 \qquad (3.19)$$. ′′

Proof. From equivalence (3.4) we obtain

$$\lambda_{ki} = \frac{b}{\hbar}\left(c_i J_k - c_k J_i\right), k \neq i, k, i = 0,1,2,3 . \qquad (3.20)$$

Considering equation (3.20) we get

$$c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = \frac{b}{2\hbar}\left[c_i\left(c_k J_v - c_v J_k\right) + c_k\left(c_v J_i - c_i J_v\right) + c_v\left(c_i J_k - c_k J_i\right)\right] = 0 .$$

Thus, we get the first of equations (3.17). Similarly, from the other two equalities of equivalence (3.4) we obtain the second and the third equation of (3.17). Since $k \neq i$ in equivalence (3.4), the physical quantities $\lambda_{\nu k}, \lambda_{i\nu}, \lambda_{ki}$ in equations (3.17) are defined for $\nu \neq k, i \neq \nu, k \neq i, k, i, \nu = 0,1,2,3$.

Differentiating equation (3.20) with respect to $x_\nu, \nu = 0,1,2,3$ we obtain

$$\frac{\partial \lambda_{ki}}{\partial x_\nu} = \frac{b}{2\hbar}\left(c_i \frac{\partial J_k}{\partial x_\nu} - c_k \frac{\partial J_i}{\partial x_\nu} \right)$$

and with equation (2.10) we get

$$\frac{\partial \lambda_{ki}}{\partial x_\nu} = \frac{b}{2\hbar}\left[c_i \left(\frac{b}{\hbar} P_\nu J_k + \lambda_{\nu k} \right) - c_k \left(\frac{b}{\hbar} P_\nu J_i + \lambda_{\nu i} \right) \right]$$
$$\frac{\partial \lambda_{ki}}{\partial x_\nu} = \frac{b}{2\hbar}\left[\frac{b}{\hbar} P_\nu \left(c_i J_k - c_k J_i \right) + c_i \lambda_{\nu k} - c_k \lambda_{\nu i} \right]$$
$$\frac{\partial \lambda_{ki}}{\partial x_\nu} = \frac{b}{\hbar} P_\nu \frac{b}{2\hbar}\left(c_i J_k - c_k J_i \right) + \frac{b}{2\hbar}\left(c_i \lambda_{\nu k} - c_k \lambda_{\nu i} \right)$$

and with equation (3.20) we obtain

$$\frac{\partial \lambda_{ki}}{\partial x_\nu} = \frac{b}{\hbar} P_\nu \lambda_{ki} + \frac{b}{2\hbar}\left(c_i \lambda_{\nu k} - c_k \lambda_{\nu i} \right)$$

and with the first of equations (3.17) we obtain

$$c_i \lambda_{\nu k} - c_k \lambda_{\nu i} = -c_\nu \lambda_{ki}$$

we get

$$\frac{\partial \lambda_{ki}}{\partial x_\nu} = \frac{b}{\hbar} P_\nu \lambda_{ki} - \frac{b c_\nu}{2\hbar} \lambda_{ki}$$

which is equation (3.18). The second equality in equation (3.18) emerges from the substitution

$$P_\nu = c_\nu - J_\nu, \nu = 0,1,2,3$$

according to equation (3.5).

Taking into account equation (3.20) we obtain

$$\lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21} =$$
$$\frac{b^2}{4\hbar^2}\left[\left(c_1 J_0 - c_0 J_1 \right)\left(c_2 J_3 - c_3 J_2 \right) + \left(c_2 J_0 - c_0 J_2 \right)\left(c_3 J_1 - c_1 J_3 \right) + \left(c_3 J_0 - c_0 J_3 \right)\left(c_1 J_2 - c_2 J_1 \right) \right] = 0$$

after the calculations. □

In the next paragraphs we investigate the external symmetry.

4. The Unified Selfvariations Interaction (USVI)

According to the law of selfvariations every material particle interacts both with the STEM emitted by itself due to the selfvariations, and with the STEM originating from other material particles. In the second case, an indirect interaction emerges between material particles through the STEM. STEM emitted by one material particle interact with another material particle. Through this mechanism the TSV predicts a unified interaction between material particles. The individual interactions only emerge from the different, for each particular case, physical quantity Q which selfvariates, resulting in the emission of the corresponding STEM.In this paragraph we study the basic characteristics of the USVI. We suppose that for the generalized particle the conservation of energy-momentum holds, hence the equations of the preceding paragraph also hold. For the rate of change of the four-vector $\frac{1}{m_0}J$ we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right) = -\frac{J_i}{m_0^2}\frac{\partial m_0}{\partial x_k} + \frac{1}{m_0}\frac{\partial J_i}{\partial x_k}$$

and with equations (2.6) and (2.10) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right) = -\frac{J_i}{m_0^2}\frac{b}{\hbar}P_k m_0 + \frac{1}{m_0}\left(\frac{b}{\hbar}P_k J_i + \lambda_{ki}\right)$$

and we finally obtain

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right) = \frac{\lambda_{ki}}{m_0}, \; k,i = 0,1,2,3. \tag{4.1}$$

According to equation (4.1), when $\lambda_{ki} \neq 0$ for at least two indices $k,i,\ \ k,i = 0,1,2,3,$ the kinetic state of the material particle is disturbed. According to equivalence (3.14) in the internal symmetry it is $\lambda_{ki} = 0$ for every $k,i = 0,1,2,3.$ Therefore, in the internal symmetry the material particle maintains its kinetic state. In an isotropic space we expect that the spontaneous emission of STEM by the material particle cannot disturb its kinetic state.

Consequently, the internal symmetry concerns the spontaneous emission of STEM by the material particle in an isotropic space.

In contrast, in the case of the external symmetry it can be $\lambda_{ki} \neq 0$ for some indices $k, i, \ \ k, i = 0,1,2,3$. Therefore, the external symmetry must be due to STEM with which the material particle interacts, and which originate from other material particles. The distribution of STEM depends on the position in space of the material particle relative to other material particles. This leads to the destruction of the isotropy of space for the material particle. The external symmetry factor will emerge in the study that follows.

The initial study of the Selfvariations concerned the rest mass and the electric charge. The study we have presented up to this point allows us to study the Selfvariations in their most general expression.

We consider a physical quantity Q which we shall call selfvariating "charge Q ", or simply charge Q, unaffected by every change of reference frame, therefore Lorentz-Einstein invariant, and obeys the law of Selfvariations, that is equation

$$\frac{\partial Q}{\partial x_k} = \frac{b}{\hbar} P_k Q, \ \ k = 0,1,2,3. \tag{4.2}$$

In equation (4.2) the momentum $P_k, \mathrm{k} = 0,1,2,3$, i.e. the four-vector P, depends on the selfvariating charge Q. Two material particles carrying a selfvariating charge of the same nature interact with each other when the STEM emitted by the charge Q_1 of one of them interacts with the charge Q of the other. In this particular case, we denote Q the charge of the material particle we are studying.

The rest mass m_0 is defined as a quantity of mass or energy divided by c^2, which is invariant according to the Lorentz-Einstein transformations. The 4-vector of the momentum J of the material particle is related to the rest mass m_0 through equation (2.7). The charge Q contributes to the energy content of the material particle and, therefore, also contributes to its rest mass. Furthermore, the charge Q modifies the 4-vector of momentum J of the material particle and, therefore, contributes to the variation of the rest mass m_0 of the material particle. Consequently, for the change of the four-vector J of the material particle due to the charge $Q,$ the four-vector P of equation (2.10) enters into equation (4.2). The consequences of this conclusion become evident when we calculate the rate of change of the four-vector $\frac{1}{Q}J$.

Theorem 4.1 Second theorem of the TSV for the external symmetry:

´´The rate of change of the four-vector $\frac{1}{Q}J$ due to the Selfvariations of the charge Q is given

by equation

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q},\quad k,i=0,1,2,3 \tag{4.3}$$

For $k\neq i$ the physical quantities $\frac{\lambda_{ki}}{Q}$ are given by

$$\frac{\lambda_{ki}}{Q}=za_{ki},\ \mathrm{k}\neq\mathrm{i},\mathrm{k},\mathrm{i}=0,1,2,3 \tag{4.4}$$

where z is the function

$$z=\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]. \tag{4.5}$$

For the constants a_{ki} the following equations hold

$$\begin{aligned} c_ia_{vk}+c_ka_{iv}+c_va_{ki}&=0\\ J_ia_{vk}+J_ka_{iv}+J_va_{ki}&=0\\ P_ia_{vk}+P_ka_{iv}+P_va_{ki}&=0 \end{aligned} \tag{4.6}$$

for every $i\neq v,v\neq k,k\neq i,\ i,k,v=0,1,2,3$.

$$\alpha_{ik}=-\alpha_{ki},k\neq i,k,i=0,1,2,3 \tag{4.7}$$

$$\alpha_{01}\alpha_{32}+\alpha_{02}\alpha_{13}+\alpha_{03}\alpha_{21}=0\,.'' \tag{4.8}$$

Proof. In order to prove the theorem, we take

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=-\frac{J_i}{Q^2}\frac{\partial Q}{\partial x_k}+\frac{1}{Q}\frac{\partial J_i}{\partial x_k}$$

and with equations (4.2) and (2.10) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=-\frac{J_i}{Q^2}\frac{b}{\hbar}P_kQ+\frac{1}{Q}\left(\frac{b}{\hbar}P_kJ_i+\lambda_{ki}\right)$$

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q}$$

which is equation (4.3). Equations (4.2) and (2.10) hold for every $k,\mathrm{i}=0,1,2,3.$ Therefore, equation (4.3) also holds for every $k,i=0,1,2,3.$

For $k\neq i,\ k,i=0,1,2,3$ and $v=0,1,2,3$ equation (3.18) holds and, since $Q\neq 0$, we obtain

$$Q\frac{\partial\lambda_{ki}}{\partial x_v}=\frac{b}{\hbar}P_vQ\lambda_{ki}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

and with equation (4.2) we get

$$Q\frac{\partial\lambda_{ki}}{\partial x_v}=\lambda_{ki}\frac{\partial Q}{\partial x_v}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

$$\frac{\partial}{\partial x_v}\left(\frac{\lambda_{ki}}{Q}\right)=-\frac{bc_v}{2\hbar}\frac{\lambda_{ki}}{Q}$$

and integrating we obtain

$$\frac{\lambda_{ki}}{Q}=a_{ki}\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]$$

where $a_{ki},k\neq i,k,i=0,1,2,3$ are the integration constants, and with (4.5) we get equation (4.4). Equations (4.6) are derived from the combination of equations (3.17) and (4.4), taking into account that $zQ\neq 0$. Equation (4.7) is derived from the combination of equation $\lambda_{ik}=-\lambda_{ki},k\neq i,k,i=0,1,2,3$ with equation (4.4). Simirarly, equation (4.8) is derived from the combination of equations (3.19) and (4.4). □

We will also use equation

$$\frac{\partial z}{\partial x_k}=-\frac{bc_k}{2\hbar}z,k=0,1,2,3 \tag{4.9}$$

which results immediately from equation (4.5).

For $k=i,k,i=0,1,2,3$ equation (4.4) does not hold. So we define the physical quantities Φ_k and T_k as given by equation

$$\Phi_k = zT_k = z\alpha_{kk} = \frac{\lambda_{kk}}{Q}, k = 0,1,2,3. \qquad (4.10)$$

Taking into account the notation of equation (4.10) the main diagonal of matrix T of equation (2.12) is given from matrix Λ

$$\Lambda = \frac{1}{Q}\begin{bmatrix} \lambda_{00} & 0 & 0 & 0 \\ 0 & \lambda_{11} & 0 & 0 \\ 0 & 0 & \lambda_{22} & 0 \\ 0 & 0 & 0 & \lambda_{33} \end{bmatrix} = \begin{bmatrix} \Phi_0 & 0 & 0 & 0 \\ 0 & \Phi_1 & 0 & 0 \\ 0 & 0 & \Phi_2 & 0 \\ 0 & 0 & 0 & \Phi_3 \end{bmatrix} = \begin{bmatrix} zT_0 & 0 & 0 & 0 \\ 0 & zT_1 & 0 & 0 \\ 0 & 0 & zT_2 & 0 \\ 0 & 0 & 0 & zT_3 \end{bmatrix}. \qquad (4.11)$$

We now define the three-vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, as given by equations (4.12) and (4.13) respectively

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \frac{1}{Q}\begin{pmatrix} ic\lambda_{01} \\ ic\lambda_{02} \\ ic\lambda_{03} \end{pmatrix} \qquad (4.12)$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} = \frac{1}{Q}\begin{pmatrix} \lambda_{32} \\ \lambda_{13} \\ \lambda_{21} \end{pmatrix}. \qquad (4.13)$$

Vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ contain all of the physical quantities λ_{ki} for $k \neq i, k, i = 0,1,2,3$ since $\lambda_{ik} = -\lambda_{ki}$.

Combining equations (4.12) and (4.13) with equation (4.4), the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are written in the form of equations (4.14) and (4.15), respectively

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = icz\begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \qquad (4.14)$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} = z\begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix}. \qquad (4.15)$$

We write equation (2.10) in the form

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + \lambda_{ki}, \mathrm{k,i} = 0,1,2,3. \tag{4.16}$$

The rate of change of the momentum of the material particle equals the sum of the two terms in the right part of equation (4.16). For $k = 0$, and since $x_0 = ict$, equation (83) gives the rate of change of the particle momentum with respect to time t, i.e. the physical quantity we call "force". By using the concept of force, as defined by Newton, we also have to use the concept of velocity. For this reason we symbolize $\mathbf{u}$ the velocity of the material particle, as given by equation

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} . \tag{4.17}$$

Also, we define the 4-vector of the four-vector u, as given by equation

$$u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} ic \\ u_x \\ u_y \\ u_z \end{bmatrix}. \tag{4.18}$$

We now prove the following theorem:

Theorem 4.2. ′′The rates of change with respect to time $t\left(x_0 = ict\right)$ of the four-vectors J and P of the momentum of the generalized particle carrying charge Q are given by equations

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J - \frac{i}{c} Q\Lambda u - \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} \tag{4.19}$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Qdx_0} J + \frac{i}{c} Q\Lambda u + \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} .'' \tag{4.20}$$

Proof. The matrix Λ is given in equation (4.11). By $\mathbf{u}\times\boldsymbol{\beta}$ we denote the outer product of vectors $\mathbf{u}$ and $\boldsymbol{\beta}$.

We now prove the first of equations (4.19):

$$\frac{d}{dt}\left(\frac{J_0}{Q}\right)=\frac{\partial}{\partial t}\left(\frac{J_0}{Q}\right)+u_1\frac{\partial}{\partial x}\left(\frac{J_0}{Q}\right)+u_2\frac{\partial}{\partial y}\left(\frac{J_0}{Q}\right)+u_3\frac{\partial}{\partial z}\left(\frac{J_0}{Q}\right)$$

and using the notation of equation (2.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right)=ic\frac{\partial}{\partial x_0}\left(\frac{J_0}{Q}\right)+u_1\frac{\partial}{\partial x_1}\left(\frac{J_0}{Q}\right)+u_2\frac{\partial}{\partial x_2}\left(\frac{J_0}{Q}\right)+u_3\frac{\partial}{\partial x_3}\left(\frac{J_0}{Q}\right)$$

and with equation (4.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right)=ic\frac{\lambda_{00}}{Q}+u_1\frac{\lambda_{10}}{Q}+u_2\frac{\lambda_{20}}{Q}+u_3\frac{\lambda_{30}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right)=\frac{\lambda_{00}}{Q}-\frac{i}{c}\left(u_1\frac{\lambda_{10}}{Q}+u_2\frac{\lambda_{20}}{Q}+u_3\frac{\lambda_{30}}{Q}\right)$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right)=\frac{\lambda_{00}}{Q}+\frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q}+u_2\frac{\lambda_{02}}{Q}+u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{1}{Q}\frac{dJ_0}{dx_0}-\frac{J_0}{Q^2}\frac{dQ}{dx_0}=\frac{\lambda_{00}}{Q}+\frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q}+u_2\frac{\lambda_{02}}{Q}+u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{dJ_0}{dx_0}=\frac{dQ}{Qdx_0}J_0+\lambda_{00}+\frac{i}{c}\left(u_1\lambda_{01}+u_2\lambda_{02}+u_3\lambda_{03}\right)$$

and with equations (4.10) and (4.12) we have

$$\frac{dJ_0}{dx_0}=\frac{dQ}{Qdx_0}J_0+Q\Phi_0-\frac{i}{c}Q\left(\frac{i}{c}u_1\alpha_1+\frac{i}{c}u_2\alpha_2+\frac{i}{c}u_3\alpha_3\right)$$

which is the first of equations (4.19) since

$$-\frac{i}{c}Q\Phi_0u_0=-\frac{i}{c}Q\Phi_0 ic=Q\Phi_0.$$

We prove the second of equations (4.19) and we can similarly prove the third and the fourth:

$$\frac{d}{dt}\left(\frac{J_x}{Q}\right)=\frac{\partial}{\partial t}\left(\frac{J_x}{Q}\right)+u_1\frac{\partial}{\partial x}\left(\frac{J_x}{Q}\right)+u_2\frac{\partial}{\partial y}\left(\frac{J_x}{Q}\right)+u_3\frac{\partial}{\partial z}\left(\frac{J_x}{Q}\right)$$

and using the notation of equations (2.3) and (2.4) we obtain

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right)=\frac{ic\partial}{\partial x_0}\left(\frac{J_1}{Q}\right)+u_1\frac{\partial}{\partial x_1}\left(\frac{J_1}{Q}\right)+u_2\frac{\partial}{\partial x_2}\left(\frac{J_1}{Q}\right)+u_3\frac{\partial}{\partial x_3}\left(\frac{J_1}{Q}\right)$$

and with equation (4.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right)=ic\frac{\lambda_{01}}{Q}+u_1\frac{\lambda_{11}}{Q}+u_2\frac{\lambda_{21}}{Q}+u_3\frac{\lambda_{31}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_1}{Q}\right)=-\frac{iu_1}{c}\frac{\lambda_{11}}{Q}+\frac{\lambda_{01}}{Q}-\frac{iu_2}{c}\frac{\lambda_{21}}{Q}+\frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{1}{Q}\frac{dJ_1}{dx_0}-\frac{J_1}{Q^2}\frac{dQ}{dx_0}=-\frac{iu_1}{c}\frac{\lambda_{11}}{Q}+\frac{\lambda_{01}}{Q}-\frac{iu_2}{c}\frac{\lambda_{21}}{Q}+\frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{dJ_1}{dx_0}=\frac{dQ}{Qdx_0}J_1-\frac{iu_1}{c}\lambda_{11}+\lambda_{01}-\frac{iu_2}{c}\lambda_{21}+\frac{iu_3}{c}\lambda_{13}$$

and with equations (4.10), (4.12) and (4.13), we obtain

$$\frac{dJ_1}{dx_0}=\frac{dQ}{Qdx_0}J_1-\frac{i}{c}Q\Phi_1-\frac{i}{c}Q\alpha_1-\frac{i}{c}Q\left(u_2\beta_3-u_3\beta_2\right)$$

which is the second of equations (4.19). Equation (4.20) results from the combination of equations (4.19) and (3.5). □

Using the symbol $\mathbf{J}$ for the momentum vector of the material particle

$$\mathbf{J}=\begin{pmatrix}J_1\\J_2\\J_3\end{pmatrix}=\begin{pmatrix}J_x\\J_y\\J_z\end{pmatrix}$$

and taking into account equations (2.3) and (2.4) and (4.11) the set of equations (4.19) can be written in the form

$$\begin{aligned}&\frac{dW}{dt}=\frac{dQ}{Qdt}W+Qc^2\Phi_0+Q\mathbf{u}\cdot\boldsymbol{\alpha}\\&\frac{d\mathbf{J}}{dt}=\frac{dQ}{Qdt}\mathbf{J}+Q\begin{pmatrix}\Phi_1u_1\\\Phi_2u_2\\\Phi_3u_3\end{pmatrix}+Q\left(\boldsymbol{\alpha}+\mathbf{u}\times\boldsymbol{\beta}\right)\end{aligned}\quad. \qquad (4.21)$$

Equations (4.21) are a simpler form of equation (4.19) with which are equivalent.

The rate of change of the four-vector J of the momentum of the material particle is given by the sum of the three terms in the right part of equation (86). The USVI and its consequences for the material particle depend on which of these terms is the strongest and which is the weakest.

The first term expresses a force parallel to four-vector J which is always different than zero due to the Selfvariations. As we will see next, the second term is related to the curvature of spacetime. The third term on the right of equation (4.19) is known as the Lorentz force, in the case of electromagnetic fields. In many cases a term or some of the terms on the right of equation (4.19) are zero, with the exception of the first term which is always different than zero.

From equation (4.19) we conclude that the pair of vectors $(\boldsymbol{\alpha},\boldsymbol{\beta})$ expresses the intensity of the field of the USVI according to the paradigm of the classical definition of the field potential. From equation (2.10) we derive tha physical quantities $\lambda_{ki}, k,i=0,1,2,3$ have units (dimensions) of $kg\cdot s^{-1}$. Thus, from equation (4.12) we derive that if Q is the rest mass, the intensity $\boldsymbol{\alpha}$ has unit of $m\cdot s^{-2}$. If Q is the electric charge, the intensity $\boldsymbol{\alpha}$ has unit of $N\cdot C^{-1}$. Now we will prove that for field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ the following equations (4.22) hold.

Theorem 4.3. ´´ For the vector pair $(\boldsymbol{\alpha},\boldsymbol{\beta})$ the following equations hold:

$$\nabla\cdot\boldsymbol{\alpha}=-\frac{icbz}{2\hbar}\left(c_1\alpha_{01}+c_2\alpha_{02}+c_3\alpha_{03}\right) \qquad \text{(a)}$$

$$\nabla\cdot\boldsymbol{\beta}=0 \qquad \text{(b)}$$

$$\nabla\times\boldsymbol{\alpha}=-\frac{\partial\boldsymbol{\beta}}{\partial t} \qquad \text{(c)} \qquad (4.22)$$

$$\nabla\times\boldsymbol{\beta}=-\frac{bz}{2\hbar}\begin{pmatrix}c_0\alpha_{01}+c_2\alpha_{21}+c_3\alpha_{31}\\ c_0\alpha_{02}+c_2\alpha_{12}+c_3\alpha_{32}\\ c_0\alpha_{03}+c_2\alpha_{13}+c_3\alpha_{23}\end{pmatrix}+\frac{\partial\boldsymbol{\alpha}}{c^2\partial t}.'' \qquad \text{(d)}$$

Proof. Differentiating equations (4.14) and (4.15) with respect to x_k, $k=0,1,2,3$ and considering equation (4.9), we obtain equations

$$\frac{\partial \boldsymbol{\alpha}}{\partial x_k} = -\frac{bc_k}{2\hbar}\boldsymbol{\alpha} \tag{4.23}$$

$$\frac{\partial \boldsymbol{\beta}}{\partial x_k} = -\frac{bc_k}{2\hbar}\boldsymbol{\beta}\,. \tag{4.24}$$

From equations (4.23) and (4.24) we can easily derive equations (4.22). Indicatively, we prove equation (4.22b). From equation (4.15) we obtain

$$\nabla \cdot \boldsymbol{\beta} = \alpha_{32}\frac{\partial z}{\partial x_1} + \alpha_{13}\frac{\partial z}{\partial x_2} + \alpha_{21}\frac{\partial z}{\partial x_3}$$

and with equation (4.9) we get

$$\nabla \cdot \boldsymbol{\beta} = -\frac{bz}{2\hbar}\left(c_1\alpha_{32} + c_2\alpha_{13} + c_3\alpha_{21}\right)$$

and with the first of equations (4.6) for $(i,v,k)=(1,3,2)$ we get

$$\nabla \cdot \boldsymbol{\beta} = 0\,.$$

The first of equations (4.6) should be taken into account for the proof of the rests of equations of (4.22). □

Considering equations (4.22) we define the scalar quantity ρ and the vector quantity $\mathbf{j}$, as given by equations

$$\begin{aligned} \rho &= \sigma \nabla \cdot \boldsymbol{\alpha} = -\sigma\frac{icbz}{2\hbar}\left(c_1 a_{01} + c_2 a_{02} + c_3 a_{03}\right) \\ \mathbf{j} &= \sigma\frac{c^2 bz}{2\hbar}\begin{pmatrix} -c_0 a_{01} - c_2\, a_{21} + c_3 a_{13} \\ -c_0 a_{02} + c_1\, a_{21} - c_3 a_{32} \\ -c_0 a_{03} - c_1\, a_{13} + c_2 a_{32} \end{pmatrix} \end{aligned} \tag{4.25}$$

where $\sigma \neq 0$ is a constant. We now prove that for the physical quantities ρ and $\mathbf{j}$ the following continuity equation holds:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0\,. \tag{4.26}$$

Proof. : From the first of equations (4.25) we obtain

$$\rho = \sigma \nabla \cdot \boldsymbol{\alpha}$$
$$\frac{\partial \rho}{\partial t} = \sigma \frac{\partial}{\partial t} \left(\nabla \cdot \boldsymbol{\alpha} \right)$$
$$\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\sigma \frac{\partial \boldsymbol{\alpha}}{\partial t} \right)$$

and with the second of equations (4.25) and equation (4.22d) we get

$$\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\sigma c^2 \nabla \times \boldsymbol{\beta} - \mathbf{j} \right)$$
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j}$$

which is equation (4.26). □

According to equation (4.26), the physical quantity ρ is the density of a conserved physical quantity q with current density $\mathbf{j}$. The conserved physical quantity q is related to field $\left(\boldsymbol{\alpha}, \boldsymbol{\beta} \right)$ through equations (4.22).We will revert to the issue of sustainable physical quantities in the next paragraphs.

The density ρ and the current density $\mathbf{j}$ have a rigidly defined internal structure as derived from equations (4.25).

We now consider the four-vector of the current density j of the conserved physical quantity q, as given by equation

$$j = \begin{bmatrix} j_0 \\ j_1 \\ j_2 \\ j_3 \end{bmatrix} = \begin{bmatrix} i \rho c \\ j_x \\ j_y \\ j_z \end{bmatrix} \tag{4.27}$$

and the 4×4 matrices M

$$M = \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix} . \tag{4.28}$$

Using matrix M equations (4.25) can be written in the form of equation

$$j=\frac{\sigma c^2 bz}{2\hbar}MC\,. \tag{4.29}$$

From equations (4.22b,c) we conclude that the potential is always defined in the $(\boldsymbol{\alpha},\boldsymbol{\beta})$- field of the USVI. That is, the scalar potential

$$V=V(t,x,y,z)=V(x_0,x_1,x_2,x_3)$$

and the vector potential $\mathbf{A}$

$$\mathbf{A}=\mathbf{A}(t,x,y,z)=\mathbf{A}(x_0,x_1,x_2,x_3)=\begin{pmatrix}A_1\\A_2\\A_3\end{pmatrix}=\begin{pmatrix}A_x\\A_y\\A_z\end{pmatrix}$$

are defined through the equations

$$\boldsymbol{\beta}=\nabla\times\mathbf{A}$$
$$\boldsymbol{\alpha}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t}=-\nabla V-\frac{ic\partial\mathbf{A}}{\partial x_0}\;.$$

We can introduce in the above equations the gauge function f. That is, we can add to the scalar potential V the term

$$-\frac{\partial f}{\partial t}=-\frac{ic\partial f}{\partial x_0}$$

and to the vector potential $\mathbf{A}$ the term

$$\nabla f$$

for an arbitrary function f

$$f=f(t,x,y,z)=f(x_0,x_1,x_2,x_3)$$

without changing the intensity $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of the field. The proof of the above equations is known and trivial and we will not repeat it here. For the field potential of the USVI the following theorem holds:

Theorem 4.4. ′′In the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field of USVI the pair of scalar-vector potentials $(V,\mathbf{A})$ is always defined through equations

$$\boldsymbol{\beta} = \nabla \times \mathbf{A}$$
$$\boldsymbol{\alpha} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = ic\nabla A_0 - \frac{ic\partial \mathbf{A}}{\partial x_0} \quad . \tag{4.30}$$

The four-vector A of the potential

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} \frac{iV}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix} \tag{4.31}$$

is given by equation

$$A_i = \begin{cases} \frac{2\hbar}{b}\frac{\alpha_{ki}}{c_k} z + \frac{\partial f_k}{\partial x_i}, \text{for } i \neq k \\ \frac{\partial f_k}{\partial x_i}, \text{for } i = k \end{cases} \tag{4.32}$$

where $c_k \neq 0, k, i = 0,1,2,3$ and f_k is the gauge function´´

Proof. Equations (4.30) are equivalent to equations (4.22b, c) as we have already mentioned. The proof of equation (4.32) can be performed through the first of equations (4.6). The mathematical calculations do not contribute anything useful to our study, thus we omit them. You can verify that the potential of equation (4.32) gives equations (4.14) and (4.15) through equations (4.30) taking also into account the first of equations (4.6). □

From equation (4.32) the following four sets of the potentials follow:

$$\begin{aligned} & c_0 \neq 0 \\ & A_0 = \frac{\partial f_0}{\partial x_0} \\ & A_1 = \frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3} \end{aligned} \tag{4.33}$$

$c_1 \neq 0$

$$A_0 = \frac{2\hbar z}{b}\frac{\alpha_{10}}{c_1} + \frac{\partial f_1}{\partial x_0}$$

$$A_1 = \frac{\partial f_1}{\partial x_1} \qquad (4.34)$$

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_1}{\partial x_2}$$

$$A_3 = \frac{2\hbar z}{b}\frac{\alpha_{13}}{c_1} + \frac{\partial f_1}{\partial x_3}$$

$c_2 \neq 0$

$$A_0 = \frac{2\hbar z}{b}\frac{\alpha_{20}}{c_2} + \frac{\partial f_2}{\partial x_0}$$

$$A_1 = \frac{2\hbar z}{b}\frac{\alpha_{21}}{c_2} + \frac{\partial f_2}{\partial x_1} \qquad (4.35)$$

$$A_2 = \frac{\partial f_2}{\partial x_2}$$

$$A_3 = \frac{2\hbar z}{b}\frac{\alpha_{23}}{c_2} + \frac{\partial f_2}{\partial x_3}$$

$c_3 \neq 0$

$$A_0 = \frac{2\hbar z}{b}\frac{\alpha_{30}}{c_3} + \frac{\partial f_3}{\partial x_0}$$

$$A_1 = \frac{2\hbar z}{b}\frac{\alpha_{31}}{c_3} + \frac{\partial f_3}{\partial x_1} \quad . \qquad (4.36)$$

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{32}}{c_3} + \frac{\partial f_3}{\partial x_2}$$

$$A_3 = \frac{\partial f_3}{\partial x_3}$$

Indicatively, we calculate the components α_1 and β_1 of the intensity $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of the USVI field from the potentials (4.33). From the second of equations (4.30) we obtain

$$\alpha_1 = ic\left(\frac{\partial A_0}{\partial x_1} - \frac{\partial A_1}{\partial x_0}\right)$$

and with equations (4.33) we get

$$\alpha_1 = ic\left[\frac{\partial}{\partial x_1}\left(\frac{\partial f_0}{\partial x_0}\right) - \frac{\partial}{\partial x_0}\left(\frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1}\right)\right]$$

$$\alpha_1 = -ic\frac{2\hbar}{b}\frac{\alpha_{01}}{c_0}\frac{\partial z}{\partial x_0}$$

and with equation (4.9) we get

$$\alpha_1 = icz\alpha_{01}$$

that is we get the intensity α_1 of the field, as given by equation (4.14).

From the first of equations (4.30) we have

$$\beta_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}$$

and with equations (4.33) we get

$$\beta_1 = \frac{\partial}{\partial x_2}\left(\frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3}\right) - \frac{\partial}{\partial x_3}\left(\frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2}\right)$$

$$\beta_1 = \frac{2\hbar}{b}\frac{\alpha_{03}}{c_0}\frac{\partial z}{\partial x_2} - \frac{2\hbar}{b}\frac{\alpha_{02}}{c_0}\frac{\partial z}{\partial x_2}$$

and with equation (4.9) we get

$$\beta_1 = -\frac{c_2\alpha_{03}}{c_0}z + \frac{c_3\alpha_{02}}{c_0}z$$

and considering that $\alpha_{02} = -\alpha_{20}$, we get

$$\beta_1 = -\frac{z}{c_0}\left(c_2\alpha_{03} + c_3\alpha_{20}\right). \tag{4.37}$$

From the first of equations (4.6) for $(i,v,k) = (2,0,3)$ we obtain

$$c_2 a_{03} + c_3 a_{20} + c_0 a_{32} = 0$$
$$c_2 a_{03} + c_3 a_{20} = -c_0 a_{32}$$

and substituting into equation (4.37), we see that

$$\beta_1 = z\alpha_{32}$$

that is, we get the intensity β_1 of the field, as given by equation (4.15).

The gauge functions $f_k, \mathrm{k} = 0,1,2,3$ in equations (4.33)-(4.36) are not independent of each other. For $c_k \neq 0$ and $c_i \neq 0$ for $k \neq i, k, i = 0,1,2,3$ equation (4.38) holds

$$f_k = f_i + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{ki}}{c_k c_i}, c_k c_i \neq 0, k \neq i,\ k, i = 0,1,2,3. \tag{4.38}$$

The proof of equation (4.38) is through the first of equations (4.6). The proof is lengthy and we omit it. Indicatively, we will prove the third of equations (4.33) from the third of equations (4.34) for $k=1$ and $i=0$ in equation (4.38).

For $c_0 \neq 0$ and $c_1 \neq 0$ both equations (4.33) and equations (4.34) hold. From equation (4.38) for $k=1$ and $i=0$ we get equation

$$f_1 = f_0 + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0 c_1} \ . \tag{4.39}$$

From the third of equations (4.33) and equation (4.39) we get

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial}{\partial x_2}\left(f_0 + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0 c_1}\right)$$

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_0}{\partial x_2} + \frac{4\hbar^2}{b^2}\frac{\alpha_{10}}{c_0 c_1}\frac{\partial z}{\partial x_2}$$

and with equation (4.9) we obtain

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_0}{\partial x_2} - \frac{2\hbar z}{b}\frac{c_2 \alpha_{10}}{c_0 c_1}$$

$$A_2 = \frac{2\hbar z}{b c_0 c_1}\left(c_0 \alpha_{12} - c_2 \alpha_{10}\right) + \frac{\partial f_0}{\partial x_2}$$

and since $\alpha_{10} = -\alpha_{01}$, we get equation

$$A_2 = \frac{2\hbar z}{b c_0 c_1}\left(c_0 \alpha_{12} + c_2 \alpha_{01}\right) + \frac{\partial f_0}{\partial x_2} \tag{4.40}$$

From the first of equations (4.6) for $(i,v,k)=(0,1,2)$ we obtain

$$c_0 a_{12} + c_2 a_{01} + c_1 a_{20} = 0$$

$$c_0 a_{12} + c_2 a_{01} = -c_1 a_{20}$$

$$c_0 a_{12} + c_2 a_{01} = c_1 a_{02}$$

and substituting into equation (4.40) we obtain equation

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2} . \tag{4.41}$$

Equation (4.41) is the third of equations (4.33).

According to equation (4.38), if $c_k \neq 0$ for more than one of the constants $c_k, k = 0,1,2,3$, the sets of equations of potential resulting from equation (4.32) have in the end a gauge function. In the application we presented assuming $c_0 \neq 0$ and $c_1 \neq 0$ for a specific gauge function f_0 in equations (4.33), the gauge function f_1 in equations (4.34) is given by equation (4.39). □

We conclude the investigation of the potential of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of USVI by proving the following corollary:

Corollary 4.1. ´´In the external symmetry, the 4-vector C of the total energy content of the generalized particle cannot vanish:

$$C=\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.'' \qquad (4.42)$$

Proof. Indeed, for $C=0$ we obtain $J=-P$ from equation (3.5). Therefore, the four-vectors J and P are parallel. According to equivalence (3.7) the parallelism of the four-vectors J and P is equivalent to the internal symmetry. Therefore, in the external symmetry it is $C\neq 0$. □

A direct consequence of these findings is that the potential of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of USVI is always defined, as given from equation (4.42). This conclusion is derived from the fact that at least one of the constants $c_k, k\in\{0,1,2,3\}$ is always different than zero.

5. The conserved physical quantities of the generalized particle and the wave equation of the TSV

The generalized particle has a set of conserved physical quantities q which we determine in this paragraph. At first, we generalize the notion of the field, as it is derived from the equations of theTSV. We prove the following theorem:

Theorem 5.1. ´´For the field $(\boldsymbol{\xi},\boldsymbol{\omega})$ of the pair of vectors

$$\boldsymbol{\xi}=ic\Psi\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} \qquad (5.1)$$

$$\boldsymbol{\omega} = \Psi \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.2}$$

where $\Psi = \Psi(x_0, x_1, x_2, x_3)$ is a function satisfying equation

$$\frac{\partial \Psi}{\partial x_k} = \frac{b}{\hbar}(\lambda J_k + \mu P_k)\Psi \tag{5.3}$$

$k = 0,1,2,3, \quad (\lambda, \mu) \neq (0,0), \quad \lambda, \mu \in \mathbb{C}$ are functions of x_0, x_1, x_2, x_3, the following equations hold

$$\begin{aligned} &\nabla \cdot \boldsymbol{\omega} = 0 \\ &\nabla \cdot \boldsymbol{\xi} = -\frac{\partial \boldsymbol{\omega}}{\partial t} \end{aligned} \tag{5.4}$$

The generalized particle has a set of conserved physical quantities q with density ρ and current density $\mathbf{j}$

$$\begin{aligned} &\rho = \sigma \nabla \cdot \boldsymbol{\xi} \\ &\mathbf{j} = \sigma c^2 \left(\nabla \times \boldsymbol{\omega} - \frac{\partial \boldsymbol{\xi}}{c^2 \partial t} \right) \end{aligned} \tag{5.5}$$

where $\sigma \neq 0$ are constants, for which conserved physical quantities the following continuity equation holds

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 . \tag{5.6}$$

The four-vectors of the current density j are given by equation

$$j = -\frac{\sigma c^2 b}{\hbar} \Psi M \left(\lambda J + \mu P \right) .'' \tag{5.7}$$

Proof. Matrix M in equation (5.7) is given by equation (4.28). We denote $\mathbf{J}$ and $\mathbf{P}$ the three-dimensional momentums as given by equations

$$\mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} \tag{5.8}$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}. \tag{5.9}$$

For the proof of the theorem we first demonstrate the following auxiliary equations (5.10)-(5.15)

$$\mathbf{J} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{5.10}$$

$$\mathbf{P} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{5.11}$$

$$\mathbf{J} \times \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -J_0 \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.12}$$

$$\mathbf{P} \times \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -P_0 \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.13}$$

$$\mathbf{J} \times \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = \begin{pmatrix} J_2 a_{21} - J_3 a_{13} \\ J_3 a_{32} - J_1 a_{21} \\ J_1 a_{13} - J_2 a_{32} \end{pmatrix} \tag{5.14}$$

$$\mathbf{P} \times \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = \begin{pmatrix} P_2 a_{21} - P_3 a_{13} \\ P_3 a_{32} - P_1 a_{21} \\ P_1 a_{13} - P_2 a_{32} \end{pmatrix}. \tag{5.15}$$

In order to prove equation (5.10) we get

$$\mathbf{J} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = J_1 a_{32} + J_2 a_{13} + J_3 a_{21}$$

and with the second of equations (4.6) for $(i, v, k) = (1, 3, 2)$, we have

$$\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0$$

Similarly, from the third of equations (4.6) we obtain equation (5.11). We now get

$$\mathbf{J}\times\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = \begin{pmatrix} J_2 a_{03} - J_3 a_{02} \\ J_3 a_{01} - J_1 a_{03} \\ J_1 a_{02} - J_2 a_{01} \end{pmatrix} = \begin{pmatrix} J_2 a_{03} + J_3 a_{20} \\ J_3 a_{01} + J_1 a_{30} \\ J_1 a_{02} + J_2 a_{10} \end{pmatrix}$$

and with the second of equations (4.6) we obtain

$$\mathbf{J}\times\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = \begin{pmatrix} -J_0 a_{32} \\ -J_0 a_{13} \\ -J_0 a_{21} \end{pmatrix}$$

which is equation (5.12). Similarly, by considering the third of equations (4.6) we derive equation (5.13). Equations (5.14) and (5.15) are derived by taking into account equations (5.8) and (5.9).

Equations (5.4) are proven with the use of equations (5.10)-(5.15). We prove the first as an example. From equation (5.2) we obtain

$$\nabla\cdot\boldsymbol{\omega} = \nabla\Psi\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix}$$

and with equation (5.3) we get

$$\nabla\cdot\boldsymbol{\omega} = \frac{b}{\hbar}\lambda\Psi\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} + \frac{b}{\hbar}\mu\Psi\mathbf{P}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix}$$

and with equations (5.10) and (5.11) we obtain

$$\nabla\cdot\boldsymbol{\omega} = 0.$$

From equations (5.4) and (5.5), the continuity equation (5.6) results. The proof is similar to the one for equation (4.26). The proof of equation (5.7) is done with the use of equations (5.10)-(5.15), and equation (4.28). □

Field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ presented in the previous paragraph is a special case of the field $(\boldsymbol{\xi},\boldsymbol{\omega})$ for $\lambda=\mu=-\frac{1}{2}$. For these values of the parameteres λ,μ we obtain from equations (5.3)

$$\frac{\partial\Psi}{\partial x_k}=\frac{b}{\hbar}\left(-\frac{1}{2}J_k-\frac{1}{2}P_k\right)\Psi$$
$$\frac{\partial\Psi}{\partial x_k}=-\frac{b}{2\hbar}\left(J_k+P_k\right)\Psi$$

and with equation (3.5) we obtain

$$\frac{\partial\Psi}{\partial x_k}=-\frac{bc_k}{2\hbar}\Psi$$

and finally we obtain

$$\Psi=z=\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]$$

and from equations (5.1),(5.2) and (4.14),(4.15) we obtain $\boldsymbol{\xi}=\boldsymbol{\alpha}$ and $\boldsymbol{\omega}=\boldsymbol{\beta}$.

From equation (2.10) it emerges that the dimensions of the physical quantities $\lambda_{ki},k,i=0,1,2,3$ are

$$\left[\lambda_{ki}\right]=kgs^{-1},k,i=0,1,2,3.$$

Thus, from equations (4.12), (4.13) and (4.14), (4.15) we obtain the dimensions of the physical quantities $Q\alpha_{ki},k,i=0,1,2,3$. Furthermore, from equation (4.11) we obtain the dimensions of the physical quantities $T_k,k=0,1,2,3.$ Thus, we get the following relationships

$$\begin{aligned}&\left[Q\alpha_{ki}\right]=kgs^{-1},k\neq i,k,i=0,1,2,3,\\&\left[QT_k\right]=kgs^{-1},k=0,1,2,3.\end{aligned}\qquad(5.16)$$

Using the first of equations (5.16) we can determine the units of measurement of the $(\boldsymbol{\xi},\boldsymbol{\omega})$-field for every selfvariating charge Q. When Q is the electric charge, we can verify that the field units are $\left(V\cdot m^{-1},\mathrm{T}\right)$. When Q is the rest mass, the field units are $\left(m\cdot s^{-2},s^{-1}\right)$. The dimensions of the field depend solely on the units of measurement of the selfvariating charge Q.

From equation (5.7) and taking into account that $\lambda, \mu \in \mathbb{C}$ we can define the dimensions of the physical quantities q through the first of equations (5.16). For $\sigma = \varepsilon_0$, where ε_0 is the electric permeability of the vacuum, q is a conserved physical quantity of electric charge. For $\sigma = \frac{\varepsilon_0}{e}$, where e the constant value we measure in the lab for the electric charge of the electron, q is a conserved physical quantity of angular momentum. For $\sigma = \frac{1}{4\pi G}$, where G is the gravitational constant, q is a conserved physical quantity of matter. Theorem 5.1 reveals the conserved physical quantities of the generalized particle.

One of the most important corollaries of the theorem 5.1 is the prediction that the generalized particle has wave-like behavior. We prove the following corollary:

Corollary 5.1. ´´For function Ψ the following equation holds

$$\begin{aligned} &\sigma c^2 \alpha_{ki} \left(\nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2} \right) = \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \\ &\sigma c^2 \alpha_{ki} \left(\nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} \right) = \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \end{aligned} \qquad (5.17)$$

$k \neq i, \quad k, i = 0,1,2,3$.´´

Proof. To prove the corollary, considering that $x_0 = ict$, we write equations (5.4) and (5.5) in the form

$$\begin{aligned} &\nabla \cdot \boldsymbol{\xi} = -\frac{i}{\sigma c} j_0 \\ &\nabla \cdot \boldsymbol{\omega} = 0 \\ &\nabla \times \boldsymbol{\xi} = -\frac{ic\partial \boldsymbol{\omega}}{\partial x_0} \\ &\nabla \times \boldsymbol{\omega} = \frac{1}{\sigma c^2} \mathbf{j} + \frac{i \partial \boldsymbol{\xi}}{c \partial x_0} \end{aligned} \quad . \qquad (5.18)$$

We will also use the identity (162) which is valid for every vector $\boldsymbol{\alpha}$

$$\nabla \times \nabla \times \boldsymbol{\alpha} = \nabla(\nabla \cdot \boldsymbol{\alpha}) - \nabla^2 \boldsymbol{\alpha} . \qquad (5.19)$$

From the third of equations (5.18) we obtain

$$\nabla\times\nabla\times\boldsymbol{\xi} = -\nabla\times\left(\frac{ic\partial\boldsymbol{\omega}}{\partial x_0}\right)$$

$$\nabla\times\nabla\times\boldsymbol{\xi} = -\frac{ic\partial}{\partial x_0}\left(\nabla\times\boldsymbol{\omega}\right)$$

and using the identity (5.19) we get

$$\nabla\left(\nabla\cdot\boldsymbol{\xi}\right)-\nabla^2\boldsymbol{\xi} = -\frac{ic\partial}{\partial x_0}\left(\nabla\times\boldsymbol{\omega}\right)$$

and with the first and fourth of equations (5.18) we get

$$\nabla\left(-\frac{i}{\sigma c}j_0\right)-\nabla^2\boldsymbol{\xi} = \frac{\partial^2\boldsymbol{\xi}}{\partial x_0^2}-\frac{i}{\sigma c}\frac{\partial\mathbf{j}}{\partial x_0}$$

and we finally get

$$\nabla^2\boldsymbol{\xi}+\frac{\partial^2\boldsymbol{\xi}}{\partial x_0^2} = \frac{i}{\sigma c}\left(\frac{\partial\mathbf{j}}{\partial x_0}-\nabla j_0\right) \tag{5.20}$$

Working similarly from equation (5.18) we obtain

$$\nabla^2\boldsymbol{\omega}+\frac{\partial^2\boldsymbol{\omega}}{\partial x_0^2} = -\frac{1}{\sigma c^2}\nabla\times\mathbf{j} \tag{5.21}$$

Combining equations (5.20) and (5.21) with equations (5.1) and (5.2), we get

$$\alpha_{ki}\left(\nabla^2\Psi+\frac{\partial^2\Psi}{\partial x_0^2}\right) = \frac{i}{\sigma c}\left(\frac{\partial j_i}{\partial x_k}-\frac{\partial j_k}{\partial x_i}\right),\ k\neq i,\quad k,i=0,1,2,3$$

which is equation (5.17).□

Equation (5.17) can be characterized as **"the wave equation of the TSV"**. The basic characteristics of equation (5.17) depend on whether the physical quantity

$$F = \nabla^2\Psi+\frac{\partial^2\Psi}{\partial^2 x_0^2} = \nabla^2\Psi-\frac{\partial^2\Psi}{c^2\partial t^2} \tag{5.22}$$

is zero or not.

This conclusion is drawn through the following theorem:

Theorem 5.2. ´´For the generalized particle the following equivalences hold

$$\nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = 0 \qquad (5.23)$$

if and only if for each $k \neq i,\ k,i = 0,1,2,3$ it is

$$\frac{\partial j_i}{\partial x_k} = \frac{\partial j_k}{\partial x_i} \qquad (5.24)$$

if and only if

$$\begin{aligned} \nabla^2\boldsymbol{\xi} - \frac{\partial^2\boldsymbol{\xi}}{c^2\partial t^2} = 0 \\ \nabla^2\boldsymbol{\omega} - \frac{\partial^2\boldsymbol{\omega}}{c^2\partial t^2} = 0 \end{aligned} \quad .´´ \qquad (5.25)$$

Proof. In the external symmetry there exists at least one pair of indices $(k,i),\ k \neq i,\ k,i \in \{0,1,2,3\}$, for which $\alpha_{ki} \neq 0$. Therefore, when equation (5.24) holds, then equation (5.23) follows from equation (5.17), and vice versa. Thus, equations (5.23) and (5.24) are equivalent. When equation (5.24) holds, then the right hand sides of equations (5.24) and (5.25) vanish, that is, equations (5.25) hold. The converse also holds, thus equations (5.24) and (5.25) are equivalent. Therefore, equations (5.23), (5.24), and (5.25) are equivalent. □

In case that $F = 0$, that is in case that equivalences (5.23), (5.24) and (5.25) hold, we shall refer to the state of the generalized particle as the "generalized photon". According to equations (5.25), for the generalized photon the $(\boldsymbol{\xi},\boldsymbol{\omega})$-field is propagating with velocity c in the form of a wave.

For the generalized photon, the following corollary holds:

Corollary 5.2: ´´ For the generalized photon, the four-vector j of the current density of the conserved physical quantities q, varies according to the equations

$$\nabla^2 j_k - \frac{\partial^2 j_k}{c^2\partial t^2} = 0,\ k = 0,1,2,3\ .´´ \qquad (5.26)$$

Proof. We prove equation (5.26) for $k = 0$, and we can similarly prove it for $k = 1,2,3$. Considering equation (4.27), we write equation (5.6) in the form

$$\frac{\partial j_0}{\partial x_0}+\frac{\partial j_1}{\partial x_1}+\frac{\partial j_2}{\partial x_2}+\frac{\partial j_3}{\partial x_3}=0. \tag{5.27}$$

Differentiating equation (5.27) with respect to x_0 we get

$$\frac{\partial^2 j_0}{\partial x_0^2}+\frac{\partial}{\partial x_0}\left(\frac{\partial j_1}{\partial x_1}\right)+\frac{\partial}{\partial x_0}\left(\frac{\partial j_2}{\partial x_2}\right)+\frac{\partial}{\partial x_0}\left(\frac{\partial j_3}{\partial x_3}\right)=0$$

$$\frac{\partial^2 j_0}{\partial x_0^2}+\frac{\partial}{\partial x_1}\left(\frac{\partial j_1}{\partial x_0}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial j_2}{\partial x_0}\right)+\frac{\partial}{\partial x_3}\left(\frac{\partial j_3}{\partial x_0}\right)=0$$

and with equation (5.24) we get

$$\frac{\partial^2 j_0}{\partial x_0^2}+\frac{\partial}{\partial x_1}\left(\frac{\partial j_0}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial j_0}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(\frac{\partial j_0}{\partial x_3}\right)=0$$

$$\frac{\partial^2 j_0}{\partial x_0^2}+\nabla^2 j_0=0$$

which is equation (5.26) for $k=0$, since $x_0=ict$.□

The way in which equations (5.25) emerge in the TSV is completely different from the way in which the electromagnetic waves emerge in Maxwell's electromagnetic theory [6-10]. Maxwell's equation predict the eqs. (5.25) for $j=0$. The TSV predicts $(\boldsymbol{\alpha},\boldsymbol{\beta})$ waves for $j\neq 0$, when eq. (5.24) is valid. Moreover the current density j in this case varies according to eq. (5.26).

One of the most important conclusions of the theorem 5.1 is that it gives the degrees of freedom of the equations of the TSV. In equation (5.7) the parameters $\lambda,\mu\in\mathbb{C},(\lambda,\mu)\neq(0,0)$ can have arbitrary values or can be arbitrary functions of x_0,x_1,x_2,x_3. Therefore, the investigation of the TSV takes place through the parameters λ and μ of equation (5.7).

If we set $(\lambda,\mu,b)=(1,0,i)$ in equation (5.7), we get equations

$$\begin{aligned}\nabla\Psi&=\frac{i}{\hbar}\mathbf{J}\Psi\\ \frac{\partial\Psi}{\partial x_0}&=\frac{i}{\hbar}J_0\Psi\end{aligned}. \tag{5.28}$$

Taking into account that $x_0 = ict$ and $J_0 = \frac{iW}{c}$, we recognize in equations (5.28) the Schrödinger operators. Using the macroscopic mathematical expressions of the momentum $\mathbf{J}$ and energy W of the material particle, we get the Schrödinger equation [11-15]. The Schrödinger equation is a special case of the wave equation of the TSV.

In Schrodinger's equations, we can slightly modify the three parameters (λ, μ, b). If we set $(\lambda, \mu, b) = (1, \alpha, i)$ in equation (5.7), where α the fine structure constant, and take into account equation (3.5), we get equations

$$\begin{aligned} \nabla\Psi &= \frac{i}{\hbar}\left(\left(1-\alpha\right)\mathbf{J} + \alpha\mathbf{C}\right)\Psi \\ \frac{\partial\Psi}{\partial x_0} &= \frac{i}{\hbar}\left(\left(1-\alpha\right)J_0 + \alpha c_0\right)\Psi \end{aligned} \quad . \tag{5.29}$$

The fine structure constant in the TSV can have the following three forms

$$\begin{aligned} \alpha &= \frac{e^2}{4\pi\varepsilon_0 c\hbar} \\ \alpha &= \frac{eQ}{4\pi\varepsilon_0 c\hbar} \\ \alpha &= \frac{Q^2}{4\pi\varepsilon_0 c\hbar} \end{aligned} \tag{5.30}$$

in the electromagnetic interaction. We denote e the constant value we measure in the lab for the electric charge of the electron . By Q we denote the electron's selfvariating charge. The difference between the two physical quantities e and Q is due to ′′the internality of the Universe to the measurement procedure′′.The unit of measurement of the charge Q is itself subject to the Selfvariations [5] (par. 4.9).

The combination of equation (5.28) with each of equations (5.29), as well as the Schrödinger equation (5.28), give the exact same results for the hydrogen atom. For the TSV, the investigation of physical reality is put on the following terms: "In the application of the TSV, and in every case except of the generalized photon, the determination of the parameters λ and μ, is sought. This determination can be either theoretical or based on experimental data." The determination of the parameter b of the law of Selfvariations is made from the boundary conditions of the differential equations of the TSV, to which we will not refer to in the present study.

6. The Lorentz-Einstein-Selfvariations Symmetry

In this paragraph we calculate the Lorentz-Einstein transformations of the physical quantities λ_{ki}, $k,i = 0,1,2,3.$ The part of spacetime occupied by the generalized particle can be flat or curved. TheLorentz-Einstein transformations give us information about this subject.

We consider an inertial frame of reference $O'\left(t',\mathrm{x}',\mathrm{y}',\mathrm{z}'\right)$ moving with velocity $\left(u,0,0\right)$ with respect to another inertial frame of reference $O\left(t,\mathrm{x},\mathrm{y},\mathrm{z}\right)$, with their origins O' and O coinciding at $t' = t = 0$. We will calculate the Lorentz-Einsteintransformations for the physical quantities λ_{ki}, $k,i = 0,1,2,3$. We begin with transformations (6.1) and (6.2)

$$
\begin{aligned}
\frac{\partial}{\partial t'} &= \gamma\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\\
\frac{\partial}{\partial x'} &= \gamma\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right)\\
\frac{\partial}{\partial y'} &= \frac{\partial}{\partial y}\\
\frac{\partial}{\partial z'} &= \frac{\partial}{\partial z}
\end{aligned}
\qquad (6.1)
$$

$$
\begin{aligned}
W' &= \gamma\left(W - uJ_x\right) & E' &= \gamma\left(E - uP_x\right)\\
J_x{}' &= \gamma\left(J_x - \frac{u}{c^2}W\right) & P_x' &= \gamma\left(P_x - \frac{u}{c^2}E\right)\\
J_y{}' &= J_y & P_y' &= P_y\\
J_z{}' &= J_z & P_z' &= P_z
\end{aligned}
\qquad (6.2)
$$

where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$.

We then use the notation (2.3), (2.4), (2.5) and obtain the transformations (6.3) and (6.4)

$$\begin{aligned}
\frac{\partial}{\partial x_0{}'} &= \gamma\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right)\\
\frac{\partial}{\partial x_1{}'} &= \gamma\left(\frac{\partial}{\partial x_1} + i\frac{u}{c}\frac{\partial}{\partial x_0}\right)\\
\frac{\partial}{\partial x_2{}'} &= \frac{\partial}{\partial x_2}\\
\frac{\partial}{\partial x_3{}'} &= \frac{\partial}{\partial x_3}
\end{aligned} \tag{6.3}$$

$$\begin{aligned}
J_0{}' &= \gamma\left(J_0 - i\frac{u}{c}J_1\right) & P_0' &= \gamma\left(P_0 - i\frac{u}{c}P_1\right)\\
J_1{}' &= \gamma\left(J_1 + i\frac{u}{c}J_0\right) & P_1' &= \gamma\left(P_1 + i\frac{u}{c}P_0\right).\\
J_2{}' &= J_2 & P_2' &= P_2\\
J_3{}' &= J_3 & P_3' &= P_3
\end{aligned} \tag{6.4}$$

We now derive the transformation of the physical quantity λ_{00}. From equation (2.10) for $k=i=0$ we get for the inertial reference frame $O'\left(t',\mathrm{x}',\mathrm{y}',\mathrm{z}'\right)$

$$\lambda_{00}{}' = \frac{\partial J_0{}'}{\partial x_0{}'} - \frac{b}{\hbar}P_0'J_0{}'$$

and with transformations (6.3) and (6.4) we obtain

$$\lambda_{00}{}' = \gamma^2\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right)\left(J_0 - i\frac{u}{c}J_1\right) - \frac{b}{\hbar}\gamma^2\left(P_0 - i\frac{u}{c}P_1\right)\left(J_0 - i\frac{u}{c}J_1\right)$$

$$\lambda_{00}{}' = \gamma^2\left(\frac{\partial J_0}{\partial x_0} - i\frac{u}{c}\frac{\partial J_1}{\partial x_0} - i\frac{u}{c}\frac{\partial J_0}{\partial x_1} - \frac{u^2}{c^2}\frac{\partial J_1}{\partial x_1} - \frac{b}{\hbar}P_0J_0 + i\frac{u}{c}\frac{b}{\hbar}P_0J_1 + i\frac{u}{c}\frac{b}{\hbar}P_1J_0 + \frac{u^2}{c^2}\frac{b}{\hbar}P_1J_1\right)$$

and replacing physical quantities

$$\frac{\partial J_0}{\partial x_0}, \frac{\partial J_1}{\partial x_0}, \frac{\partial J_0}{\partial x_1}, \frac{\partial J_1}{\partial x_1}$$

from equation (2.10) we get

$$\lambda_{00}{}' = \gamma^2 (\frac{b}{\hbar} P_0 J_0 + \lambda_{00} - i\frac{u}{c}\frac{b}{\hbar} P_0 J_1 - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\frac{b}{\hbar} P_1 J_0 - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\frac{b}{\hbar} P_1 J_1$$
$$- \frac{u^2}{c^2}\lambda_{11} - \frac{b}{\hbar} P_0 J_0 + i\frac{u}{c}\frac{b}{\hbar} P_0 J_1 + i\frac{u}{c}\frac{b}{\hbar} P_1 J_0 + \frac{u^2}{c^2}\frac{b}{\hbar} P_1 J_1)$$

and we finally obtain equation

$$\lambda_{00}{}' = \gamma^2 \left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11} \right).$$

Following the same procedure for $k, \mathrm{i} = 0,1,2,3$ we obtain the following 16 equations (27) for the Lorentz-Einstein transformations of the physical quantities λ_{ki} :

$$\lambda_{00}{}' = \gamma^2 \left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11} \right)$$
$$\lambda_{01}{}' = \gamma^2 \left(\lambda_{01} + i\frac{u}{c}\lambda_{00} - i\frac{u}{c}\lambda_{11} + \frac{u^2}{c^2}\lambda_{10} \right)$$
$$\lambda_{02}{}' = \gamma \left(\lambda_{02} - i\frac{u}{c}\lambda_{12} \right)$$
$$\lambda_{03}{}' = \gamma \left(\lambda_{03} - i\frac{u}{c}\lambda_{13} \right)$$

$$\lambda_{10}{}' = \gamma^2 \left(\lambda_{10} - i\frac{u}{c}\lambda_{11} + i\frac{u}{c}\lambda_{00} + \frac{u^2}{c^2}\lambda_{01} \right)$$
$$\lambda_{11}{}' = \gamma^2 \left(\lambda_{11} + i\frac{u}{c}\lambda_{10} + i\frac{u}{c}\lambda_{01} - \frac{u^2}{c^2}\lambda_{00} \right) \qquad (6.5)$$
$$\lambda_{12}{}' = \gamma \left(\lambda_{12} + i\frac{u}{c}\lambda_{02} \right)$$
$$\lambda_{13}{}' = \gamma \left(\lambda_{13} + i\frac{u}{c}\lambda_{03} \right)$$

$$\lambda_{20}{}' = \gamma \left(\lambda_{20} - i\frac{u}{c}\lambda_{21} \right)$$
$$\lambda_{21}{}' = \gamma \left(\lambda_{21} + i\frac{u}{c}\lambda_{20} \right)$$
$$\lambda_{22}{}' = \lambda_{22}$$
$$\lambda_{23}{}' = \lambda_{23} \qquad .$$

The first two of equations (6.5) is self-consistent when equation

$$\lambda_{00} = \lambda_{11} \qquad (6.6)$$

Then by the second of equations (6.5) we obtain

$\lambda_{01}{}' = \lambda_{01}$.

According to equivalence (3.14) these transformations relate to the external symmetry, in which it holds that $\lambda_{ik} = -\lambda_{ki}$ for $i \neq k, i, k = 0,1,2,3$. Thus, we obtain the following transformations for the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$

$$\begin{matrix} \lambda_{00}{}' = \lambda_{00} \\ \lambda_{11}{}' = \lambda_{11} \\ \lambda_{22}{}' = \lambda_{22} \\ \lambda_{33}{}' = \lambda_{33} \end{matrix} \qquad \begin{matrix} \lambda_{01}{}' = \lambda_{01} \\ \lambda_{02}{}' = \gamma\left(\lambda_{02} + i\dfrac{u}{c}\lambda_{21}\right) \\ \lambda_{03}{}' = \gamma\left(\lambda_{03} - i\dfrac{u}{c}\lambda_{13}\right) \\ \lambda_{32}{}' = \lambda_{32} \\ \lambda_{13}{}' = \gamma\left(\lambda_{13} + i\dfrac{u}{c}\lambda_{03}\right) \\ \lambda_{21}{}' = \gamma\left(\lambda_{21} - i\dfrac{u}{c}\lambda_{02}\right) \end{matrix} . \tag{6.7}$$

Taking into account equations (4.4), (4.10) and that the physical quantity zQ is invariant under the Lorentz-Einstein transformations, we obtain the following transformations for the constants $\alpha_{ki}, k \neq i, k, i = 0,1,2,3$ and the physical quantities $T_k, k = 0,1,2,3$

$$\begin{matrix} T_0{}' = T_0 \\ T_1{}' = T_1 \\ T_2{}' = T_2 \\ T_3{}' = T_3 \end{matrix} \qquad \begin{matrix} \alpha_{01}{}' = \alpha_{01} \\ \alpha_{02}{}' = \gamma\left(\alpha_{02} + i\dfrac{u}{c}\alpha_{21}\right) \\ \alpha_{03}{}' = \gamma\left(\alpha_{03} - i\dfrac{u}{c}\alpha_{13}\right) \\ \alpha_{32}{}' = \alpha_{32} \\ \alpha_{13}{}' = \gamma\left(\alpha_{13} + i\dfrac{u}{c}\alpha_{03}\right) \\ \alpha_{21}{}' = \gamma\left(\alpha_{21} - i\dfrac{u}{c}\alpha_{02}\right) \end{matrix} . \tag{6.8}$$

Equation (6.6) correlates the physical quantities λ_{00} and λ_{11} in the same inertial frame of reference. Taking into account equation (4.10) we obtain $T_0 = T_1$. Thus, when transformations

(6.8) hold, $T_0 = T_1$ also holds. The reference frame $\mathrm{O}'(t', x', y', z')$ moves with respect to the reference frame $\mathrm{O}(t, x, y, z)$ with constant velocity along the $x.$-axis. If we assume that the motion is along the y - or z -axis, the generalization of equation $T_0 = T_1$ follows; the Lorentz-Einstein transformations lead to the following equation $T_0 = T_1 = T_2 = T_3 = 0$. Thus, we derive the following two corollaries.

Corollary 6.1. ´´ When the portion of spacetime occupied by the generalized particle is flat, it is

$$T_0 = T_1 = T_2 = T_3 = 0 \,.´´ \tag{6.9}$$

Corollary 6.2.´´When

$$T_k \neq 0 \tag{6.10}$$

for at least one $k \in \{0,1,2,3\}$ the portion of spacetime occupied by the generalized particle is curver and not flat.´´

In the external symmetry it is $\alpha_{ki} \neq 0$ for at least on pair of indices $k, i \in \{0,1,2,3\}$. Thus, in external symmetry it is $\alpha_{ki} = 0$ only for some pairs of indices $k, i \in \{0,1,2,3\}$. The Lorentz-Einstein transformations reveal that in flat spacetime this cannot be arbitrary. Let's assume that it is

$$\alpha_{02} = 0$$

for every inertial frame of reference. Then, we obtain

$$\alpha_{02}' = 0$$

and with transformations (6.8) we obtain

$$\gamma\left(\alpha_{02} + i\frac{u}{c}\alpha_{21}\right) = 0$$

and since it is $\alpha_{02} = 0$ we obtain that it also holds

$$\alpha_{21} = 0 \,.$$

Working similarly with all of the transformations (6.8) we end up with the following four sets of equations of external symmetry in the flat spacetime:

$$\begin{aligned} &\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\ &\alpha_{02} \neq 0 \\ &\alpha_{03} \neq 0 \\ &\alpha_{32} \neq 0 \\ &\alpha_{13} \neq 0 \\ &\alpha_{21} \neq 0 \end{aligned} \tag{6.11}$$

$$\begin{aligned} &\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\ &\alpha_{02} = 0 \\ &\alpha_{03} = 0 \\ &\alpha_{32} \neq 0 \vee \alpha_{32} = 0 \\ &\alpha_{13} = 0 \\ &\alpha_{21} = 0 \end{aligned} \tag{6.12}$$

$$\begin{aligned} &\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\ &\alpha_{02} \neq 0 \vee \alpha_{02} = 0 \\ &\alpha_{03} = 0 \\ &\alpha_{32} \neq 0 \vee \alpha_{32} = 0 \\ &\alpha_{13} = 0 \\ &\alpha_{21} \neq 0 \vee \alpha_{21} = 0 \end{aligned} \tag{6.13}$$

$$\begin{aligned} &\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\ &\alpha_{02} = 0 \\ &\alpha_{03} \neq 0 \vee \alpha_{03} = 0 \\ &\alpha_{32} \neq 0 \vee \alpha_{32} = 0 \\ &\alpha_{13} \neq 0 \vee \alpha_{13} = 0 \\ &\alpha_{21} = 0 \end{aligned} . \tag{6.14}$$

The symmetry that equations (6.11)-(6.14) express will be referred to as the symmetry of the Lorentz-Einstein-Selfvarlations. These symmetries hold only in case that the part of spacetime occupied by the generalized particle is flat.

7. The Fundamental Study for The Corpuscular Structure of Matter in external symmetry. The Π-Plane.

The material particles are in a constant interaction between them (via the USVI) because of STEM. This interaction has consequences in the internal structure of the generalized particle, including the distribution of its total energy and momentum between the material particle and the surrounding spacetime.

The internal structure of the generalized particle is determined by the relations among the elements of the matrix T. The same holds for the rest mass m_0 of the material particle, the rest energy E_0 of STEM, with which the material particle interacts, and the total rest mass M_0 of the generalized particle. In this paragraph, we study this relation among the elements of the matrix T. We now prove the following theorem:

Theorem 7.1. ´´ For the elements of the T matrix it holds that:

$$T_0T_1T_2T_3 + T_0T_1\alpha_{32}^2 + T_0T_2\alpha_{13}^2 + T_0T_3\alpha_{21}^2 + T_1T_2\alpha_{03}^2 + T_1T_3\alpha_{02}^2 + T_2T_3\alpha_{01}^2 = 0 \text{ .´´} \quad (7.1)$$

Proof. We develop equation (2.13), obtaining the set of equations

$$\begin{aligned} &J_0\lambda_{00} + J_1\lambda_{01} + J_2\lambda_{02} + J_3\lambda_{03} = 0 \\ &-J_0\lambda_{01} + J_1\lambda_{11} - J_2\lambda_{21} + J_3\lambda_{13} = 0 \\ &-J_0\lambda_{02} + J_1\lambda_{21} + J_2\lambda_{22} - J_3\lambda_{32} = 0 \\ &-J_0\lambda_{03} - J_1\lambda_{13} + J_2\lambda_{32} + J_3\lambda_{33} = 0 \end{aligned}$$

and from equations (4.4) and (4.10) we have

$$\begin{aligned} &J_0zQT_0 + J_1zQ\alpha_{01} + J_2zQ\alpha_{02} + J_3zQ\alpha_{03} = 0 \\ &-J_0zQ\alpha_{01} + J_1zQT_1 - J_2zQ\alpha_{21} + J_3zQ\alpha_{13} = 0 \\ &-J_0zQa_{02} + J_1zQa_{21} + J_2zQ\mathrm{T}_2 - J_3zQa_{32} = 0 \\ &-J_0zQ\alpha_{03} - J_1zQ\alpha_{13} + J_2zQ\alpha_{32} + J_3zQT_3 = 0 \end{aligned}$$

and since it holds that $zQ \neq 0$, we take the set of equations

$$\begin{aligned} &J_0T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \\ &-J_0\alpha_{01} + J_1T_1 - J_2\alpha_{21} + J_3\alpha_{13} = 0 \\ &-J_0\alpha_{02} + J_1\alpha_{21} + J_2T_2 - J_3\alpha_{32} = 0 \\ &-J_0\alpha_{03} - J_1\alpha_{13} + J_2\alpha_{32} + J_3T_3 = 0 \end{aligned} . \quad (7.2)$$

The set of equations given in (7.2) comprise a 4×4 homogeneous linear system of equations with unknowns the momenta J_0, J_1, J_2, J_3. In order for the material particle to exist, the system of equations (7.2) must obtain non-vanishing solutions. Therefore, its determinant must vanish. Thus, we obtain equation

$$T_0T_1T_2T_3 + T_0T_1\alpha_{32} + T_0T_2\alpha_{13} + T_0T_3\alpha_{21} + T_1T_2\alpha_{03} + T_1T_3\alpha_{02} + T_2T_3\alpha_{01}$$
$$+(\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21})^2 = 0$$

and with equation (4.8) we arrive at equation (7.1). □

We consider the 4×4 N matrix, given as:

$$N = \begin{bmatrix} 0 & \alpha_{32} & \alpha_{13} & \alpha_{21} \\ -\alpha_{32} & 0 & -\alpha_{03} & \alpha_{02} \\ -\alpha_{13} & \alpha_{03} & 0 & -\alpha_{01} \\ -\alpha_{21} & -\alpha_{02} & \alpha_{01} & 0 \end{bmatrix} . \qquad (7.3)$$

Using the matrix N, we now write equation (4.6) in the form of

$$\begin{aligned} NC &= 0 \\ NJ &= 0 \, . \\ NP &= 0 \end{aligned} \qquad (7.4)$$

We now prove Lemma 7.1:

Lemma 7.1. ′′The four-vectors C, J, P satisfy the set of equations

$$\begin{aligned} N^2C &= 0 \\ N^2J &= 0 \, .'' \\ N^2P &= 0 \end{aligned} \qquad (7.5)$$

Proof. We multiply the set of equations (7.4) from the left with the matrix N, and equations (7.5) follow. □

Using lemma 7.1 we prove theorem 7.2 :

Theorem 7.2. ′′For $M \neq 0$ it holds that:

1. $MN = NM = 0$. (7.6)

2. $M^2 + N^2 = -\alpha^2 I$ (7.7)

$$\alpha^2 = \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 + \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 . \quad (7.8)$$

Here, I is the 4×4 identity matrix.

3. For $\alpha \neq 0$ the matrix M has two eigenvalues τ_1 and τ_2, with corresponding eigenvectors v_1 and v_2, given by:

$$\tau_1 = i\alpha$$

$$v_1 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} - \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \\ \alpha_{03}\alpha_{13} - \alpha_{02}\alpha_{21} \\ \alpha_{01}\alpha_{21} - \alpha_{03}\alpha_{32} \\ \alpha_{02}\alpha_{32} - \alpha_{01}\alpha_{13} \end{bmatrix} \quad (7.9)$$

$$\tau_2 = -i\alpha$$

$$v_2 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} + \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \\ \alpha_{03}\alpha_{13} - \alpha_{02}\alpha_{21} \\ \alpha_{01}\alpha_{21} - \alpha_{03}\alpha_{32} \\ \alpha_{02}\alpha_{32} - \alpha_{01}\alpha_{13} \end{bmatrix} . \quad (7.10)$$

4. For $\alpha \neq 0$ the matrix N has the same eigenvalues with the matrix M, and two corresponding eigenvectors n_1 and n_2, given by:

$$\tau_1 = i\alpha$$

$$n_1 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{bmatrix} - \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \\ \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{bmatrix} \quad (7.11)$$

$$\tau_2 = -i\alpha$$

$$n_2 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{bmatrix} + \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \\ \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{bmatrix} . \quad (7.12)$$

5. $\alpha^2 = \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 + \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0$. (7.13)

$M^2C=0$

6. $M^2J=0$.′′ (7.14)

$M^2P=0$

Proof. The matrices M and N are given by equations (4.28) and (7.3). The proof of equations (7.6), (7.7), (7.9), (7.10), (7.11) and (7.12) can be performed by the appropriate mathematical calculations and the use of equation (4.8).

We multiply equation (7.7) from the left with the column matrices C, J, P , and obtain

$$M^2C+N^2C=-\alpha^2C$$
$$M^2J+N^2J=-\alpha^2J$$
$$M^2P+N^2P=-\alpha^2P$$

and from equations (7.5) we obtain

$$\begin{aligned}M^2C&=-\alpha^2C\\ M^2J&=-\alpha^2J\ .\\ M^2P&=-\alpha^2P\end{aligned} \qquad (7.15)$$

According to the set of equations (7.15) , and for $\alpha\neq 0$, the matrix M^2 has as eigenvalue $\alpha^2\neq 0$ with corresponding eigenvector $v\neq 0$. From equations (7.15) it is evident that the four-vectors C, J, P are parallel to the four-vector v, hence they are also parallel to each other. This is imposssible in the case of the external symmetry, according to Theorem 3.3. Therefore, $\alpha^2=0$, so that the matrix M^2 does not have the four-vector v as an eigenvector. Thus, we arrive at equation (7.13). Then, from equations (7.15) we arrive at equations (7.14), since it holds that $\alpha^2=0$.□

The matrix M^2 , for $M\neq 0$, is a 4×4 symmetric matrix. Furthermore, according to theorem 7.2 , it holds that $tr\left(M^2\right)=2\alpha^2=0$. An immediate consequence of theorem 7.2 is corollary 7.1.

Corollary 7.1. ′′For the matrix T of external symmerty, it is not possible that exactly one of the physical quantities $\alpha_{ki}, k\neq i, k, i\in\{0,1,2,3\}$ is nonzero.′′

Proof. Let us suppose that the matrix T has exactly one nonzero element physical quantity $\alpha_{ki} \neq 0, k \neq i, k, i \in \{0,1,2,3\}$. From equation (4.28) we see that $M \neq 0$, and from equation (7.8) we obtain $\alpha^2 = \alpha_{ki}^2 \neq 0$. This cannot hold, according to equation (7.13).□

From theorem 7.2 it follows:

Corollary 7.2. ´´For the four-vector j of the conserved physical quantities q it holds that:

$$Mj = 0 \tag{7.16}$$

$$Nj = 0 .´´ \tag{7.17}$$

Proof. We multiply equation (5.7) by matrix M from the left and obtain

$$Mj = -\frac{\sigma c^2 b}{\hbar} \Psi \left(\lambda M^2 J + \mu M^2 P \right)$$

and with the second and the third of equations (7.14) we have

$$Mj = 0 .$$

We multiply the terms of equation (5.7) from the left with the matrix N, and obtain

$$Nj = -\frac{\sigma c^2 b}{\hbar} \Psi NM \left(\lambda J + \mu P \right)$$

and with equation (7.6) we take

$$Nj = 0 .$$ □

In the equations of the TSV there appear sums of squares that vanish, like the ones appearing in equations (3.6) and (7.13). Writing these equations in a suitable manner, we can introduce into the equations of the TSV complex numbers. From equation (3.6) , and for $M_0 \neq 0$, we obtain

$$\left(\frac{c_0}{M_0 c} \right)^2 + \left(\frac{c_1}{M_0 c} \right)^2 + \left(\frac{c_2}{M_0 c} \right)^2 + \left(\frac{c_3}{M_0 c} \right)^2 + 1 = 0$$

Therefore, the physical quantities

$$\frac{c_0}{M_0 c}, \frac{c_1}{M_0 c}, \frac{c_2}{M_0 c}, \frac{c_3}{M_0 c}$$

belong in general to the set of complex numbers $\mathbb{C}$. This transformation of the equations of the TSV is not necessary. It suffices to remember that within the equations of the TSV there are sums of squares that vanish. We now prove theorem 7.3, which also intercorrelates the elements of the matrix T:

Theorem 7.3. ′′In the external symmetry and for the elements of the matrix T it holds that:

$$\begin{aligned} &T_i a_{\nu k} = 0 \\ &i \neq \nu, \nu \neq k, k \neq i, i, \nu, k = 0,1,2,3 \end{aligned} .'' \tag{7.18}$$

Proof. We differentiate the second equation of the set of equations (4.6)

$$\begin{aligned} &J_i \alpha_{\nu k} + J_k \alpha_{i\nu} + J_\nu \alpha_{ki} = 0 \\ &i \neq \nu, \nu \neq k, k \neq i, i, \nu, k = 0,1,2,3 \end{aligned}$$

with respect to x_j, $j = 0,1,2,3$. Considering equations (2.10) and (4.4), we have

$$\alpha_{\nu k}\left(\frac{b}{\hbar} P_j J_i + zQ\alpha_{ji}\right) + \alpha_{i\nu}\left(\frac{b}{\hbar} P_j J_k + zQ\alpha_{jk}\right) + \alpha_{ki}\left(\frac{b}{\hbar} P_j J_\nu + zQ\alpha_{j\nu}\right) = 0$$

$$\frac{b}{\hbar} P_j \left(J_i \alpha_{\nu k} + J_k \alpha_{i\nu} + J_\nu \alpha_{ki}\right) + zQ\left(\alpha_{\nu k}\alpha_{ji} + \alpha_{i\nu}\alpha_{jk} + \alpha_{ki}\alpha_{j\nu}\right) = 0$$

and with the second equation of the set of equations (4.6), and taking into account that $zQ \neq 0$, we obtain

$$\begin{aligned} &\alpha_{\nu k}\alpha_{ji} + \alpha_{i\nu}\alpha_{jk} + \alpha_{ki}\alpha_{j\nu} = 0 \\ &i \neq \nu, \nu \neq k, k \neq i, i, \nu, k, j = 0,1,2,3 \end{aligned} . \tag{7.19}$$

Inserting into equation (7.19) successively $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ and $j = 0,1,2,3$, we arrive at the set of equations

$$
\begin{aligned}
&T_0\alpha_{32} = 0\\
&T_0\alpha_{13} = 0\\
&T_0\alpha_{21} = 0\\
&T_1\alpha_{02} = 0\\
&T_1\alpha_{03} = 0\\
&T_1\alpha_{32} = 0\\
&T_2\alpha_{01} = 0\\
&T_2\alpha_{03} = 0\\
&T_2\alpha_{13} = 0\\
&T_3\alpha_{01} = 0\\
&T_3\alpha_{02} = 0\\
&T_3\alpha_{21} = 0
\end{aligned}
. \qquad (7.20)
$$

The set of equations (7.20) is equivalent to equation (7.18).□

Theorem 7.3 is one of the most powerful tools for investigating the external symmetry. This results from corollary 7.3 :

Corollary 7.3. ′′For the elements of the matrix T of the external symmetry the following hold:

1. For every $k \neq i, \nu \neq k, \nu \neq i, k, i, \nu \in \{0,1,2,3\}$ it holds that

$$
\left.
\begin{aligned}
&\alpha_{ki} \neq 0\\
&k \neq i\\
&\nu \neq k, i
\end{aligned}
\right\} \Rightarrow \mathrm{T}_\nu = 0 . \qquad (7.21)
$$

2. If $\alpha_{ki} \neq 0$ for at least four pairs of the physical quantities $\alpha_{ki}, k \neq i, k, i \in \{0,1,2,3\}$, then all the elements of the main diagonal of the matrix T are equal to zero:

$$
\left.
\begin{aligned}
&\alpha_{ki} = 0\\
&\alpha_{\nu j} = 0\\
&k \neq i, \nu, j\\
&i \neq \nu, j\\
&\nu \neq j\\
&k, i, \nu, j \in \{0,1,2,3\}
\end{aligned}
\right\} \Rightarrow T_0 = T_1 = T_2 = T_3 = 0 .'' \qquad (7.22)
$$

Proof. Corollary 7.3 is an immediate consequence of theorem 7.3.□

From theorem 7.3 the following corollary follows, regarding the elements of the main diagonal of the matrices of the external symmetry:

Corollary 7.4. ´´At least one of the elements of the main diagonal of the matrix T is equal to zero.´´

Proof. If $T_\nu \neq 0$ for every $\nu \in \{0,1,2,3\}$, from equations (7.20) we obtain $\alpha_{ki} = 0$ for every set of indices $k \neq i, k, i = 0,1,2,3$, and from equation (7.1) we have

$$T_0 T_1 T_2 T_3 = 0 .$$

This cannot hold, since we assumed that $T_\nu \neq 0$ for every $\nu = 0,1,2,3$. Therefore, at least one element of the main diagonal of the matrix T is equal to zero.

We present a second way for proving this result. In the case of $T_\nu \neq 0$ for every $\nu \in \{0,1,2,3\}$, we obtain from equations (7.20) that $\alpha_{ki} = 0$, for every $k \neq i, k, i = 0,1,2,3$ .Thus, the matrix T takes the form

$$T = zQ \begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix} .$$

From equation (2.13) we take

$$T_0 J_0 = T_1 J_1 = T_2 J_2 = T_3 J_3 = 0$$

Since we assumed that

$$T_0 T_1 T_2 T_3 \neq 0$$

we obtain

$$J_0 = J_1 = J_2 = J_3 = 0 .$$

Thus, the material particle does not exist.□

We consider now the three-dimensional vectors

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} \tag{7.23}$$

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \quad . \tag{7.24}$$

In the case of the T matrices with $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$, we define the vector $\boldsymbol{\mu} \neq \mathbf{0}$ from equation

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{pmatrix} \quad . \tag{7.25}$$

Combining equations (5.1), (5.2) with equations (7.23) and (7.24) we obtain

$$\boldsymbol{\xi} = ic\Psi\,\mathbf{n} \tag{7.26}$$

$$\boldsymbol{\omega} = \Psi\boldsymbol{\tau} \quad . \tag{7.27}$$

The field $\boldsymbol{\xi}$ is parallel to the vector $\mathbf{n}$ and the field $\boldsymbol{\omega}$ is parallel to the vector $\boldsymbol{\tau}$.

For every vector

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

which is determined by the physical quantities of the TSV, we define the physical quantity

$$\|\boldsymbol{\alpha}\| = \left(\boldsymbol{\alpha}^T\boldsymbol{\alpha}\right)^{\frac{1}{2}} = \left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)^{\frac{1}{2}} . \tag{7.28}$$

Here, the matrix $\boldsymbol{\alpha}^T$ is the transposed matrix of the column matrix $\boldsymbol{\alpha}$.

From equations (7.23) and (7.24) we obtain

$$\boldsymbol{\tau} \cdot \mathbf{n} = \alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21} .$$

Also, from equation (4.8) we have

$$\boldsymbol{\tau} \cdot \mathbf{n} = 0 . \tag{7.29}$$

Therefore, the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$ are perpendicular to each other. Considering also equation (7.25), we see that the triple of the vectors $\{\boldsymbol{\mu}, \mathbf{n}, \boldsymbol{\tau}\}$ forms a right-handed vector basis.

From equation (7.13) we have

$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = -\left(\alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2\right)$$

and with equations (7.23), (7.24), and using the notation of equation (7.28), we obtain

$$\| \mathbf{n} \|^2 = -\| \boldsymbol{\tau} \|^2$$

and finally we obtain

$$\| \mathbf{n} \| = \pm i \| \boldsymbol{\tau} \| . \qquad (7.30)$$

From equation (7.25) we have

$$\boldsymbol{\mu}^2 = \left(\mathbf{n} \times \boldsymbol{\tau}\right)^2$$

and since the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$ are perpendicular to each other, we obtain from equation (7.29) that

$$\boldsymbol{\mu}^2 = \mathbf{n}^2 \boldsymbol{\tau}^2$$

and using the notation of equation (7.28) we have

$$\| \boldsymbol{\mu} \|^2 = \| \mathbf{n} \|^2 \| \boldsymbol{\tau} \|^2$$

$$\| \boldsymbol{\mu} \| = \pm \| \mathbf{n} \| \| \boldsymbol{\tau} \|$$

and from equation (7.30) we take

$$\| \boldsymbol{\mu} \| = \pm i \| \mathbf{n} \|^2 = \mp \| \boldsymbol{\tau} \|^2 . \qquad (7.31)$$

In the case of the T matrices, where $\| \mathbf{n} \| \neq \mathbf{0}$, and from equation (7.31), it follows that $\| \boldsymbol{\tau} \| \neq 0, \| \boldsymbol{\mu} \| \neq 0$. In these cases we can define the set of unit vectors $\{\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3\}$, given by

$$
\begin{aligned}
&\boldsymbol{\varepsilon}_1 = \frac{\boldsymbol{\mu}}{||\boldsymbol{\mu}||} \\
&\boldsymbol{\varepsilon}_2 = \frac{\mathbf{n}}{||\mathbf{n}||} \quad . \\
&\boldsymbol{\varepsilon}_3 = \frac{\boldsymbol{\tau}}{||\boldsymbol{\tau}||} \\
&||\mathbf{n}|| \neq 0
\end{aligned} \tag{7.32}
$$

The triple of vectors $\{\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3\}$ forms a right-handed orthonormal vector basis.

In the cases of the T matrices with $\boldsymbol{\tau} \neq \mathbf{0}$, we define with Π the plane perpendicular to the vector $\boldsymbol{\tau} \neq \mathbf{0}$. In the cases where moreover $\mathbf{n} \neq \mathbf{0}$, we obtain from equation (7.25) that $\boldsymbol{\mu} \neq \mathbf{0}$.

In these cases the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$ are perpendicular to the vector $\boldsymbol{\tau}$, as implied by equations (7.25) and (7.29). Therefore, the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$ belong to the plane Π, and they also form an orthogonal basis of this plane. We note that the vectors of the TSV, which may belong to the plane Π, are given as a linear combination of the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$. Therefore, the condition for $\boldsymbol{\tau} \neq \mathbf{0}$ is not sufficient, in order for the plane Π to acquire a physical meaning. Also, we note that because of equation (7.13), the plane Π, when it is defined, is not a vector subspace of $\mathbb{R}^3$.

We now prove theorem 7.4:

Theorem 7.4. ′′In the case of the T matrices with $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$ and $\boldsymbol{\tau} \neq \pm\mathbf{n} \neq \mathbf{0}$, the vectors

$\mathbf{J}, \mathbf{P}, \mathbf{C}, \mathbf{j}, \nabla\Psi$ belong to the same plane Π.′′

Proof. From equations (4.6), for $(i, \nu, k) = (1, 3, 2)$, we obtain

$$
\begin{aligned}
&c_1\alpha_{32} + c_2\alpha_{13} + c_3\alpha_{21} = 0 \\
&J_1\alpha_{32} + J_2\alpha_{13} + J_3\alpha_{21} = 0 \\
&P_1\alpha_{32} + P_2\alpha_{13} + P_3\alpha_{21} = 0
\end{aligned}
$$

and from equations (5.8),(5.9) and (7.23) we get

$$\boldsymbol{\tau}\cdot\mathbf{C}=0$$
$$\boldsymbol{\tau}\cdot\mathbf{J}=0 \quad (7.33)$$
$$\boldsymbol{\tau}\cdot\mathbf{P}=0$$

where

$$\mathbf{C}=\mathbf{J}+\mathbf{P} \quad (7.34)$$

as implied by equation (3.5). From equation (7.33) we conclude that the vectors $\mathbf{C},\mathbf{J},\mathbf{P}$, being perpendicular to vector $\boldsymbol{\tau}$, belong to the plane Π. From equation (5.3) and equations (5.8) and (5.9) we obtain

$$\nabla\Psi=\frac{b}{\hbar}\left(\lambda\mathbf{J}+\mu\mathbf{P}\right).$$

Therefore, the vector $\nabla\Psi$, as a linear combination of the vectors $\mathbf{J},\mathbf{P}$, belongs to the plane Π. By developing the terms of equation (7.17), the first obtained equation is

$$\alpha_{32}j_1+\alpha_{13}j_2+\alpha_{21}j_3=0$$

and using equation (7.23) we have

$$\boldsymbol{\tau}\cdot\mathbf{j}=0\,. \quad (7.35)$$

Therefore, the vector $\mathbf{j}$, being perpendicular to the vector $\boldsymbol{\tau}$, belongs to the plane Π. The vectors $\mathbf{J},\mathbf{P},\mathbf{C},\mathbf{j},\nabla\Psi$ vary according to the equations of the TSV, while staying on the plane Π. □

From this study we can obtain a method about the determination of the four-vectors $\mathbf{J},\mathbf{P},\mathbf{C}$, as well as for the rest masses $m_0,\frac{E_0}{c^2},M_0$. This method is applied in the case the matrix M does not vanish, that is $M\neq 0$. We shall refer to this method as the $SV-M$ method.

The steps of the *SV-M* method:

Step 1. We choose external symmetry matrix T we want to study.

Step 2. We apply Theorem 7.3.

Step 3. We use equation (7.13).

Step 4. We use equation (2.13), or the equivalent equations (7.2).

Step 5. We use the second of the set of equations (4.6).

Step 6. We use the first of the set of equations (7.14).

Step 7. We use the first of the set of equations (4.6).

Step 8. We use equation (3.5).

As an example, we apply this method on the matrix T :

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix} \qquad (7.36)$$

where, $\alpha_{01}\alpha_{21} \neq 0$.

From equations (7.20), and since $\alpha_{01} \neq 0$ and $\alpha_{21} \neq 0$, we have $T_0 = T_2 = T_3 = 0$, and the matrix (7.36) becomes

$$T = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (7.37)$$

For $T_1 \neq 0$, according to corollary 6.2 the portion of spacetime occupied by the generalized particle is curved. From equation (4.10) we obtain $\Phi_1 \neq 0$. Therefore, the second term of the second part of the second equation in the set of equations (4.21) is nonzero.

In the case the portion of spactime occupied by the generalized particle is flat, we obtain from corollary 6.1 that $T_1 = 0$. Therefore, $T_0 = T_1 = T_2 = T_3 = 0$. In this case, and from equation (4.11), we obtain $\Lambda = 0$, and the second term of the second part of equation (4.19) vanishes.

From equation (7.13) we take

$$\alpha^2 = \alpha_{01}^2 + \alpha_{21}^2 = 0$$

$$\alpha_{21} = \pm i\alpha_{01}. \qquad (7.38)$$

From equations (7.2) we obtain

$$\begin{aligned} &J_1\alpha_{01} = 0 \\ &-J_0\alpha_{01} + J_1T_1 - J_2\alpha_{21} = 0 \\ &J_1\alpha_{21} = 0 \end{aligned}$$

and since $\alpha_{01}\alpha_{21} \neq 0$, we have that

$$\begin{aligned} &J_1 = 0 \\ &J_2 = -\frac{\alpha_{01}}{\alpha_{21}} J_0 \end{aligned}. \qquad (7.39)$$

From the second of equations (4.6), for $(i,\nu,k) = (3,0,1)$ we have

$$J_3\alpha_{01} + J_1\alpha_{30} + J_0\alpha_{13} = 0$$

and since

$$\alpha_{01} \neq 0, \alpha_{30} = -\alpha_{03} = 0, \alpha_{13} = 0$$

we obtain

$$J_3 = 0. \qquad (7.40)$$

From equations (7.39) and (7.40), and from equation (2.4), we get the four-vector J

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ -\dfrac{\alpha_{01}}{\alpha_{21}} \\ 0 \end{bmatrix} = J_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix} = \frac{W}{c} \begin{bmatrix} i \\ 0 \\ \mp 1 \\ 0 \end{bmatrix}. \qquad (7.41)$$

For the second equality in equation (7.41) we applied the second equation of equations (7.38).

From equations (4.29) and (7.37) we have

$$M = \begin{bmatrix} 0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & 0 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7.42}$$

$$M^2 = \begin{bmatrix} -\alpha_{01}^2 & 0 & -\alpha_{01}\alpha_{21} & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_{01}\alpha_{21} & 0 & -\alpha_{21}^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{7.43}$$

From the first of equations (7.14) we see that

$$M^2 C = 0$$

and with equations (3.5) and (7.43) we obtain

$$-a_{01}^2 c_0 - a_{01}a_{21}c_2 = 0$$
$$-a_{01}a_{21}c_0 - a_{21}^2 c_2 = 0$$

and taking into account that $\alpha_{01}\alpha_{21} \neq 0$, we obtain

$$c_2 = -\frac{\alpha_{01}}{\alpha_{21}} c_0 . \tag{7.44}$$

From the first of the equations (4.6), for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we have

$$c_0\alpha_{12} + c_2\alpha_{01} + c_1\alpha_{20} = 0$$
$$c_0\alpha_{13} + c_3\alpha_{01} + c_1\alpha_{30} = 0$$
$$c_0\alpha_{23} + c_3\alpha_{02} + c_2\alpha_{30} = 0$$
$$c_1\alpha_{23} + c_3\alpha_{12} + c_2\alpha_{31} = 0$$

and taking into account the zero elements of the matrix T we have

$$c_0\alpha_{21} + c_2\alpha_{01} = 0$$
$$c_3\alpha_{01} = 0$$
$$c_3\alpha_{12} = 0$$

and since

$$j_0\alpha_{12} + j_2\alpha_{01} = 0$$
$$j_3\alpha_{01} = 0$$
$$j_3\alpha_{12} = 0$$

we obtain

$$c_2 = \frac{\alpha_{21}}{\alpha_{01}} c_0 \quad . \qquad (7.45)$$
$$c_3 = 0$$

The first of equations (7.45) is equation (7.44), because of equation (7.38).

From equations (3.5) and (7.45) we obtain the four-vector C

$$C = \begin{bmatrix} c_0 \\ c_1 \\ \dfrac{\alpha_{21}}{\alpha_{01}} c_0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ \pm i c_0 \\ 0 \end{bmatrix}. \qquad (7.46)$$

Combining equation (3.5)

$$P = C - J$$

with equations (7.41) and (7.46) we obtain the four-vector P

$$P = \begin{bmatrix} c_0 - J_0 \\ c_1 \\ \mp i\left(c_0 - J_0\right) \\ 0 \end{bmatrix}. \qquad (7.47)$$

After having determined the four-vectors J, P, C, we can calculate the rest masses $m_0, \frac{E_0}{c^2}, M_0$. From equations (2.7) and (7.41) we get

$$m_0 = 0 \,. \qquad (7.48)$$

From equations (2.8) and (7.47) we have

$$E_0 = \pm icc_1 \,. \qquad (7.49)$$

From equations (3.6) and (7.46) we also have

$$M_0 = \pm \frac{ic_1}{c}. \tag{7.50}$$

The calculation of the four-vector j of the current density of the conserved physical quantities q is dfone from corollary 7.2. This method is applied for $M \neq 0$, and is performed in two steps. We shall refer to this method as the SV_q -method.

The steps of the SV_q - method:

Step 1. We use equation (7.17), or the equivalent equation:

$$\begin{gathered} j_i\alpha_{\nu k} + j_k\alpha_{i\nu} + j_\nu\alpha_{ki} = 0 \\ i \neq \nu, \nu \neq k, k \neq i, i, \nu, \mathrm{k} = 0,1,2,3 \end{gathered}. \tag{7.51}$$

Step 2. We use equation (7.16).

We apply the SV_q -method on the matrix T given by equation (7.37). From equation (7.51), for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we have

$$\begin{gathered} j_0\alpha_{12} + j_2\alpha_{01} + j_1\alpha_{20} = 0 \\ j_0\alpha_{13} + j_3\alpha_{01} + j_1\alpha_{30} = 0 \\ j_0\alpha_{23} + j_3\alpha_{02} + j_2\alpha_{30} = 0 \\ j_1\alpha_{23} + j_3\alpha_{12} + j_2\alpha_{31} = 0 \end{gathered}$$

and taking into account the elements of the matrix T we have

$$\begin{gathered} j_0\alpha_{12} + j_2\alpha_{01} = 0 \\ j_3\alpha_{01} = 0 \\ j_3\alpha_{12} = 0 \end{gathered}$$

and since $\alpha_{01} \neq 0, \alpha_{30} = -\alpha_{03} = 0, \alpha_{13} = 0$, we get

$$\begin{gathered} j_2 = \frac{\alpha_{21}}{\alpha_{01}} j_0 \\ j_3 = 0 \end{gathered}. \tag{7.52}$$

The matrix M is given by equation (7.42). Thus, from equations (4.27) and (7.16) we have

$$\begin{gathered} j_1\alpha_{01} = 0 \\ -j_0\alpha_{01} - j_2\alpha_{21} = 0 \\ j_1\alpha_{21} = 0 \end{gathered}$$

and since $\alpha_{01} \neq 0$ and $\alpha_{21} \neq 0$, we have

$$\begin{aligned} j_1 &= 0 \\ j_2 &= -\frac{\alpha_{01}}{\alpha_{21}} j_0 \end{aligned} . \tag{7.53}$$

The first of the equations (7.52) and the second of the equations (7.53) are identical due to equations (7.38). From equations (7.52) and (7.53) we obtain the four-vector j

$$j = j_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{21}}{\alpha_{01}} \\ 0 \end{bmatrix} = j_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix} = \rho c \begin{bmatrix} i \\ 0 \\ \mp 1 \\ 0 \end{bmatrix}. \tag{7.54}$$

We now summarize the obtained information for the generalized particle of the matrix T of equation (7.36):

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix} \qquad P = \begin{bmatrix} c_0 - J_0 \\ c_1 \\ \mp i\left(c_0 - J_0\right) \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} c_0 \\ c_1 \\ \pm i c_0 \\ 0 \end{bmatrix} \qquad j = j_0 \begin{bmatrix} 1 \\ 0 \\ \mp i \\ 0 \end{bmatrix} \tag{7.55}$$

$$m_0 = 0 \, , E_0 = \pm i c c_1 \quad , M_0 = \pm \frac{i c_1}{c} .$$

$$T_1 \neq 0 \Rightarrow \text{curved spacetime} \tag{7.56}$$

$$\text{flat spacetime} \Rightarrow T_1 = 0 .$$

From equations (5.7) and (7.41), (7.47), (7.54) we have

$$j_0 = -\frac{\sigma c^2 b}{\hbar} \Psi \lambda c_1 \alpha_{01} = \pm \frac{i \sigma c^2 b}{\hbar} \Psi \lambda c_1 \alpha_{21} \tag{7.57}$$

for the matrix T of our study. Also, from equations (5.17) and (7.54) we obtain

$$\frac{\partial j_0}{\partial x_1} = -\sigma c^2 F \alpha_{01} = \pm i\sigma c^2 F \alpha_{21}$$
$$\frac{\partial j_0}{\partial x_2} = \pm i \frac{\partial j_0}{\partial x_0} \qquad . \qquad (7.58)$$
$$\frac{\partial j_0}{\partial x_3} = 0$$
$$F = \nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2}$$

Equations (7.57) and (7.58) correlate the function Ψ with the four-vector j of the current density of the conserved physical quantities q. These equations hold for the matrix T of equation (7.37).

The presented method about the study of the generalized particle is possibly the simplest, but surely not the only one. The TSV stems from one equation which nonetheless generates an extremely complex network of equations. We present one method, which serves as a test for the self-consistency of the TSV. With the same method we can check for calculational errors of the obtained equations, as we proceed from one set of equations of the TSV into another set. We shall refer to this method as the $SV-T$-method.

The internal structure of every generalized particle depends on the corresponding matrix T. The $SV-T$ method consists of the following steps:

The $SV-T$ - Method:

We choose an equation (E_1), which holds for the matrix T, and for which there exist at least two different components of the four-vector J, or one component and the rest mass m_0. By differentiating equation (E_1) with respect to $x_k, k = 0,1,2,3$ we obtain a second equation (E_2).

With the help of equation (2.10)

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + \lambda_{ki} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki}$$
$$k, i = 0,1,2,3$$

the constants $\alpha_{ki}, k, i = 0,1,2,3$ are introduced into equation (E_2). Equation (E_2) has to be compatible with the elements of the matrix T. In the case equation (E_1) contains the rest mass m_0 we apply equation (2.6)

$$\frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k J_i m_0, k = 0,1,2,3.$$

We apply the method for the matrix T of equation (7.37). From equation (7.41) we obtain

$$J_2 = \pm i J_0. \quad (7.59)$$

This equation contains the components J_0, J_2 of the four-vector J. We differentiate equation (7.59) with respect to $x_k, k = 0,1,2,3$, to obtain

$$\frac{b}{\hbar} P_k J_2 + zQ\alpha_{k2} = \pm i\left(\frac{b}{\hbar} P_k J_0 + zQ\alpha_{k0}\right)$$

and using equation (7.59) we have

$$zQ\alpha_{k2} = \pm izQ\alpha_{k0}$$

and since $zQ \neq 0$ we get

$$\alpha_{k2} = \pm i\alpha_{k0}, k = 0,1,2,3. \quad (7.60)$$

In equation (7.60) we insert successively $k = 0,1,2,3$

For $k = 0$ we obtain

$$\alpha_{02} = \pm i\alpha_{00} = \pm iT_0$$

which holds, since $\alpha_{02} = 0, T_0 = 0$.

For $k = 1$ we get

$$\alpha_{12} = \pm i\alpha_{10}$$

and since $\alpha_{10} = -\alpha_{01}$, we get

$$\alpha_{12} = \pm i\alpha_{01}$$
$$\alpha_{01}^2 + \alpha_{21}^2 = 0$$

which are equations (7.38).

For $k = 2$ we obtain

$$a_{22} = \pm i a_{20}$$
$$T_2 = \pm i a_{20}$$

which holds for the matrix T, since $a_{02} = 0, T_2 = 0$.

For $k = 3$ we have

$$\alpha_{32} = \pm i\alpha_{30}$$
$$\alpha_{32} = \mp i\alpha_{03}$$

which holds for the matrix T, since $\alpha_{32} = 0, \alpha_{03} = 0$.

For the matrix we study it holds that $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$ and $\boldsymbol{\tau} \neq \pm\mathbf{n} \neq \mathbf{0}$,and therefore plane Π is defined. From equations (7.32) we have

$$\boldsymbol{\varepsilon}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$
$$\boldsymbol{\varepsilon}_2 = \begin{pmatrix} i \\ 0 \\ 0 \end{pmatrix}. \tag{7.61}$$
$$\boldsymbol{\varepsilon}_3 = \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix}$$

From equations (7.46) and (7.61) we have

$$\mathbf{C} = \begin{pmatrix} c_1 \\ \pm i c_0 \\ 0 \end{pmatrix} = \pm i c_0 \boldsymbol{\varepsilon}_1 + c_1 \boldsymbol{\varepsilon}_2 . \tag{7.62}$$

Equations (7.62) contains the components $(\pm ic_0, c_1)$ of the vector $\mathbf{C}$ with respect to the vector basis $(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2)$ of the Π-plane. Considering that the vectors $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2$ are perpendicular to each other, we obtain from equation (7.62)

$$\boldsymbol{\varepsilon}_1 \cdot \mathbf{C} = \pm ic_0$$
$$\boldsymbol{\varepsilon}_2 \cdot \mathbf{C} = ic_1$$

and from equations (7.49) and (7.50) we have

$$\begin{aligned} \boldsymbol{\varepsilon}_1 \cdot \mathbf{C} &= \pm ic_0 \\ \boldsymbol{\varepsilon}_2 \cdot \mathbf{C} &= \pm M_0 = \pm \frac{E_0}{c^2} \end{aligned} \quad . \qquad (7.63)$$

From the first of the equations (7.63) we obtain the equivalence

$$\boldsymbol{\varepsilon}_1 \cdot \mathbf{C} = 0 \Leftrightarrow c_0 = 0 . \qquad (7.64)$$

The total energy ic_0 of the generalized particle is zero, if and only if the vector $\mathbf{C}$ is perpendicular to the vector $\boldsymbol{\varepsilon}_1$, and therefore parallel to the vector $\boldsymbol{\varepsilon}_2$. Similarly, from the second of the equations (7.63) we obtain the equivalence

$$\boldsymbol{\varepsilon}_2 \cdot \mathbf{C} = 0 \Leftrightarrow M_0 = 0 \Leftrightarrow E_0 = 0 . \qquad (7.65)$$

The rest masses M_0 and $\frac{E_0}{c^2}$ are equal to zero, if and only if the vector $\mathbf{C}$ of the total momentum of the generalized particle is parallel to the vector $\boldsymbol{\varepsilon}_1$.

As a consequence of theorem 7.3, and for a large set of matrixes of the external symmetry, which contain many non-zero elements $\alpha_{ki} \neq 0, k, i = 0,1,2,3$, it is

$$T_0 = T_1 = T_2 = T_3 = 0 .$$

For these matrices we prove the following theorem 7.5:

Theorem 7.5. ´´The matrices T of the external symmetry, for which all the elements of the main diagonal are zero, the four-vectors J and j are parallel to each other.´´

Proof. From equations (2.12) and (4.4),(4.10) we have

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & T_2 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & T_3 \end{bmatrix}. \tag{7.66}$$

In the case where

$$T_0 = T_1 = T_2 = T_3 = 0$$

we get

$$T = zQM\ . \tag{7.67}$$

from equations (4.28) and (7.66). Combining equations (2.13), (7.17), and since $zQ \neq 0$, we have

$$MJ = 0$$

and taking into account the second of the equations (7.4) we have

$$\begin{matrix} MJ = 0 \\ NJ = 0 \end{matrix}. \tag{7.68}$$

Because the equations (7.68) and (7.16) hold simultaneously, the four-vectors J and j are parallel to each other.□

An immediate consequence of theorem 7.5 is corollary 7.5 :

Corollary 7.5 ′′For the symmetries where all of the elements of the main diagonal of the matrix T are zero, there exists a function

$$V = V\left(x_0, x_1, x_2, x_3\right) \neq 0$$

satisfying the continuity equation

$$\nabla \cdot \left(\frac{\mathbf{J}}{V}\right) + \frac{\partial}{\partial t}\left(\frac{W}{V}\right) = 0\ .'' \tag{7.69}$$

Proof. From theorem 7.5 there exists a function

$$V = V\left(x_0, x_1, x_2, x_3\right) \neq 0$$

such that

$$J = Vj\ . \tag{7.70}$$

Then equation (7.69) results from the combination of equations (4.27), (5.6) and (7.70), since

$$J_0 = \frac{iW}{c}\ .\ \square$$

We shall not present in the present work the physical content of equation (7.69).

The next theorem 7.6 relates the four-vector J with the elements of the main diagonal of the external symmetry matrix T.

Theorem 7.6. ´´For every external symmetry matrix T it holds that

$$T_0 J_0^2 + T_1 J_1^2 + T_2 J_2^2 + T_3 J_3^2 = 0.'' \tag{7.71}$$

Proof. Since the material particle exists, at least one component of the four-vector J is nonzero. We prove the theorem for $J_0 \neq 0$. The proof for $J_i \neq 0, i = 1,2,3$ follows similar lines. For $J_0 \neq 0$, we obtain from equations (7.2)

$$J_0 T_0 + J_1 \alpha_{01} + J_2 \alpha_{02} + J_3 \alpha_{03} = 0$$
$$\alpha_{01} = \frac{1}{J_0}\left(J_1 T_1 - J_2 \alpha_{21} + J_3 \alpha_{13}\right)$$
$$\alpha_{02} = \frac{1}{J_0}\left(J_1 \alpha_{21} + J_2 T_2 - J_3 \alpha_{32}\right)$$
$$\alpha_{03} = \frac{1}{J_0}\left(-J_1 \alpha_{13} + J_2 \alpha_{32} + J_3 T\right)$$

and replacing the terms $\alpha_{01}, a_{02}, a_{03}$ in the first equation we obtain

$$J_0 T_0 + \frac{J_1}{J_0}\left(J_1 T_1 - J_2 \alpha_{21} + J_3 \alpha_{13}\right) + \frac{J_2}{J_0}\left(J_1 \alpha_{21} + J_2 T_2 - J_3 \alpha_{32}\right)$$
$$+ \frac{J_3}{J_0}\left(-J_1 \alpha_{13} + J_2 \alpha_{32} + J_3 T_3\right) = 0$$

$$J_0^2 T_0 + J_1^2 T_1 - J_1 J_2 \alpha_{21} + J_1 J_3 \alpha_{13} + J_2 J_1 \alpha_{21} + J_2^2 T_2$$
$$- J_2 J_3 \alpha_{32} - J_3 J_1 \alpha_{13} + J_3 J_2 \alpha_{32} + J_3^2 T_3 = 0$$

$$T_0 J_0^2 + T_1 J_1^2 + T_2 J_2^2 + T_3 J_3^2 = 0\ .\ \square$$

An immediate consequence of theorem 7.6 is corollary 7.5.

Corollary 7.6. ´´For every matrix T of the external symmetry the following holds:

1. $T_0 = T_1 = T_2 = T_3 \neq 0 \Rightarrow m_0 = 0$. (7.72)

2. $$\left.\begin{array}{l} T_0 = T_1 = T_2 = T_3 \\ m_0 \neq 0 \end{array}\right\} \Rightarrow T_0 = T_1 = T_2 = T_3 = 0 \text{ .´´} \quad (7.73)$$

Proof. For $T_0 = T_1 = T_2 = T_3$ we obtain from equation (7.71)

$$T_0\left(J_0^2 + J_1^2 + J_2^2 + J_3^2\right) = 0$$

and with equation (2.7) we have

$$T_0 m_0^2 c^2 = 0. \quad (7.74)$$

1. Since $T_0 \neq 0$, from equation (7.74) we have $m_0 = 0$.

2. Since $m_0 \neq 0$, from equation (7.74) we have $T_0 = 0$. Since $T_0 = T_1 = T_2 = T_3$, we obtain

$T_0 = T_1 = T_2 = T_3 = 0$. □

We calculate the number of the external symmetry matrices. This number is determined by theorem 7.3 and corollaries 7.1 and 7.4. Also notice that the external symmetry matrices are non-zero. Applying simple combinatorial rules, we see that altogether there exist

$$N_1 = 14$$

external symmetry matrices with $\alpha_{ki} = 0$ for every $k \neq i, k, i = 0,1,2,3$. These matrices contain non-zero elements only on the main diagonal. The number $N_2^{'}$ of matrices with two elements, $\alpha_{ki} \neq 0, k \neq i, k, i \in \{0,1,2,3\}$ is

$$N_2^{'} = 27$$

with three elements it is

$$N_3^{'} = 23$$

with four elements it is

$N_4^{'} = 15$

with five elements it is

$N_5 = 6$

with six elements it is

$N_6 = 1$.

From equation (2.13) and the second of the equations (4.6) we can prove that some of these matrices give the four-vector $J = 0$, thus are rejected. Therefore, we obtain

$$
\begin{aligned}
&N_1 = 14 \\
&N_2 = N_2^{'} - 3 = 24 \\
&N_3 = N_3^{'} - 17 = 6 \\
&N_4 = N_4^{'} - 12 = 3 \\
&N_5 = 6 \\
&N_6 = 1
\end{aligned}
$$

Thus the total number N_T of external symmetry matrices is

$$N_T = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 = 54 \qquad (7.75)$$

The matrix $T = 0$ is unique

$N_O = 1$

and according to theorem 3.3 this matrix expresses the internal symmetry. Therefore, the total number of the matrices of the internal and external symmetry predicted by the Law of Selfvariations is

$$N_{OT} = N_O + N_T = 55. \qquad (7.76)$$

There exist

$$N_J = N_T - 14 = 40 \qquad (7.77)$$

external symmetry matrices with different four-vectors J, P, C, j.

We now prove for example that the following matrix

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & T_2 & 0 \\ -\alpha_{03} & -\alpha_{13} & 0 & T_3 \end{bmatrix}$$

is not an external symmetry matrix.

Applying theorem 7.3 for the above matrix we have

$$T_0 = T_1 = T_2 = T_3 = 0$$

and therefore it takes the form

$$T = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & 0 & 0 \\ -\alpha_{03} & -\alpha_{13} & 0 & 0 \end{bmatrix}$$

and with equation (2.13) we obtain

$$\begin{aligned} &J_1\alpha_{01} + J_3\alpha_{03} = 0 \\ &-J_0\alpha_{01} - J_2\alpha_{21} + J_3\alpha_{13} = 0 \\ &J_1\alpha_{21} = 0 \\ &-J_0\alpha_{03} - J_1\alpha_{13} = 0 \end{aligned}$$

and since

$$\alpha_{01}\alpha_{03}\alpha_{13}\alpha_{21} \neq 0$$

we have

$$J_0 = J_1 = J_2 = J_3 = 0$$

which is impossible since there is no material particle in this case.

We present now a notation for the matrices of the external symmetry. In every matrix T we use an upper and a lower index. As lower indices we use the pairs $(k,i), k \neq i, k,i = 0,1,2,3$ of the constants $\alpha_{ki} \neq 0$, which are nonzero. These indices, which appear always in pairs, are placed in the order of the following constants: $\alpha_{01}, \alpha_{02}, \alpha_{03}, \alpha_{32}, \alpha_{13}, \alpha_{21}$, which are nonzero. As upper indices we use the indices of the nonzero elements of the main diagonal, in the following order: T_0, T_1, T_2, T_3. For example, the matrix T given in equation (7.37) is denoted as T^1_{0121}.

With this notation, the $N_T = 54$ external symmetry matrices are given from the following six sets Ω:

$$
\begin{aligned}
&\Omega_1 = \left\{T^0, T^1, T^2, T^3, T^{01}, T^{02}, T^{03}, T^{12}, T^{13}, T^{23}, T^{012}, T^{013}, T^{023}, T^{123}\right\} \\
&\Omega_2 = \left\{T^0_{0102}, T_{0102}, T^0_{0103}, T_{0103}, T^0_{0203}, T_{0203}, T^3_{3213}, T_{3213}, T^2_{3221}, T_{3221}, T^1_{1321}, T_{1321},\right. \\
&\left. T^1_{0113}, T_{0113}, T^1_{0121}, T_{0121}, T^2_{0232}, T_{0232}, T^2_{0221}, T_{0221}, T^3_{0332}, T_{0332}, T^3_{0313}, T_{0313}\right\} \\
&\Omega_3 = \left\{T^0_{010203}, T_{010203}, T_{010232}, T_{010221}, T^3_{033213}, T_{033213}\right\} \\
&\Omega_4 = \left\{T_{01023213}, T_{01033221}, T_{02031321}\right\} \\
&\Omega_5 = \left\{T_{0102033221}, T_{0102033213}, T_{0102031321}, T_{0102321321}, T_{0103321321}, T_{0203321321}\right\} \\
&\Omega_6 = \left\{T_{010203321321}\right\}
\end{aligned}
\quad . \tag{7.78}
$$

From the combination of eqs. (2.10), (3.5) and (4.4), and the eq. (2.13) we get

$$
\begin{aligned}
&\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki}, k, i = 0,1,2,3 \\
&J + P = C \\
&TJ = 0
\end{aligned}
\quad . \tag{7.79}
$$

It is easy to verify that the TSV could be formulated starting from eqs. (7.79). The reason we refer to this is that in the external symmetries of the sets $\Omega_2, \Omega_3, \Omega_4, \Omega_5, \Omega_6$ of eq. (7.78), as for the symmetries T^1_{0121} and T_{0121} we have studied, it is $m_0 = 0$. Thus we cannot start our disposition from eq. (1.1). In contrast eqs. (7.79) are generally valid. We chose to demonstrate the formulation of the TSV starting with eq. (1.1) for the reason that there is no other way to determine the eqs. (7.79). Moreover its physical content would not be clear. Eqs. (7.79) give the selfvariations of the 4-vectors J and P. The only constant quantities of the generalized particle is its total momentum, as expressed by the 4-vector C, and its total rest mass M_0.

The study of the external symmetry matrix $T_{010203321321}$ with elements $\alpha_{ki} \neq 0, \forall k \neq i, k, i \in \{0,1,2,3\}$ is algebraically demanding. Hence we shall finish this paragraph by stating the elements of this matrix:

$$T = T_{010203321321} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & \mp\alpha_{03} & \pm\alpha_{02} \\ -\alpha_{02} & \pm\alpha_{03} & 0 & \mp\alpha_{01} \\ -\alpha_{03} & \mp\alpha_{02} & \pm\alpha_{01} & 0 \end{bmatrix} \qquad (7.80)$$

$$\alpha_{01}\alpha_{02}\alpha_{03}\alpha_{32}\alpha_{13}\alpha_{21} \neq 0$$

$$\boldsymbol{\tau} = \pm\mathbf{n} \neq \mathbf{0} \qquad (7.81)$$

$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0 \qquad (7.82)$$

$$J = J_0\begin{bmatrix} 1 \\ 0 \\ \pm\dfrac{a_{03}}{a_{01}} \\ \mp\dfrac{a_{02}}{a_{01}} \end{bmatrix} + J_1\begin{bmatrix} 0 \\ 1 \\ \dfrac{a_{02}}{a_{01}} \\ \dfrac{a_{03}}{a_{01}} \end{bmatrix} \qquad (7.83)$$

$$C = c_0\begin{bmatrix} 1 \\ 0 \\ \pm\dfrac{\alpha_{03}}{\alpha_{01}} \\ \mp\dfrac{\alpha_{02}}{\alpha_{01}} \end{bmatrix} + c_1\begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} \\ \dfrac{\alpha_{03}}{\alpha_{01}} \end{bmatrix} \qquad (7.84)$$

$$c_0 J_1 \neq c_1 J_0 \qquad (7.85)$$

$$P = C - J \qquad (7.86)$$

$$j = 0 \tag{7.87}$$

$$m_0 = \frac{E_0}{c^2} = M_0 = 0 \tag{7.88}$$

$$\nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial^2 x_0} = 0 \tag{7.89}$$

$$\begin{aligned}
\frac{\partial \Psi}{\partial x_0} &= \frac{b}{\hbar}\left(\mu J_0 + \lambda P_0\right)\Psi \\
\frac{\partial \Psi}{\partial x_1} &= \frac{b}{\hbar}\left(\lambda J_1 + \mu P_1\right)\Psi \\
\frac{\partial \Psi}{\partial x_2} &= \pm \frac{\alpha_{03}}{\alpha_{01}}\frac{\partial \Psi}{\partial x_0} + \frac{\alpha_{02}}{\alpha_{01}}\frac{\partial \Psi}{\partial x_1} \\
\frac{\partial \Psi}{\partial x_3} &= \mp \frac{\alpha_{02}}{\alpha_{01}}\frac{\partial \Psi}{\partial x_0} + \frac{\alpha_{03}}{\alpha_{01}}\frac{\partial \Psi}{\partial x_1}
\end{aligned} . \tag{7.90}$$

The USVI of this symmetry is given by equation

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J + zQ \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & \mp\alpha_{03} & \pm\alpha_{02} \\ -\alpha_{02} & \pm\alpha_{03} & 0 & \mp\alpha_{01} \\ -\alpha_{03} & \mp\alpha_{02} & \pm\alpha_{01} & 0 \end{bmatrix} \begin{bmatrix} -1 \\ \frac{iu_1}{c} \\ \frac{iu_2}{c} \\ \frac{iu_3}{c} \end{bmatrix} . \tag{7.91}$$

Using as a base the TSV theorems we can study all external symmetry matrices. In the following paragraphs we present the detailed study of two other external symmetry matrices.

8. The Symmetry T=QΛ

In this paragraph we study the T matrices, which have all their elements equal to zero, except the elements on the main diagonal. Thus we study matrices of the form

$$T = Q\Lambda = zQ\begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix} \tag{8.1}$$

using the notation of equation (4.11). From equations (4.28) ,(7.3) and (8.1) we have

$$\begin{matrix} M = 0 \\ N = 0 \end{matrix}. \tag{8.2}$$

The matrices M and N are zero; as a consequence the matrices of the symmetries $T = Q\Lambda$ share common properties, which we shall study in the following.

According to corollary 7.4 , at least one of the diagonal elements of the matrices of equation (8.1) is zero. Also they cannot be all zero, since in the case of the external symmetry it holds that $T \neq 0$. Therefore, there is a number of

$$N_1 = \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 14$$

different matrices for which the relation $T = Q\Lambda$ holds.

In the symmetry $T = Q\Lambda$ at least one element of the matrix Λ is different from zero, that is $\Lambda \neq 0$. Furthermore $\alpha_{ki} = 0$ for every $k \neq i.k, i \in \{0,1,2,3\}$, therefore also $\boldsymbol{\alpha} = \mathbf{0}, \boldsymbol{\beta} = \mathbf{0}$, which follows from equations (4.14) and (4.15). Thus the USVI in the case of the symmetry $T = Q\Lambda$ is given by the equations

$$\begin{aligned} \frac{dJ}{dx_0} &= \frac{dQ}{Qdx_0} J - \frac{i}{c} Q\Lambda u \\ \frac{dP}{dx_0} &= -\frac{dQ}{Qdx_0} J + \frac{i}{c} Q\Lambda u \end{aligned} \tag{8.3}$$

implied by the equations (4.19) and (4.20).

Another common characteristic for the 14 kinds of symmetries $T = Q\Lambda$ is that $\boldsymbol{\tau} = \mathbf{0}$, and therefore the plane Π is not defined. Similarly, the vectors $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3$ of equations (7.32) are not defined.

A fundamental characteristic of the symmetries $T = Q\Lambda$ is that the four-vector j of the conserved physical quantites q vanishes. Combining the first of equations (8.2) with equation (5.7) we obtain

$$j = 0 . \tag{8.4}$$

Therefore, in the part of spacetime occupied by the generalized particle, there is no flow of conserved physical quantities q.

Another common characteristic is that the rest mass m_0 of the material particle can be diferent from zero

$$m_0 = 0 \vee m_0 \neq 0 \tag{8.5}$$

for all 14 matrices of the symmetry. The form of the four-vector J is different for each matrix of the symmetry.

We calculate now the four-vector of momentum J of the matrix T^{12}. According to our notation we have

$$T^{12} = zQ\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8.6}$$

$$T_1 T_2 \neq 0$$

From equation (2.13), and since $T_1 T_2 \neq 0$, $T_0 = T_3 = 0$, we obtain for the four-vector J, in the form

$$J = \begin{bmatrix} J_0 \\ 0 \\ 0 \\ J_3 \end{bmatrix}. \tag{8.7}$$

Combining equations (2.7) and (8.7), we obtain for the rest mass m_0 the equation

$$-m_0^2c^2 = J_0^2 + J_3^2 . \tag{8.8}$$

We apply now the $SV-T$ method :

We differentiate equation (8.8) with respect to $x_k, k=0,1,2,3$ and taking into account equations (2.6), (2.10) and (4.4) we obtain

$$-\frac{b}{\hbar}P_k m_0^2 c^2 = J_0\left(\frac{b}{\hbar}P_k J_0 + zQ\alpha_{k0}\right) + J_3\left(\frac{b}{\hbar}P_k J_3 + zQ\alpha_{k3}\right)$$

and from equation (8.8) we have

$$zQJ_0\alpha_{k0} + zQJ_3\alpha_{k3} = 0$$

and since $zQ \neq 0$, we have

$$J_0\alpha_{k0} + J_3\alpha_{k3} = 0, k = 0,1,2,3 . \tag{8.9}$$

We insert successively $k=0,1,2,3$ into equation (8.9), hence:

For $k=0$ we have

$$J_0 T_0 + J_3\alpha_{03} = 0$$

which holds since for the matrix T^{12} it is $T_0 = \alpha_{03} = 0$.

For $k=1$ we have

$$J_0\alpha_{10} + J_3\alpha_{13} = 0$$

which holds since for the matrix T^{12} it is $\alpha_{10} = \alpha_{13} = 0$.

For $k=2$ we have

$$J_0\alpha_{20} + J_3\alpha_{23} = 0$$

which holds since for the matrix T^{12} it is $\alpha_{20} = \alpha_{32} = 0$.

For $k=3$ we have

$$J_0\alpha_{30} + J_3 T_3 = 0$$

which holds since for the matrix T^{12} it is $\alpha_{30} = T_3 = 0$.

According to the proof of eq. (8.4) it is possible that $J_0 = 0$ or $J_3 = 0$, but it is not possible that $J_0 = J_3 = 0$, since in this case the material particle does not exist. Therefore from equation (8.8) we conclude that

$$m_0 \neq 0 \vee \left\{ m_0 = 0 \wedge J_3 = \pm i J_0 \right\}. \qquad (8.10)$$

Simirarly we can prove that relations analogous to relation (8.10), hold for all matrices of the symmetry $T = Q\Lambda$.

For the matrix T^{12} it is $T_1 \neq 0$. Therefore the part of spacetime occupied by the generalized particle in the symmetry T^{12} is curved, according to corollary 6.2.

Because of equation (8.4) the wave equation (5.17) holds identically $(0 = 0)$. Therefore for the symmetries $T = Q\Lambda$ the study of the wave behavior of matter is done via eq. (5.3).

From equation (4.11) we obtain

$$\Lambda = z \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (8.11)$$

$$T_1 T_2 \neq 0$$

for the symmetry T^{12}. From equations (8.7) and (8.11), and from the first of equations (8.3), we obtain

$$\frac{dJ}{dx_0} = \frac{dQ}{Q dx_0} J$$

and with equation (8.7) we have

$$\frac{dJ_0}{dx_0} = \frac{dQ}{Q dx_0} J_0$$

$$\frac{dJ_3}{dx_0} = \frac{dQ}{Q dx_0} J_3$$

and finally we obtain

$$
\begin{aligned}
&J_0 = \sigma_0 Q \\
&J_3 = \sigma_3 Q \\
&(\sigma_0, \sigma_3) \neq (0,0) \\
&\sigma_0, \sigma_3 = \text{constants}
\end{aligned} \quad . \tag{8.12}
$$

Thus the four-vector J is given by equation

$$
J = Q\begin{bmatrix} \sigma_0 \\ 0 \\ 0 \\ \sigma_3 \end{bmatrix} \tag{8.13}
$$

$$
\begin{aligned}
&(\sigma_0, \sigma_3) \neq (0,0) \\
&\sigma_0, \sigma_3 = \text{constants}
\end{aligned}
$$

as implied by equation (8.7). Therefore, for the symmetry T^{12} the momentum of the material particle is proportional to the charge Q. This feature is a common characteristic for all matrices of the symmetry $T = Q\Lambda$.

Combining eqs. (3.5) and (8.13) we have

$$
P = \begin{bmatrix} c_0 - \sigma_0 Q \\ c_1 \\ c_2 \\ c_3 - \sigma_3 Q \end{bmatrix} \quad . \tag{8.14}
$$

$$
\begin{aligned}
&(\sigma_0, \sigma_3) \neq (0,0) \\
&\sigma_0, \sigma_3 = \text{constants}
\end{aligned}
$$

Now from eqs. (4.2) and (8.14) we have

$$
\begin{aligned}
&\frac{\partial Q}{\partial x_0} = \frac{b}{\hbar}(c_0 - \sigma_0 Q)Q \\
&\frac{\partial Q}{\partial x_1} = \frac{b}{\hbar}c_1 Q \\
&\frac{\partial Q}{\partial x_2} = c_2 Q \\
&\frac{\partial Q}{\partial x_3} = \frac{b}{\hbar}(c_3 - \sigma_3 Q)Q
\end{aligned} \quad . \tag{8.15}
$$

From the identity

$$\frac{\partial}{\partial x_k}\left(\frac{\partial Q}{\partial x_i}\right)=\frac{\partial}{\partial x_i}\left(\frac{\partial Q}{\partial x_k}\right), k\neq i, k, i=0,1,2,3$$

and eqs. (8.15) we have after the calculations

$$\begin{aligned}
&\sigma_0 c_1 = 0\\
&\sigma_0 c_2 = 0\\
&\sigma_3 c_1 = 0\\
&\sigma_3 c_2 = 0\\
&c_0\sigma_3 = c_3\sigma_0
\end{aligned}$$

and because of

$$(\sigma_0,\sigma_3)\neq(0,0)$$

we finally get

$$\begin{aligned}
&c_1 = c_2 = 0\\
&c_0\sigma_3 = c_3\sigma_0
\end{aligned} \tag{8.16}$$

From eqs. (8.15) and (8.16) we have

$$\begin{aligned}
&Q = Q(x_0, x_3)\\
&\frac{\partial Q}{\partial x_0} = \frac{b}{\hbar}(c_0-\sigma_0 Q)Q\,.\\
&\frac{\partial Q}{\partial x_3} = \frac{b}{\hbar}(c_3-\sigma_3 Q)Q
\end{aligned} \tag{8.17}$$

From eq. (8.17) we have

$$\begin{aligned}
&Q = \frac{c_0}{\sigma_0}\frac{1}{1-K_{12}\exp\left(-\frac{b}{\hbar}(c_0x_0+c_3x_3)\right)}\\
&c_0\sigma_3 = c_3\sigma_0\\
&\sigma_0\neq 0
\end{aligned} \tag{8.18}$$

$$Q = \frac{c_3}{\sigma_3} \frac{1}{1 - K_{12} \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3 \right) \right)}$$

$$c_0 \sigma_3 = c_3 \sigma_0 \tag{8.19}$$

$$\sigma_3 \neq 0$$

where $K_{12} \in \mathbb{C}, K_{12} \neq 0$ constant. For $\sigma_0 \sigma_3 \neq 0$ the eqs. (8.18) and (8.19) are equivalent, because of the 2nd eq. of (8.16).

From the combination of eqs. (5.3) and (3.5) we have

$$\frac{\partial \Psi}{\partial x_k} = \left(\left(\lambda - \mu \right) J_k + \mu c_k \right) \Psi, k = 0,1,2,3$$

and with eq. (8.15) and the 1st eq. of (8.16) we have

$$\Psi = \Psi\left(x_0, x_3 \right)$$

$$\frac{\partial \Psi}{\partial x_0} = \frac{b}{\hbar}\left(\left(\lambda - \mu \right) \sigma_0 Q + \mu c_0 \right) \Psi$$

$$\frac{\partial \Psi}{\partial x_3} = \frac{b}{\hbar}\left(\left(\lambda - \mu \right) \sigma_3 Q + \mu c_3 \right) \Psi \quad . \tag{8.20}$$

$$c_0 \sigma_3 = c_3 \sigma_0$$

$$\left(\sigma_0, \sigma_3 \right) \neq \left(0, 0 \right)$$

The eq. (8.20) is the wave eq. of the symmetry T^{12}. The charge Q is given from eqs. (8.18), (8.19), while the physical quantities λ and μ express the degrees of freedom of the TSV.

From eq. (8.14) and the 1st eq. of (8.16) we have

$$P = \begin{bmatrix} c_0 - \sigma_0 Q \\ 0 \\ 0 \\ c_3 - \sigma_3 Q \end{bmatrix}. \tag{8.21}$$

From eq. (3.5) and the 1st eq. of (8.16) we have

$$C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_3 \end{bmatrix}. \tag{8.22}$$

From eqs. (8.18), (8.19) and (8.13) we have

$$J = J\left(x_0, x_3, c_0, c_3\right) = \frac{1}{1 - K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3\right)\right)}\begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_3 \end{bmatrix} \tag{8.23}$$

and from eqs. (8.21), (8.13) and (8.18), (8.19) we have

$$P = P\left(x_0, x_3, c_0, c_3\right) = \left(1 - \frac{1}{1 - K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3\right)\right)}\right)\begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_3 \end{bmatrix}. \tag{8.24}$$

From eqs. (8.22), (8.23) and (8.24) it follows that the 4-vectors J, P, C are parallel. According to the equivalence (3.4) and eq. (4.4) this parallelism is expected for the symmetries $T = Q\Lambda$, since it is $\alpha_{ki} = 0, \forall k \neq i, k, i = 0,1,2,3$. However the parallelism of the 4-vectors J, P, C we have met in the theorem 3.3 as a characteristic of internal symmetry. Hence we will finish the paragraph for the symmetries $T = Q\Lambda$ with the refutation of this apparent inconsistency.

From eq. (8.13) we get $J_1 = J_2 = 0$ for the symmetry T^{12}, hence the initial eq. (2.7) is written

$$J_0^2 + J_3^2 + m_0^2 c^2 = 0. \tag{8.25}$$

Subsequently we perform the same procedure as for the proof of eq. (2.10), from eq. (2.7). After the calculations and because in symmetry T^{12} it holds that $\alpha_{ki} = 0, \forall k \neq i, k, i = 0,1,2,3$ equation (8.6) follows from eq. (8.25). During the procedure of proof, the physical quantities T_1 and T_2 do not follow from eq. (8.25). In contrast from eq. (2.7) for $J_1 \neq 0, J_2 \neq 0$ and $\alpha_{ki} = 0, \forall k \neq i, k, i = 0,1,2,3$ we get $T_1 = T_2 = 0$, as is predicted from the internal symmetry theorem 3.3. Exactly at this point we find the differences of the symmetries $T = Q\Lambda$ with internal symmetry. In internal symmetry it is $T_0 = T_1 = T_2 = T_3 = 0$, and according to corollary 6.1 the part of spacetime occupied by the generalized particle may be a plane. Moreover space is isotropic, in the part of spacetime occupied by the generalized particle. The momentum vectors $\mathbf{J}$, $\mathbf{P}$ and $\mathbf{C}$ are 3-dimensional, and it is not possible to let vanish some component J_1, J_2, J_3 of the momentum from eq. (2.7), with an appropriate rotation of the reference system we use. There is a very specific inertial reference frame in which $J_1 = J_2 = J_3 = 0$ ([5], paragraph 5.3). In contrast with the symmetries $T = Q\Lambda$ spacetime is curved as implied by the corollary 6.2. Moreover in symmetries $T = Q\Lambda$ space is **intensely anisotropic**, in the part of spacetime which is occupied by the generalized particle. According to eqs. (8.22), (8.23) and (8.24) the momentums $\mathbf{C}$, $\mathbf{J}$ and $\mathbf{P}$ in symmetry T^{12} are 1-dimensional, towards the direction of the axis $x_3 = z$. The intense anisotropy of space, in the part of spacetime which is occupied by the generalized particle, is a basic characteristic of the symmetry $T = Q\Lambda$. This anisotropy varies for the symmetries of the set Ω_1 in eq. (7.78).

One symmetry $T = Q\Lambda$ is characterized by the symmetries of the 4-vector J which are absend in the eq. (2.7). For symmetry T^{12} the components are J_1 and J_2.

From eqs. (8.13) and (8.25) we have

$$\begin{gathered}\left(\sigma_0^2+\sigma_3^2\right)Q^2+m_0^2c^2=0\\ \left(\sigma_0,\sigma_3\right)\neq\left(0,0\right)\end{gathered}\quad . \tag{8.26}$$

Eq. (8.26) gives the contribution of charge Q to the rest mass m_0 of the material particle.

We now calculate the distribution of the total rest mass M_0 of the generalized particle between the material particle and STEM. From eqs. (8.23) and (8.25) we have

$$\frac{c_0^2+c_3^2}{\left(1-K_{13}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)\right)^2}+m_0^2c^2=0$$

and from eq. (8.22) and (3.5) we have

$$-\frac{M_0^2c^2}{\left(1-K_{13}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)\right)^2}+m_0^2c^2=0$$

and finally we get

$$m_0 = \pm \frac{M_0}{1 - K_{13} \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3 \right) \right)} . \tag{8.27}$$

Analogous from eqs. (8.24), (2.8), and (3.5) we have

$$E_0 = \pm \frac{M_0 c^2 K_{13} \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3 \right) \right)}{1 - K_{13} \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3 \right) \right)} . \tag{8.28}$$

Eqs. (8.27) and (8.28) give the distribution of rest mass M_0 between the material particle and STEM. The study of the remaining 13 symmetries $T = Q\Lambda$ is done in the same way as the one we demonstrated for symmetry T^{12}.

We now set $K_{12} = -K$ in equations (8.23) and (8.24), where K the constant of equation (3.9). Comparing equations (8.23), (8.24) and (3.9, (3.12), (3.13) we come to the conclusion that the external symmetry T^{12} can emerge from the internal symmetry for $J_1 = J_2 = 0$. This can occur when an external cause blocks the emmision of STEM along the axes x_1 and x_2. In this way the isotropic emmision of the internal symmetry is converted into the anisotropic external symmetry T^{12}. In general the following corollary of theorem 3.3 holds:

Corollary 8.1 : ′′The external symmetry $T = Q\Lambda$ can emerge from the internal symmetry when an external cause blocks the emmision of STEM along one or more axes $x_i, i = 0,1,2,3$. These axes define the kind of external symmetry $T = Q\Lambda$ that results.′′

Corollary 8.1 gives us a mechanism through which the symmetry $T = Q\Lambda$ can emerge. The external cause is necessary, since the internal symmetry expresses the spontaneous isotropic emmision of STEM due to the selfvariations.

9. The Symmetries T^{0}_{010203} and T_{010203}.

In this chapter we study the generalized particle of the matrix

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & 0 & 0 \\ -\alpha_{02} & 0 & T_2 & 0 \\ -\alpha_{03} & 0 & 0 & T_3 \end{bmatrix}. \qquad (9.1)$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

From theorem 7.3 we have that for this matrix it is

$$T_1 = T_2 = T_3 = 0$$

and thus it is written in the form

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}. \qquad (9.2)$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

From the matrix in equation (9.2) we obtain the symmetries

$$T = T^{0}_{010203} = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \qquad (9.3)$$

$$\alpha_{01}\alpha_{02}\alpha_{03}T_0 \neq 0$$

$$T = T_{010203} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}. \qquad (9.4)$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

First we study the symmetry T^{0}_{010203}. For this symmetry it is $M \neq 0$, hence we apply the $SV-M$ -method. From equation (7.3) we have

$$\alpha_{01}^2+\alpha_{02}^2+\alpha_{03}^2=0. \tag{9.5}$$

From equations (7.2) we obtain

$$\begin{aligned}
&J_0T_0+J_1\alpha_{01}+J_2\alpha_{02}+J_3\alpha_{03}=0\\
&J_0\alpha_{01}=0\\
&J_0\alpha_{02}=0\\
&J_0\alpha_{03}=0
\end{aligned}$$

and since $\alpha_{01}\alpha_{02}\alpha_{03}\neq 0$ and $T_0\neq 0$ we have

$$\begin{aligned}
&J_0=0\\
&J_1\alpha_{01}+J_2\alpha_{02}+J_3\alpha_{03}=0
\end{aligned}. \tag{9.6}$$

From the second of the equations (4.6), and for $(i,\nu,\kappa)=(0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we obtain

$$\begin{aligned}
&J_2\alpha_{01}-J_1\alpha_{02}=0\\
&J_3\alpha_{01}-J_1\alpha_{03}=0\ .\\
&J_3\alpha_{02}-J_2\alpha_{03}=0
\end{aligned} \tag{9.7}$$

From equations (9.6),(9.7), and since it holds that $a_{01}a_{02}a_{03}\neq 0$, we have

$$\begin{aligned}
&J_0=0\\
&J_2=\frac{\alpha_{02}}{\alpha_{01}}J_1\ .\\
&J=\frac{\alpha_{03}}{\alpha_{01}}J_1
\end{aligned} \tag{9.8}$$

From equations (9.8) we obtain the four-vector J

$$J=J_1\begin{bmatrix}0\\1\\\dfrac{\alpha_{02}}{\alpha_{01}}\\\dfrac{\alpha_{03}}{\alpha_{01}}\end{bmatrix}. \tag{9.9}$$

From equations (4.28) and (9.3) we have

$$M=\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \tag{9.10}$$

$$M^2=\begin{bmatrix} -\alpha_{01}^2-\alpha_{02}^2-\alpha_{03}^2 & 0 & 0 & 0 \\ 0 & -\alpha_{01}^2 & -\alpha_{01}\alpha_{02} & -\alpha_{01}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{02} & -\alpha_{02}^2 & -\alpha_{02}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{03} & -\alpha_{02}\alpha_{03} & -\alpha_{03}^2 \end{bmatrix}$$

and with equation (9.5) we obtain

$$M^2=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha_{01}^2 & -\alpha_{01}\alpha_{02} & -\alpha_{01}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{02} & -\alpha_{02}^2 & -\alpha_{02}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{03} & -\alpha_{02}\alpha_{03} & -\alpha_{03}^2 \end{bmatrix}. \tag{9.11}$$

From the first of the equations (7.14) and the equation (9.11), we get after the calculations

$$c_1\alpha_{01}+c_2\alpha_{02}+c_3\alpha_{03}=0\,. \tag{9.12}$$

From the first of the equations (4.6), and for $(i,\nu,\kappa)=(0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we obtain

$$\begin{aligned} c_0\alpha_{01}-c_1\alpha_{02}&=0 \\ c_3\alpha_{01}-c_1\alpha_{03}&=0\ . \\ c_3\alpha_{02}-c_2\alpha_{03}&=0 \end{aligned} \tag{9.13}$$

From equations (9.12) and (9.13) we have

$$\begin{aligned} c_1&=\frac{\alpha_{01}}{\alpha_{02}}c_0 \\ c_2&=c_0 \qquad . \\ c_3&=\frac{\alpha_{03}}{\alpha_{02}}c_0 \end{aligned} \tag{9.14}$$

From equations (9.14) we obtain the four-vector C

$$C = c_0 \begin{bmatrix} 1 \\ \dfrac{\alpha_{01}}{\alpha_{02}} \\ 1 \\ \dfrac{\alpha_{03}}{\alpha_{02}} \end{bmatrix}. \tag{9.15}$$

From equation (3.5) and equations (9.9) and (9.15) we obtain the four-vector P

$$P = \begin{bmatrix} c_0 \\ \dfrac{\alpha_{01}}{\alpha_{02}} c_0 - J_1 \\ c_0 - \dfrac{\alpha_{02}}{\alpha_{01}} J_1 \\ \dfrac{\alpha_{03}}{\alpha_{02}} c_0 - \dfrac{\alpha_{03}}{\alpha_{01}} J_1 \end{bmatrix}. \tag{9.16}$$

With the knowledge of the four-vectors J, P, C we can calculate the rest masses $m_0, \dfrac{E_0}{c^2}, M_0$. From equations (2.7) and (9.9) we get

$$-m_0^2 c^2 = J_1^2 \left[1 + \left(\frac{\alpha_{02}}{\alpha_{01}} \right)^2 + \left(\frac{\alpha_{03}}{\alpha_{01}} \right)^2 \right]$$

and using equation (9.5) we obtain

$$m_0 = 0. \tag{9.17}$$

From equations (2.8) and (9.16) we have

$$E_0 = \pm icc_0. \tag{9.18}$$

For the proof of equation (9.18) we used also equation (9.5). From equations (3.6) and (9.15) we have

$$M_0 = \pm \frac{ic_0}{c}. \tag{9.19}$$

The vector $\boldsymbol{\tau}$ vanishes

$$\boldsymbol{\tau} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and thus the plane Π is not defined. For the same reason it also holds that $\boldsymbol{\mu} = \mathbf{0}$. On the contrary the vector $\mathbf{n}$ is nonzero

$$\mathbf{n} = \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \qquad (9.20)$$

From equations (9.9),(9.15) and (9.16) we see that the vectors $\mathbf{J}, \mathbf{P}, \mathbf{C}$ are parallel to the vector $\mathbf{n}$. We write the vectors $\mathbf{J}$,$\mathbf{C}$, as given by equations (9.9), (9.15) and (9.20)

$$\mathbf{J} = \frac{J_1}{\alpha_{01}} \mathbf{n} \qquad (9.21)$$

$$\mathbf{C} = \frac{c_0}{\alpha_{01}} \mathbf{n} \ .$$

The vector $\mathbf{C}$ is a constant vector aligned to the direction of the vector $\mathbf{n}$. From equivalence (3.4) we obtain

$$\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), k \neq i, k, i = 0,1,2,3$$

and with equation (4.4) we have

$$zQ\alpha_{ki} \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right)$$

and for $k = 0, i = 0,1,2,3$ we obtain

$$zQ\alpha_{01} = \frac{b}{2\hbar}\left(c_1 J_0 - c_0 J_1\right)$$
$$zQ\alpha_{02} = \frac{b}{2\hbar}\left(c_2 J_0 - c_0 J_2\right)$$
$$zQ\alpha_{03} = \frac{b}{2\hbar}\left(c_3 J_0 - c_0 J_3\right)$$

and with equations (9.9) and (9.15) we have

$$zQ\alpha_{01} = -\frac{bc_0}{2\hbar} J_1$$
$$zQ\alpha_{02} = -\frac{bc_0}{2\hbar} J_2$$
$$zQ\alpha_{03} = -\frac{bc_0}{2\hbar} J_3$$

and solving with respect to J_1, J_2, J_3 we obtain

$$J_1 = -\frac{2\hbar}{bc_0} zQ\alpha_{01}$$
$$J_2 = -\frac{2\hbar}{bc_0} zQ\alpha_{02}$$
$$J_3 = -\frac{2\hbar}{bc_0} zQ\alpha_{03}$$

and using equation (9.20) we take

$$\mathbf{J} = -\frac{2\hbar}{bc_0} zQ\mathbf{n}$$

and taking into account that $J_0 = 0$, we have

$$\begin{aligned} &J_0 = 0 \\ &\mathbf{J} = -\frac{2\hbar}{bc_0} zQ\mathbf{n} \quad . \\ &J = -\frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \mathbf{n} \end{bmatrix} \end{aligned} \tag{9.22}$$

In equation (9.22) the function z is given by equation (4.5). Equation (9.22) expresses the dependence of the four-vector J on the charge Q in the case of the external symmetry T_{010203}^{0}.

For the matrix T_{010203}^{0} it is $M \neq 0$ therefore we apply the SV_q - method for the determination of the four-vector j. For $(i,\nu,\kappa) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ in equation (7.51), and considering the elements of the matrix T_{010203}^{0}, we obtain

$$j_2 = \frac{\alpha_{02}}{\alpha_{01}} j_1$$
$$j_3 = \frac{\alpha_{03}}{\alpha_{01}} j_1 \quad . \tag{9.23}$$

From equations (4.27), (7.16) and (9.10) we have

$$\alpha_{01} j_1 + \alpha_{02} j_2 + \alpha_{03} j_3 = 0 \quad . \tag{9.24}$$

From equations (9.23) and (9.24) we obtain the four-vector j

$$j = \begin{bmatrix} j_0 \\ j_1 \\ \frac{\alpha_{02}}{\alpha_{01}} j_1 \\ \frac{\alpha_{03}}{\alpha_{01}} j_1 \end{bmatrix} \quad . \tag{9.25}$$

From equations (9.25) and (9.20) we obtain the current density

$$\mathbf{j} = \frac{j_1}{\alpha_{01}} \mathbf{n} \quad . \tag{9.26}$$

Therefore the current density $\mathbf{j}$ has the same direction as the vector $\mathbf{n}$.

From the wave equation (5.17) and equations (9.25) and (9.20) we obtain

$$\sigma c^2 F \mathbf{n} = \frac{1}{\alpha_{01}} \frac{\partial j_1}{\partial x_0} \mathbf{n} - \nabla j_0$$
$$\frac{1}{\alpha_{01}} \frac{\partial j_1}{\partial x_1} = \frac{1}{\alpha_{02}} \frac{\partial j_1}{\partial x_2} = \frac{1}{\alpha_{03}} \frac{\partial j_1}{\partial x_3} \quad . \tag{9.27}$$
$$F = \nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2}$$

From the second of equations (9.27) we have

$$\nabla j_1 = \left(\frac{\partial j_1}{\partial x_1}, \frac{\partial j_1}{\partial x_1} \frac{\alpha_{02}}{\alpha_{01}}, \frac{\partial j_1}{\partial x_1} \frac{\alpha_{03}}{\alpha_{01}} \right)$$
$$\nabla j_1 = \frac{1}{\alpha_{01}} \frac{\partial j_1}{\partial x_1} \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix}$$

and with equation (9.20) we have

$$\nabla j_1 = \frac{1}{\alpha_{01}} \frac{\partial j_1}{\partial x_1} \mathbf{n} \ . \tag{9.28}$$

From equations (9.28) and (9.20) we have

$$\nabla \cdot \mathbf{j} = \frac{1}{\alpha_{01}^2} \frac{\partial}{\partial x_1} \left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \right)$$

and with equation (9.5) we get

$$\nabla \cdot \mathbf{j} = 0 \, . \tag{9.29}$$

Combining the continuity equation (5.6) with equation (9.29) we obtain

$$\frac{\partial j_0}{\partial x_0} = 0 \ . \tag{9.30}$$

Therefore the charge density $j_0 = i\rho c$ does not depend on time in the symmetry we study. Finally after combining equations (5.3), (9.9) and (9.16) we have

$$\begin{aligned} &\frac{\partial \Psi}{\partial x_0} = \frac{b c_0 \mu}{\hbar} \Psi \\ &\nabla \Psi = \frac{b}{\hbar} \left(\frac{\lambda - \mu}{\alpha_{01}} J_1 + \frac{\mu c_0}{\alpha_{02}} \right) \Psi \mathbf{n} \ . \\ &\lambda, \mu \in \mathbb{C}, \left(\lambda, \mu \right) \neq \left(0, 0 \right) \end{aligned} \tag{9.31}$$

Let us remind that the parameters λ, μ appearing in equation (9.31) express the two degrees of freedom of the TSV.

The portion of space-time occupied by the generalized particle is curved, since $T_0 \neq 0$, according to corollary 6.2. Also from the combination of equations (9.9), (4.19), and taking into account that $T_1 = T_2 = T_3 = 0$, we obtain

$$\frac{dJ}{dx_0} = \frac{dQ}{Q dx_0} J + zQ \begin{bmatrix} T_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \frac{i}{c} Q \begin{bmatrix} \frac{i}{c} \mathbf{u} \cdot \boldsymbol{\alpha} \\ \boldsymbol{\alpha} \end{bmatrix} \tag{9.32}$$

for the *USVI* of the external symmetry T_{010203}^{0} .

In symmetry T_{010203} it holds that $T_0 = T_1 = T_2 = T_3 = 0$. Hence from theorem 7.6 we could have that $J_0 \neq 0$, and the four-vector J could take the form

$$J = \begin{bmatrix} J_0 \\ J_1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} J_1 \\ \dfrac{\alpha_{03}}{\alpha_{01}} j_1 \end{bmatrix} \qquad (9.33)$$

$$J_0 \neq 0$$

in the symmetry T_{010203}. However equation (9.33) is rejected. Following the same procedure as the one for proving equation (9.17), we obtain from equation (9.33) that

$$m_0^2 c^2 = -J_0^2 \neq 0. \qquad (9.34)$$

Applying the $SV-T$ method, we conclude that equation (9.34) cannot hold. Therefore, the symmetries T^0_{010203} and T_{010203} have the same four-vectors J, P, C and j. The only difference lies in the vanishing or non-vanishing of the physical quantity T_0. As derived from equation (9.32) this difference has consequences for the *USVI* of the two symmetries. The symmetry T_{010203} is one of the $N_T - N_J = 14$ symmetries, according to the ordering of the external symmetry matrices in paragraph 7.

10. The Generalized Particle of the Field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ and the Confinement Equation.

In this paragraph we study the generalized particle of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, for which the function Ψ is known. This shall allow us to perform a particular application of theorem (5.1).

For $\lambda = \mu = -\dfrac{1}{2}$ in equation (5.3) we obtain

$$\frac{\partial \Psi_k}{\partial x_k} = -\frac{b}{2\hbar}\left(J_k + P_k\right)\Psi, k = 0,1,2,3$$

and with equation (3.5) we have

$$\frac{\partial \Psi_k}{\partial x_k} = -\frac{bc_k}{2\hbar}\Psi, k = 0,1,2,3$$

and using the notation of equation (4.9) we have

$$\Psi = z = \exp\left[-\frac{b}{2\hbar}\left(c_0 x \ + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]. \qquad (10.1)$$

From equation (10.1) and equations (5.1), (5.2) and (4.14), (4.15), we obtain

$$\begin{aligned} \boldsymbol{\xi} = \boldsymbol{\alpha} = icz\begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} = icz\mathbf{n} \\ \boldsymbol{\omega} = \boldsymbol{\beta} = z\begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} = z\boldsymbol{\tau} \end{aligned} \quad . \qquad (10.2)$$

The field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is a special case of the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$ for $\lambda = \mu = -\frac{1}{2}$.

The fact that the function Ψ of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is known allows us to derive two impotant results about the total rest mass M_0 of the generalized partcle. From equation (10.1) we obtain

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = \frac{b^2}{4\hbar^2}\left(c_0^2 + c_1^2 + c_1^2 + c_1^2\right)$$

and with equation (3.6) we have

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = -\frac{b^2}{4\hbar^2}M_0^2 c^2\Psi \ . \qquad (10.3)$$

According to equation (10.3) and theorem 5.2 the generalized photon in the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ exists, if and only if

$$M_0 = 0 , \qquad (10.4)$$

that is in the case the total rest mass of the generalized particle is zero. For $M_0 \neq 0$ the generalized particle appears.

Setting $\lambda = \mu = -\frac{1}{2}$ in the equations of paragraph 5, we arrive at the equations of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. For example, by setting $\lambda = \mu = -\frac{1}{2}$ into equation (5.7) we obtain

$$j = \frac{\sigma c^2 bz}{2\hbar} MC .$$

This is equation (4.29), as we have proved in paragraph 4, for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. On the other hand, equation (10.3) results only because the function Ψ is known, as given by equation (10.1) for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$.

By knowing the function Ψ we can study the consequences for a material particle that is confined within a constant volume V . The conserved physical quantity q, is constant within the volume V occupied by the generalized particle. Therefore, it holds that

$$\begin{aligned} &\frac{dq}{dt} = \frac{icdq}{dx_0} = 0 \\ &V = \text{constant} \end{aligned} . \tag{10.5}$$

The total conserved physical quantity q contained within the volume V occupied by the generalized particle is

$$q = \int_V \rho dV . \tag{10.6}$$

The equation (10.6) holds independently of the fact, whether the volume V of the generalized particle variates or not. The density ρ for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by the first of the equations (4.25)

$$\rho = -\sigma \frac{icbz}{2\hbar}\left(c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03}\right). \tag{10.7}$$

In the case of

$$c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03} = 0 ,$$

that is in the case of

$\mathbf{n} \cdot \mathbf{C} = 0$,

as derived from equations (3.5) and (7.24), we obtain from equation (10.7) that $\rho = 0$. That is, for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ the following equivalence holds

$$\rho = 0 \Leftrightarrow \mathbf{n} \cdot \mathbf{C} = 0 \Leftrightarrow c_1 \alpha_{01} + c_2 \alpha_{02} + c_3 \alpha_{03} = 0 . \tag{10.8}$$

In the case of $\rho \neq 0$, and from the combination of equations (10.6) and (10.7), we have

$$q = -\sigma \frac{icb(\mathbf{n} \cdot \mathbf{C})}{2\hbar} \int_V z dV . \tag{10.9}$$

The integration in the second part of equation (10.9) is performed within the total volume V occupied by the generalized particle. Therefore, in the case the volume V is constant, the integral in the second part of equation (10.9) is independent of the quantiites $x_1 = x, x_2 = y, x_3 = z$. Therefore, in the case volume V is constant, the physical quantity q in equation (10.9) depends only on time.

Thus by combining equations (10.5) and (10.9) for a constant volume V , we obtain

$$\begin{aligned} &\frac{d}{dt} \int_V z dV = 0 \\ &V = \text{constant} \end{aligned} . \tag{10.10}$$

Working with equation (10.10) in the general case presents some mathematical difficulties. Therefore in the present work we will restrict our study on the simplest case. We shall study the case for which the total momentum $\mathbf{C}$ of the generalized particle is aligned on the direction of the x - axis, that is for the case of $c_1 \neq 0, c_2 = c_3 = 0$. In this case we obtain from equation (3.6) that $M_0^2 c^2 = -c_0^2 - c_1^2$. Furthermore it must also hold that $\rho \neq 0$, that is $c_1 \alpha_{01} \neq 0$, according to equivalence (10.8), and since $c_1 \neq 0$, it must also hold that $\alpha_{01} \neq 0$. Therefore our study refers to the particular case where

$$\begin{aligned} &c_1 \neq 0 \\ &c_2 = c_3 = 0 \\ &\alpha_{01} \neq 0 \\ &M_0^2 c^2 = -c_0^2 - c_1^2 \end{aligned} . \tag{10.11}$$

We suppose that the generalized particle occupies the constant volume V defined by the relations (10.12) in a frame of reference $O(t, x_1 = x, x_2 = y, x_3 = z)$.

$$
\begin{aligned}
&\alpha \le x_1 \le \beta \\
&0 \le x_2 \le L_2 \\
&0 \le x_3 \le L_3 \\
&\alpha < \beta \\
&L = \beta - \alpha > 0 \\
&L_2, L_3 > 0, L_2, L_3 = \text{constants}
\end{aligned}
\quad . \qquad (10.12)
$$

For the quantities α, β it holds that

$$
\frac{d\alpha}{dt} = \frac{d\beta}{dt} = u < c \qquad (10.13)
$$

where u is the velocity with which the volume V is moving in the chosen frame of reference.

From equation (10.1), relations (10.11), and since $x_0 = ict$, we have

$$
z = \exp\left(-\frac{icbc_0}{2\hbar} t\right) \exp\left(-\frac{bc_1}{2\hbar} x\right)
$$

$$
\int_V z dV = -\frac{2\hbar L_2 L_3}{bc_1} \exp\left(-\frac{icbc_0}{2\hbar} t\right) \left[\exp\left(-\frac{bc_1 \beta}{2\hbar}\right) - \exp\left(-\frac{bc_1 \alpha}{2\hbar}\right)\right] . \qquad (10.14)
$$

From equation (10.14) we see that equation equation (10.10) holds, if and only if

$$
\exp\left(-\frac{bc_1 \beta}{2\hbar}\right) - \exp\left(-\frac{bc_1 \alpha}{2\hbar}\right) = 0
$$
$$
\exp\left(-\frac{bc_1 (\beta - \alpha)}{2\hbar}\right) = 1
$$

$$
\exp\left(\frac{bc_1 L}{2\hbar}\right) = 1 . \qquad (10.15)
$$

Equation (10.15) holds only in the case the constant b of the Law of Selfvariations is an imaginary number, $b = i \| b \|, \| b \| \in \mathbb{R}$. In this case we obtain

$$\cos\left(\frac{bc_1L}{2\hbar}\right)=1$$
$$\sin\left(\frac{bc_1L}{2\hbar}\right)=0$$
$$b=i\|b\|, \|b\|\in\mathbb{R}$$

and finally, we get

$$c_1=n\frac{4\pi\hbar}{L\|b\|}, n=\pm1,\pm2,\pm3,... \quad (10.16)$$

Combining equation (10.16) with the last of the equations (10.11) we have

$$M_0^2c^2=-c_0^2-n^2\frac{16\pi^2\hbar^2}{L^2\|b\|^2}, n=1,2,3... \quad (10.17)$$

Therefore the momentum c_1 and the rest mass M_0 of the confined generalized particle is quantized.

In the case of the generalized photon, that is for $M_0=0$, and according to equation (10.17) we have

$$\begin{aligned} &c_0=n\frac{i4\pi\hbar}{L\|b\|}, n=\pm1,\pm2,\pm3,... \\ &M_0=0 \end{aligned} \quad . \quad (10.18)$$

Combining equations (10.1),(10.16) and (10.18) we have

$$\begin{aligned} &\Psi=z=\exp\left[n\frac{4\pi i}{L}(ct-x)\right] \\ &n=\pm1,\pm2,\pm3,... \\ &M_0=0 \end{aligned} \quad . \quad (10.19)$$

The function Ψ expresses a harmonic wave of wavelenght $\lambda=\frac{L}{2n}$ propagating along the x -axis.

We now calculate the equation corresponding to the equation (10.10) for the field $(\boldsymbol{\xi},\boldsymbol{\omega})$, in general. The reason of not having calculated the general equation (in the case of the one spatial dimension) already in paragraph 5 is that the relation of the confinement of the generalized particle with the appearance of the quantization would not have become obvious.

From equations (4.27), (4.28) and (5.7), and since it holds that $j_0 = i\rho c$, we obtain:

$$\rho = \frac{icb}{\hbar}\Psi\left[\lambda\left(\alpha_{01}J_1 + \alpha_{02}J_2 + \alpha_{03}J_3\right) + \mu\left(\alpha_{01}P_1 + \alpha_{02}P_2 + \alpha_{03}P_3\right)\right]$$

and together with equations (5.8), (5.9) and (7.24) we have

$$\rho = \frac{icb}{\hbar}\Psi\left(\lambda\mathbf{J}\cdot\mathbf{n} + \mu\mathbf{P}\cdot\mathbf{n}\right). \tag{10.20}$$

From equations (10.5), (10.20) for the generalized particle occupying a constant volume V we obtain

$$\frac{d}{dt}\int_V \Psi\left(\lambda\mathbf{J}\cdot\mathbf{n} + \mu\mathbf{P}\cdot\mathbf{n}\right)dV = 0. \tag{10.21}$$

For $\lambda = \mu = -\frac{1}{2}$ equation (10.21) gives equation (10.10), after considering equations (3.5) and (10.1).

For the internal symmetry $T = 0$ it holds that $M = 0$, and from equation (5.7) we obtain $j = 0$. Hence equation (10.5) degenerates into the identity, $0 = 0$, therefore the confinement equation (10.21) does not hold. The same holds also for all the external symmetries $T = Q\Lambda$ as follows from equation (8.4). Therefore the confinement eq. (10.21) is valid for the $N_j = N_J - N_{\Omega_1} = 40 - 1 = 39$ generalized particles of the external symmetries of the sets $\Omega_2, \Omega_3, \Omega_4, \Omega_5$ of eq. (7.78).

11. The Cosmological Data as a Consequence of the Theorem of Internal Symmetry

The theorem 3.3., that is the theorem of internal symmetry, predicts and justifies the cosmological data. We present the relevant study in this paragraph.

The emission of the electromagnetic spectrum of the far-distant astronomical objects we observe today has taken place a long time interval ago. At the moment of the emission the rest mass and the electric charge of the material particles had smaller values than the corresponding ones measured in the laboratory, “now”, on Earth, due to the manifestation of the Selfvariations. The consequences resulting from this difference are recorded in the cosmological data. The cosmological data have a microscopic and not a macroscopic cause.

Due to the Selfvariations of the rest masses of the material particles the gravitational interaction cannot play the role attributed to it by the Standard Cosmological Model (SCM). The gravitational interaction cannot cause neither the collapse, nor the expansion of the universe, since it decreases on a cosmological scale according to the factor $\frac{1}{1+z}$. The gravitational interaction exercised on our galaxy by a far-distant astronomical object with redshift $z=9$ is only the $\frac{1}{10}$ of the expected one. The universe is static and flat, according to the law of Selfvariations.

For a non- moving particle, that is for $J_1 = J_2 = J_3 = 0$, from equation (3.12) we get that $c_1 = c_2 = c_3 = 0$ and from equation (3.9) we obtain

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0 \right)$$

and since $x_0 = ict$, we have

$$\Phi = K \exp\left(-\frac{icc_0}{\hbar} t \right)$$

and from equation (3.10) we obtain

$$m_0 = m_0(t) = \pm \frac{M_0}{1 + K \exp\left(-\frac{icc_0}{\hbar} t \right)} \quad . \tag{11.1}$$

The rest mass m_0 of the material particle is a function of time t .

We now denote by k the constant

$$k = -\frac{icc_0}{\hbar}$$

and from eq. (3.5) we have

$$k = -\frac{icc_0}{\hbar} = \frac{W+E}{\hbar}. \tag{11.2}$$

We also denote by A the time-dependent function

$$A = A(t) = -K\exp(kt) = -\Phi. \tag{11.3}$$

Following this notation, equation (11.1) is written as

$$m_0 = m_0(t) = \pm\frac{M_0}{1-A} \ . \tag{11.4}$$

From equation (11.3) we have

$$\frac{dA}{dt} = \dot{A} = kA \ . \tag{11.5}$$

for the expression of the parameter $A = A(t)$. Similarly, using the above notation equation (3.11) is written as

$$E_0 = E_0(t) = \mp\frac{M_0c^2A}{1-A}. \tag{11.6}$$

We consider an astronomical object at distance r from Earth. The emission of the electromagnetic spectrum of the far-distant astronomical objects we observe “now”on Earth has taken place before a time interval $\delta t = t - \frac{r}{c}$. From equation (11.3) we have that the parameter A obtained the value

$$A = A(r) = A(t)exp\left(-k\frac{r}{c}\right)$$

and from equation (11.4) we have

$$m_0(r) = \pm\frac{M_0}{1-A\exp\left(-k\frac{r}{c}\right)} \ . \tag{11.7}$$

Similarrly from equation (11.6) we have

$$E_0(r) = \mp \frac{M_0 c^2 A \exp\left(-k\frac{r}{c}\right)}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \tag{11.8}$$

From equations (11.4) and (11.7) we have

$$m_0(r) = m_0 \frac{1-A}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \tag{11.9}$$

We can prove that for the electric charge q of the material particles an equation analogous to equation (11.7) is valid. From equation (4.2) we derive an equation corresponding to equation (11.9), which is the following equation

$$q(r) = q \frac{1-B}{1 - B\exp\left(-k_1\frac{r}{c}\right)} \quad . \tag{11.10}$$

The fine structure constant α is defined as

$$\alpha = \frac{q^2}{4\pi c\hbar} \quad . \tag{11.11}$$

and using equation (11.10) we obtain

$$\alpha(r) = \alpha \left(\frac{1-B}{1 - B\exp\left(-k_1\frac{r}{c}\right)} \right)^2 \quad . \tag{11.12}$$

The wave length λ of the linear spectrum is inversely proportional to the factor $m_0 q^4$, where m_0 is the rest mass and q is the electric charge of the electron. If we denote by λ_0 the wavelength of a photon emitted by an atom "now"on Earth, and by λ the same wavelength of the same atom received "now" on Earth from the far-distant astronomical object, the following relation holds:

$$\frac{\lambda}{\lambda_0} = \frac{m_0 q^4}{m_0(r) q^4(r)}$$

and from equations (11.9) and (11.10) we obtain

$$\frac{\lambda}{\lambda_0} = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1 - A}\left(\frac{1 - B\exp\left(-k_1\frac{r}{c}\right)}{1 - B}\right)^4 . \tag{11.13}$$

From equation (11.13) we have for the redshift

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1$$

of the astronomical object that

$$z = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1 - A}\left(\frac{1 - B\exp\left(-k_1\frac{r}{c}\right)}{1 - B}\right)^4 - 1 . \tag{11.14}$$

Equation (11.14) can also be written as

$$z = \frac{1 - A exp\left(-k\frac{r}{c}\right)}{1 - A}\left(\frac{\alpha(r)}{\alpha}\right)^2 - 1 \tag{11.15}$$

after considering equation (11.12).

From the cosmological data and from measurements conducted on Earth, we know that the variation of the fine structure constant is extremely small. Therefore, from equation (11.15), we obtain with extremely accurate approximation

$$z = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1 - A} - 1$$

$$z = \frac{A}{1 - A}\left(1 - e^{-\frac{kr}{c}}\right). \tag{11.16}$$

Equation (11.16) holds with great accuracy. The variation of the fine structure constant is so small, so that any contribution t redshift is overlapped by the same contributions from the far-distant astronomical objects, due to Doppler's effect.

For small distances r, we obtain from equation (11.16)

$$z=\frac{A}{1-A}\left(1-1+\frac{kr}{c}\right)$$
$$z=\frac{kA}{c(1-A)}r$$

and comparing this with Hubble's law

$$cz=Hr$$

we get

$$\frac{kA}{1-A}=H \tag{11.17}$$

where H is Hubble's parameter.

From equation (11.17) we have

$$\frac{dH}{dt}=\dot{H}=\frac{k\dot{A}(1-A)+kA\dot{A}}{(1-A)^2}$$

$$\dot{H}=\frac{k\dot{A}}{(1-A)^2}$$

and with equation (11.5) we obtain

$$\dot{H}=\frac{k^2A}{(1-A)^2}$$

and from equation (11.17) we have

$$\dot{H}=\frac{H}{A}\quad . \tag{11.18}$$

For $m_0>0$ amd $E_0<0$ we have

$$m_0E_0<0$$

and with eqs. (3.10) and (3.11) we have

$$\frac{M_0^2\Phi}{(1+\Phi)^2}<0$$

$\Phi < 0$

and with eq. (11.3) we finally get

$$\begin{aligned} &\Phi < 0 \\ &A = -\Phi > 0 \end{aligned} \tag{11.19}$$

From eq. (11.17) we have

$$\frac{kA}{1-A} > 0$$

and considering relation (11.19) we get two combinations for the constant k and the parameter A:

$$\begin{aligned} &0 < A < 1 \wedge k > 0 \Leftrightarrow 0 < 1 + \Phi < 1 \wedge k > 0 \\ &A > 1 \wedge k < 0 \Leftrightarrow 1 + \Phi < 0 \wedge k < 0 \end{aligned} \tag{11.20}$$

From eqs. (11.7) and (11.8) it follows that the sign change of the constant k is equivalent with the interchange of the roles of the rest masses m_0 and $\frac{E_0}{c^2}$. Hence it suffices to present the conclusions resulting from the first case of (11.20).

For $k > 0$ for eq. (11.16) we have

$$\lim_{r \to \infty} z = \frac{A}{1-A} \quad . \tag{11.21}$$

The redshift has an upper limit which depends on the value of the parameter A, even in the case that the universe extends to infinity. In the case the universe has finite extension, let $r_{\max} = R$ and from eq. (11.16) we have

$$z_{\max} = \frac{A}{1-A}\left(1 - e^{-\frac{kR}{c}}\right). \tag{11.22}$$

Thus redshift has a maximum value. The upper redshift limit of eq. (11.21) and $z_{\max}$ of eq. (11.22) are almost equal. Hence in the following we will use eq. (11.21).

From eqs. (11.16) and (11.5) we get after the calculations

$$\frac{dz}{dt} = \dot{z} = \frac{kA}{(1-A)^2}\left(1 - e^{-\frac{kr}{c}}\right)$$

and with eq. (11.17) we have

$$\dot{z} = \frac{H}{1-A}\left(1-e^{-\frac{kr}{c}}\right) = \frac{H}{A}z > 0 \tag{11.23}$$

Thus the redshift of far distant astronomical objects increases slightly with the passage of time.

According to eq. (11.21) it is

$$z < \frac{A}{1-A}$$

and because of $1-A>0$ we have

$$\frac{z}{1+z} < A$$

and because of $A<1$ we have

$$\frac{z}{1+z} < A < 1. \tag{11.24}$$

From the inequality (11.24) it follows that

$$A \to 1^{-} . \tag{11.25}$$

We prove now that as $A \to 1^{-}$ the equation (11.16) tends to Hubble's law $cz = Hr$.

Let $x = \frac{1-A}{A}$ then $x \to 0^{+}$ for $A \to 1^{-}$, while from eq. (11.17) we get $k = xH$ and eq. (11.6) may be written as

$$z = \frac{1}{x}\left[1-\exp\left(-\frac{Hr}{c}\right)\right].$$

Hence we get

$$\lim_{A\to 1^{-}} z = \lim_{x\to 0^{+}} \frac{1}{x}\left[1-\exp\left(-x\frac{Hr}{c}\right)\right] = \frac{Hr}{c}.$$

From relation (11.5) follows the conclusion that

$$\frac{dA}{dt} = kA > 0\ . \tag{11.26}$$

Thus the parameter A increases with the passage of time. Hence according to the forementioned proof, the eq. (11.16) tends to the Hubble law with the passage of time.

Combining eqs. (11.9) and (11.16) we have

$$m_0(z) = \frac{m_0}{1+z}\ . \tag{11.27}$$

The eq. (11.27) has multiple consequences on cosmological scale.

According to eq. (11.27) the gravitational interaction between two astronomical objects is smaller than expected by the factor $\frac{1}{1+z}$. The redshift z depends on their distance r as given in eq. (11.16). This is the redshift that an observer on one object would measure by observing the other object.

For the solar system or for the structure of a galaxy or a galaxy cluster, eq. (11.27) has no consequences. On this distance scale we practically have $z=0$. However we can seek consequences on this scale from another equation

From eq. (11.4) we have

$$\frac{dm_0}{dt} = \dot{m}_0 = \pm\frac{M_o\,\dot{A}}{(1-A)^2}$$

and from eq. (11.4) we have

$$\dot{m}_0 = m_0\,\frac{\dot{A}}{1-A}$$

and with eq. (11.5) we get

$$\frac{\dot{m}_0}{m_0} = \frac{kA}{1-A}$$

and with eq. (11.17) we get

$$\frac{\dot{m}_0}{m_0} = H . \qquad (11.28)$$

Eq. (11.28) concerns the mass $m_0 = m_0(t)$. Therefore its consequences can be found in our galaxy or even in the solar system. We notice that the value of the Hubble parameter H is probably smaller than the one accepted today, but we will not continue the analysis to this matter in this publication. In any case the experimental verification of eq. (11.28) requires measurements with sensitive instruments of observation. For the conduct of these measurements eq. (4.19) of the USVI must also be taken into account.

Eq. (11.27) has important consequences on cosmological scale distances. For such distances the gravitational interaction diminishes quickly and beyond some distance it practically vanishes. It has however played an important role for the creation of all large structures in the universe.

As we will see further down, the very early universe differed only slightly from vacuum. The gravitational interaction strengthens with the passage of time, as the rest masses of material particles increase. Moreover, its strength depends on distance as predicted by the law of universal gravitation, but also for cosmological distances, as predicted by eq. (11.27). Both these factors played an important role for the creation of all large structures in the universe and have not been both accounted for in the interpretation of the cosmological data via the SCM.

From eqs. $E = mc^2$ and (11.27) we have

$$E(z) = \frac{E}{1+z} . \qquad (11.29)$$

In every case of transformation of mass to energy. The production of energy in the universe is mainly achieved via hydrogen fusion and nuclear reactions. Therefore the energy produced in the past in the far distant astronomical objects was smaller than the corresponding energy produced today in our galaxy through the same mechanism. This fact has two immediate consequences.

The first is that eq. (11.16) is valid for the redshift z_a of the radiation which stems from accelerated/ decelerated electric charges

$$z_a = \frac{A}{1-A}\left(1-e^{-\frac{kr}{c}}\right) \ . \qquad (11.30)$$

And hence for the continuous spectrum. Similar mechanisms which accelerate electric charges in our galaxy and in far distant astronomical objects do not give the same amount of energy to the electric charges. According to eq. (11.29) the energy which is supplied to the electric charges in far distant astronomical objects is less than the corresponding energy in our galaxy.

The second consequence concers the luminosity distance D of far distant astronomical objects. The overall decrease of the energy produced in the past, due to eq. (11.29) has as consequence the overall decrease of luminositites of distant astronomical objects. From the definition of the luminosity distance D it follows easy that

$$D = r\sqrt{1+z} \ . \qquad (11.31)$$

Between the distance r of the astronomical object and the distance D measured from its luminosity. The luminosity distance D is measured always larger than the real distance of the astronomical object. The real distance r of the distant astronomical object is given by eq.

$$r = \frac{c}{k}\ln\left(\frac{A}{1-z(1-A)}\right) \qquad (11.32)$$

which follows from eq. (11.16). The distance measurement from eq. (11.32) can be made if we know the constant k and the parameter A. Generally, due to eq. (11.17) it suffices to know two of the parameters k, A, H .

The ionization energy as well as the excitation energy X_n of atoms is proportional to the factor $m_0 q^4$, where m_0 is the rest mass and q the electric charge of the electron. Hence we get

$$\frac{X_n(r)}{X_n} = \frac{m_0(r)}{m_0}\left(\frac{q(r)}{q}\right)^4$$

$$\frac{X_n(r)}{X_n} = \frac{m_0(r)}{m_0}\left(\frac{\alpha(r)}{\alpha}\right)^2$$

And because of

$$\frac{\alpha(r)}{\alpha} \approx 1$$

we have

$$\frac{X_n(r)}{X_n} = \frac{m_0(r)}{m_0}$$

and with eq. (11.27) we have

$$\frac{X_n(r)}{X_n} = \frac{X_n(z)}{X_n} = \frac{1}{1+z}$$

$$X_n(r) = X_n(z) = \frac{X_n}{1+z} \quad . \tag{11.33}$$

From eq. (11.33) we conclude tha the ionization and excitation energies of atoms decrease with inceasing redshift. This fact has consequences on the degree of ionization of atoms in the distant astronomical objects

The number of excited atoms in a gas in a state of thermodynamic equilibrium is given by Boltzmann's eq.

$$\frac{N_n}{X_1} = \frac{g_n}{g_1} \exp\left(-\frac{X_n}{KT} \right) \tag{11.34}$$

where N_n is the number of atoms at energy level n, X_n the excitation energy from the the 1st to the n^{th} energy level, $K = 1.38 \times 10^{-23} JK^{-1}$ Boltzmann's constant, T the temperature in degrees Kelvin, and g_n the multiplicity of level n, i.e. the number of levels into which level n is split apart inside a magnetic field.

Combining eqs. (11.33) and (11.34) we get

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} \exp\left(-\frac{X_n}{KT(1+z)} \right). \tag{11.35}$$

For the hydrogen atom for $n = 2, X_2 10.5eV = 16.4 \times 10^{-19} J, g_1 = 2, g_2 = 8$ and at the surface of the Sun where $T \sim 6000K$ eq. (11.32) implies that just one in 10^8 atoms is at state $n = 2$.

Correspondingly from eq. (11.33) and for $z=1$ we have $\frac{N_2}{N_1}=2.2\times10^{-4}$, for $z=2$ we have $\frac{N_2}{N_1}=5.8\times10^{-3}$, and for $z=5$ we have $\frac{N_2}{N_1}=0.15$.

Considering eq. (11.21) we get from eq. (11.33)

$$X_n(r\to\infty)=X_n(1-A). \tag{11.36}$$

Considering relations (11.24) and (11.25) we conclude that the ionization and excitation energies of atoms tend to zero in the very early universe. The universe went through an ionization phase in its initial phase of evolution.

The laboratory value of the Thomson scattering coefficient is given by eq.

$$\sigma_{\mathrm{T}}=\frac{8\pi}{3}\frac{q^4}{m_0^2c^4} \tag{11.37}$$

where m_0 the rest mass and q the electric charge of the electron. Thus we have

$$\frac{\sigma_T(z)}{\sigma_T}=\left(\frac{m_0}{m_0(z)}\right)\left(\frac{\alpha(z)}{\alpha}\right)^2$$

and because of $\alpha(z)\sim\alpha$ we get

$$\frac{\sigma_T(z)}{\sigma_T}=\left(\frac{m_0}{m_0(z)}\right)^2$$

and with eq. (11.27) we have

$$\frac{\sigma_T(z)}{\sigma_T}=(1+z)^2. \tag{11.38}$$

The Thomson coefficient concerns the scattering of photons with low energy E. For photons with high energy E the photon scattering is determined from the Klein-Nishina coefficient :

$$\sigma=\frac{3}{8}\sigma_T\frac{m_0}{E}\left[\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right] \tag{11.39}$$

in the laboratory and

$$\sigma(z)=\frac{3}{8}\sigma_T(z)\frac{m_0(z)c^2}{E(z)}\left[\ln\left(\frac{2E(z)}{m_0(z)c^2}\right)+\frac{1}{2}\right] \tag{11.40}$$

in astronomical objects with redshift z .

From eqs. (11.27) and (11.29) we have

$$\frac{m_0(z)}{E(z)}=\frac{m_0}{E}$$

hence from eq. (11.40) we get

$$\sigma(z)=\frac{3}{8}\sigma_T(z)\frac{m_0c^2}{E}\left[\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right]$$

and with eq. (11.38) we have

$$\frac{\sigma(z)}{\sigma}=\frac{\sigma_T(z)}{\sigma_T}=(1+z)^2. \tag{11.41}$$

From eq. (11.41) we conclude that the Thomson andKlein-Nishina scattering coefficients increase with redshift and indeed in the same manner. Considering eq. (11.21) we have

$$\frac{\sigma(r\to\infty)}{\sigma}=\frac{\sigma_T(r\to\infty)}{\sigma_T}=\frac{1}{(1-A)^2}. \tag{11.42}$$

Considering eqs. (11.24) and (11.25) we conclude that the Thomson andKlein-Nishina scattering coefficients had enormous values in the vary early universe. In its initial phase the universe was totally opaque. From this initial phase stems the cosmic microwave background radiation (CMBR) we observe today.

The internal symmetry theorem (3.3) predicts that the initial universe was at a 'vacuum state' with temperature $T=0K$. Due to the Selfvariations the universe evolved to the state we observe today. This evolution agrees with the fact that the CMRB corresponds to a black body radiation with temperature $T\sim 2.73K$.

Combining eqs. (3.11) and (11.3) we have in the laboratory

$$J_i = \frac{c_i}{1-A(t)}, i = 0,1,2,3$$

and

$$J_i(r) = \frac{c_i}{1-A\left(t-\frac{r}{c}\right)} = \frac{c_i}{1-A\exp\left(-\frac{kr}{c}\right)}, i = 0,1,2,3$$

for an astronomical object at distance r, and combining these two eqs. with eq. (11.9) we get

$$\frac{J_i(r)}{J_i} = \frac{m_0(r)}{m_0}$$

and with eq. (11.27) we have

$$\frac{J_i(z)}{J_i} = \frac{1}{1+z}$$

$$J_i(z) = \frac{J_i}{1+z}, i = 0,1,2,3. \qquad (11.43)$$

From the Heisenberg uncertainty principle for the axis $x_1 = x$ we have

$$J_1 \Delta x \sim \hbar$$

in the lab, and

$$J_1(z)\Delta x(z) \sim \hbar$$

for the astronomical object, and combining these two relations we get

$$J_1(z)\Delta x(z) = J_1 \Delta x$$

and with eq. (11.43) we have

$$\Delta x(z) = (1+z)\Delta x. \qquad (11.44)$$

From eq. (11.44) we conclude that the uncertainty $\Delta x(z)$ of position of a material particle increases with redshift. Moreover as the universe evolved towards the state we observe today, the uncertainty of position of material particles was decreasing.

From eqs. (11.44) and (11.21) we have

$$\Delta x(r \to \infty) = \frac{\Delta x}{1-A}. \tag{11.45}$$

Considering relations (11.24) and (11.25) we conclude that in the very early universe there existed great uncertainty of position of material particles. The same conclusions arise for the Bohr radius. The TSV is agrees with the uncertainty principle. In the next paragraph we will see that the uncertainty of position of a material particle is one more consequence of theorem 3.3.

From eq. (11.33) it follows that as the universe evolved to the state we observe today the ionization energy increased. This prediction is generally valid for any kind of negative dynamical energies which bind together material particles to produce more complex particles. From eq. (11.27) we have

$$\Delta m_0(z)c^2 = \frac{\Delta m_0 c^2}{1+z} \tag{11.46}$$

for the energy $\Delta m_0 c^2$, the mass deficiency, which ties together the particles which constitute the nuclei of the elements. According to eq. (11.46) the energy $\Delta m_0 c^2$, like the ionization energies, increased as the universe evolved towards its present state.

Particle like the electron, which today are considered fundamental may in fact be composed of other particles. Our inability to break them apart could be due to the strengthening of the binding energies of the constituent particles. The mass M_0 in eq. (3.10) has many chances to the only realy fundamental rest mass, from which the masses of all other particles are composed.

From eqs. (11.27) and (11.21) we have

$$m_0(r \to \infty) = m_0(1-A) \neq 0. \tag{11.47}$$

Considering the relations (11.24) and (11.25) we conclude that, towards the initial state of the universe, the rest masses of material particles tend to zero:

$$m_0(r \to \infty) = m_0(1-A) \to 0. \tag{11.48}$$

From eq. (11.8) we have

$$E_0(r \to \infty) = 0. \tag{11.49}$$

According to the relations (11.48) and (11.49) the initial state of the universe slightly differed from vacuum. The same conclusion arises in the case the universe is finite, taking $r_{\max} = R < \infty$ instead of the condition $r \to \infty$ we have used.

We have studied the case of $J_1 = J_2 = J_3 = 0$ in equation (3.12) in order to bypass the consequences on the redshift produced by the proper motion of the electron. Thus, from equation (3.6) we obtain

$$M_0 = \pm \frac{ic_0}{c} .$$

From equation (11.2) we also have

$$M_0 = \pm \frac{k\hbar}{c^2} . \qquad (11.50)$$

From equation (11.17) we obtain that the constant k obtains an extremely small value. Therefore, the same holds and for the rest mass M_0, as a result of equation (11.50).

From equation (11.5) we conclude that the parameter A varies only very slightly with the passage of time. The age of the Universe is correlated at a greater degree with the value of the parameter A we measure today, and less with Hubble's parameter H. In any case the two parameters A and H are correlated via eq. (11.17).

All of the presented consequences of theorem (3.3) are recorded within the cosmological data [16-26]. For the confirmation of the predictions of the theorem for the initial state of the Universe the improvement of our observational instruments is demanded.

In the observations conducted for distances of cosmological scales, we observe the Universe as it was in the past. That is, we observe directly the consequences of the Selfvariations. We do not possess this possibility for the distances of smaller scales. The cosmological data are the result of the immediate observation of the Selfvariations and their consequences.

12. Other Consequences of the Theorem of the Internal Symmetry

The consequences of the theorem of the internal symmetry cover a wider spectrum, than the one already stated for the cosmological data. In these, the consequences of the dependence of the function Φ on time $x_0 = ict$ are recorded. The function Φ, according to equation (3.9), is a function of the set of the coordinates and also of the constants $x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3$, and given as

$$\Phi\left(x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3\right) = K \exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]. \tag{12.1}$$

As in the previous paragraph, we refrain our study in the case of $\Phi \neq -1$, as included in theorem 3.3.. This case is equivalent with the relation $C \neq 0$.

Equations (3.10) and (3.13) express the rest mass m_0 of the material particle and the rest energy E_0 as a function of Φ

$$m_0 = \pm \frac{M_0}{1+\Phi} \tag{12.2}$$

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi} \tag{12.3}$$

$$E_0 + m_0 c^2 = \pm M_0 c^2 . \tag{12.4}$$

In these equations the only constant is the rest mass M_0 of the generalized particle. Additionally the rest masses m_0 and E_0 depend on the constants c_0, c_1, c_2, c_3, according to eqs. (12.2), (12.3) and (3.9), in the following sense: For a constant rest mass M_0 of a generalized particle there are infinite values of the constants c_0, c_1, c_2, c_3, i.e. infinite states of the 4-vector C, for which eq. (3.6) is valid:

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2 c^2 . \tag{12.5}$$

According to eq. (3.12) the different states of the 4-vector C are equivalent with the ability of the material particle to have different momentums at the same point $A\left(x_0, x_1, x_2, x_3\right)$.

Therefore the evolution of the generalized particle depends on all physical quantities $x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3$. We now deduce corollary 12.1 of theorem 3.3.

Corollary 12.1. ´´ The only constant physical quantity for a material particle is its total rest mass M_0. The evolution of the Universe, or of a system of particles, or of one particle, does not depend only on time. Its evolution is determined by the Selfvariations, as this manifestation is expressed through the function Φ.´´

Proof. Corollary 12.1 is an immediate consequence of theorem 3.3.□

According to corollary 12.1, each material particle is uniquely defined from the rest mass M_0 of equations (12.2) and (12.3).

From equations (2.4), (2.5) and (3.5) , and since it holds that $x_0 = ict$, we can write the function Φ in the form

$$\Phi = \Phi(t, x_1, x_2, x_3) = K \exp\left(-\frac{b}{\hbar}\left[-(W+E)t + c_1x_1 + c_2x_2 + c_3x_3\right]\right) \tag{12.6}$$

with the sum $W + E = -icc_0$ being constant. This equation gives Φ as a function of time t, instead of the variable $x_0 = ict$.

In the afterword we present the reasons, according to which the TSV strenghtens at an important degree the Theory of Special Relativity [27-28]. In contrast, the theorem of internal symmerty highlights a fundamental difference between the TSV and the Theory of General Relativity. According to equations (12.1) and (12.2), the physical quantity, which is being introduced into the equations of the TSV and remains invariant with repsect to all systems of reference, is the quantity given by

$$\delta = \frac{b}{\hbar}(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3) \in \mathbb{C} \ . \tag{12.7}$$

Therefore, the TSV studies the physical quantity δ, and not, the also invariant with respect to all systems of reference, physical quantity of the four-dimensional arc length

$$dS^2 = (dx_0)^2 + (dx_1)^2 + (dx_2)^2 + (dx_3)^2 \ . \tag{12.8}$$

This arc length is studied by the Theory of General Relativity. The study of dS^2 can be interpreted in the manner that the Theory of General Relativity is a macroscopic theory. On the contrary, in the TSV a differentiation between the levels of the macrocosm and the microcosm does not exist. In equations (3.12), and for the energy and the momentum of the material particle,

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3$$

the concept of velocity does not exist. With the exception of equations (4.19) and (4.20), within the totality of the equations of the TSV we already presented, the concept of velocity does not enter. As we will see in the following, theorem 3.3 which justifies the cosmological data, predicts the uncertainty of the postion-momentum of the material particles. The difference among these two theories is highlighted in a concrete manner by the comparison of equations (12.7) and (12.8). In the first, spacetime appers together with the four-vector C . The second equation refers only to spacetime.

We present an example which highlights the diffrences among these two theories. It is the famous Twin Paradox. We consider that the reader is familiar with this thought experiment, as well as the result of the Theory of General Relativity [29]. The Theory of General Relativity predicts correctly the time difference in the time duration counted by the two twins. On the other hand, according to corollary 12.1, this time difference does not suffice for providing a difference in the evolution of the twins. The twins have the same generalized particles, which acquire the same rest masses M_0, at the time they meet together. At the beginning and at the end of the travel the two twins are identical. Einstein drives the wrong conclusion, not because the Theory of General Relativity is wrong, but because he regards that this time difference implies a different evolution of the twins. But, this is not a characteristic of the Theory of General Relativity. This is a common characteristic of all the physical theories preceding the TSV.

At this point let me commentate. Einstein refers to this thought experiment as the "Twin Paradox", and not as a consequence of the Theory of General Relativity. According to my opinion, Einstein understood that something was missing from the Theory of General Relativity. To this point advocates also his peristance for determining the cause of the quantum phenomena.

General relativity has been experimentaly verified from a large number of experiments. Moreover on a distance scale of a few hundred kpc its predictions are not affected by eq.

(11.27). We expect that the combination of the two theories on this distance scale will give important results for the physical reality.

We consider a generalized particle with rest mass M_0. The material particle of the generalized particle (together with STEM) can be at the spacetime point $A(x_0, x_1, x_2, x_3)$ with its energy-momentum having any value. According to eqs. (3.9) and (3.12) this can happen only with the variation of the 4-vector C. For a generalized particle the rest mass M_0 is constant, which means that through the variation of the 4-vector C, equation (12.5) remains valid.

From eq. (3.1) we have

$$\frac{\partial J_i}{\partial c_k} + \frac{\partial P_i}{\partial c_k} = \frac{\partial c_i}{\partial c_k}, k, i = 0,1,2,3 \tag{12.9}$$

and from eq. (3.9) we have

$$\frac{\partial \Phi}{\partial c_k} = -\frac{b}{\hbar}\Phi\left(x_k + \sum_{\substack{j=0\\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right), k = 0,1,2,3. \tag{12.10}$$

Eqs. (12.9) and (12.10) remain valid for any variation of the 4-vector C. We will know prove the following corollary of theorem 3.3:

Corollary 12.2 ′′The variation of the 4-vector C of a generalized particle with rest mass M_0,

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2 c^2 \tag{12.11}$$

implies the variation of the 4-vectors J and P according to eqs.

$$\frac{\partial J_i}{\partial c_k} = \frac{1}{1+\Phi}\left(\frac{\partial c_i}{\partial c_k} + \frac{b}{\hbar}\frac{c_i}{1+\Phi}\Phi\left(x_k + \sum_{\substack{j=0\\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right)\right), k, i = 0,1,2,3 \tag{12.12}$$

$$\frac{\partial P_i}{\partial c_k} = \frac{\Phi}{1+\Phi}\left(\frac{\partial c_i}{\partial c_k} - \frac{b}{\hbar}\frac{c_i}{1+\Phi}\left(x_k + \sum_{\substack{j=0\\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right)\right), k, i = 0,1,2,3 \text{ .′′} \tag{12.13}$$

Proof: From eq. (12.11) it follows that any change of one of the constants $c_k, k=0,1,2,3$, induces a change to the others. Taking the derivatives with respect to $c_k \neq 0, k=0,1,2,3$ we have

$$c_k = -\sum_{\substack{j=0\\ j\neq k}}^{3} c_j \frac{\partial c_j}{\partial c_k}, c_k \neq 0, k=0,1,2,3. \qquad (12.14)$$

The corollary is then implied from the combination of the eqs. (3.12), (12.10) and (12.9). □

According to the relation (11.19) we have $\Phi<0$, while according to the relations (11.20) the physical quantity $1+\Phi$ can be a positive or negative number. Hence from eqs. (12.12) and (12.13) we may determine the consequences for the material particle depending on whether the rates of change of

$$\frac{\partial J_i}{\partial c_k}, \frac{\partial P_i}{\partial c_k}, k,i=0,1,2,3$$

are negative, positive or zero.

As the 4-vector C variates there arises an uncertainty for the position of the material particle. Corollary (12.2) predicts many cases for this uncertainty. We will restrict ourselves to one of them. For $\frac{\partial J_i}{\partial c_k}>0$ and $1+\Phi<0$ from eq. (12.12) we have

$$\frac{\partial c_i}{\partial c_k} + \frac{b}{\hbar}\frac{c_i}{1+\Phi}\Phi\left(x_k + \sum_{\substack{j=0\\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right) < 0, k,i=0,1,2,3$$

and from eq. (3.12) we have

$$\frac{\partial c_i}{\partial c_k} + \frac{b}{\hbar} J_i \Phi\left(x_k + \sum_{\substack{j=0\\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right) < 0, k,i=0,1,2,3 \; . \qquad (12.15)$$

From inequality (12.15) for $i=k, k,i=0,1,2,3$ we have

$$1 + \frac{b}{\hbar} J_k \Phi\left(x_k + \sum_{\substack{j=0\\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right) < 0, k=0,1,2,3$$

and because $\Phi<0$ we have

$$J_\kappa\left(x_k+\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\right)>-\frac{\hbar}{b\Phi},k=0,1,2,3. \quad (12.16)$$

In the case where

$$\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\leq 0 \quad (12.17)$$

from inequality (12.16) we have

$$J_k x_k>-\frac{\hbar}{b\Phi},k=0,1,2,3. \quad (12.18)$$

$$\Phi<0$$

From eqs. (3.12) and (3.13) it follows that $P_k=\Phi J_k, k=0,1,2,3$, and considering that $\Phi<0$ we get from inequality (12.18) that

$$P_k x_k<-\frac{b}{\hbar},k=0,1,2,3. \quad (12.19)$$

The inequalities (12.16), (12.17), (12.18) and (12.19) reverse direction for $1+\Phi>0$. The inequalities (12.18) and (12.19) correspond to Heisenberg's uncertainty principle [30]. Corollary (12.2) gives rise to restrictions in the position of the material particle, in the spacetime area occupied by the generalized particle. These restrictions concern the position-momentum product.

The USVI gives the variation of the 4-vectors J and P in spacetime. However the internal symmetry theorem brings about a 'hidden' parameter of the interactions: The 4-vectors J and P may variate according to the variation of the 4-vector C. One of the consequences of the variation of the 4-vector C is the intense uncertainty of position-momentum showing up in the laboratory.

The theorem of internal symmetry, as well as the two degrees of freedom appearing in equations (5.3) and (5.7), foundain on a novel basis the manipulation of quantum information. The generalized particle is a sustained state, with constant total rest mass M_0. We may however interfere with the internal structure of the generalized particle changing the momentum of the material particle. According to eq. (3.12) the variation of the momentum of the material particle can be effected either with the change of position $A(x_0,x_1,x_2,x_3)$ of the material particle, or with the change of the 4-vector C. The first variation is determined by the USVI and the second from corollary 12.2. With a periodic variation of either the energy of the material particle or the 4-vector C we can achieve the redistribution of the physical quatities m_0, E_0, J, P, j in the spacetime area occupied by the generalized particle. Through the variation of the physical quantities m_0, E_0, J, P, j we can transmit information in spacetime. Until now the transmission of information was achieved only with the first approach. Moreover we did not know the origin or structure of STEM. Corollary 12.2 permits us to study the possibility of information transmission through the variation of the 4-vector C. The two degrees of freedom in eqs. (5.3) and (5.7) refer to the function Ψ, which has a fundamental role for the transmission of information in either way. This role is clearly visible in eq. (5.7) for the 4-vector j of the current density of the preserved quantities of the generalized particle.

Analogous conclusions with the corollary 12.2 arise for the external symmetry also. Before we can make the corresponding study, it is necessary to express the 4-vectors J and P as a function of $x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3$. The functions $J = J\left(x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3\right)$ and $P = P\left(x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3\right)$ are different for every external symmetry.

13. Afterword

As an afterword we make some general comments about the TSV. Having concluded our study, it is clear that the whole network of the equations of the TSV stems from combination of the axiom of Selfvariations, as given in equation (4.2) with the principle of conservation of the four-vector of momentum, and equation (2.7). The principle of conservation of the four-vector of momentum has been derived and tested empirically, from the experimental data. Equation (2.7) is probably derived by the other two axioms. The TSV bases axiomatically the theoretical Physics with only three axioms. As far as I know, this is a minimum number of axioms, including the axiomatization of many mathematical or physical theories. Equation (2.7) is derived by the Theory of Special Relativity. For this reason, we begin our comments from the relation between the TSV and the Theory of Special Relativity.

The Theory of Special Relativity imposes constraints on the mathematical formulation of the physical laws. All mathematical expressions of the physical laws must remain invariant with respect to the Lorentz-Einstein transformations. The TSV imposes further constraints on the mathematical formulation of the physical laws. If we denote by L the set of equations remaining invariant according to the Lorentz-Einstein transformations, and by S the set of equations compatible with the Law of Selfvariations, it holds that $S \neq L$, with $S \subset L$.

One indicative example refers to the Lienard-Wiechert electromagnetic potentials. These potentials were proposed by Lienard and Wiechert in 1899 and give the correct form of the electromagnetic field and the electromagnetic radiation for an arbitrarily moving electric charge. After the formulation of the Theory of Special Relativity by Einstein in 1905, it has been proved that the Lienard-Wiechert potentials remain invariant under the Lorentz-Einstein transformations. After the formulation of the TSV it has been proved that they are not compatible with the Selfvariations. The TSV replaces the Lienard-Wiechert electromagnatic potentials with the macroscopic potentials of the TSV, which give exactly the same field, as the one produced by the Lienard-Wiechert potentials. The macroscopic potentials of the TSV are compatible with the Selfvariations, and with the Lorentz-Einstein transformations ($S \subset L$). Another characteristic of the macroscopic potentials of the TSV is the following:

We can consider that the Selfvariations are manifested, or that the electric charge is constant, and obtain exactly the same field. This is another expression of the "internality of the Universe during the process of measurement".

For the derivation of the Lorentz-Einstein transformations we consider two observers exchanging signals with velocity c. If the observers move with equal velocities to each other, the Lorentz-Einstein transformations result. If the observes exhange signals with a velocity different than c, for example acoustic signals, we derive another set of transformations, which are wrong. Einsten's answer was that, generally, we choose the exchange of signals propagating with c by the obtained result, that is because in this way we derive the correct form of transformations.

At this point, the TSV reinforces greatly the Theory of Special Relativity. Among the material particles one constant exchange of generalized photons exists, which propagates in the macrocosm with velocity c. According to the TSV, the exchange of signals with velocity c is not just an assumption undertaken in order to derive the Lorentz-Einstein transformations, but constitutes a continuous physical reality.

The Selfvariations of the rest masses are realized if and only if they are balanced by a corresponding emission of negative energy (STEM) in the surrounding space of the material particle, so that the conservation of energy-momentum holds. This energetic content of spacetime is expressed by the four-vector P given in equation (2.5). Microscopically, the energy of STEM is expressed by equation (11.9), which is a result of equation (3.12), that is of the theorem of internal symmetry. This is expected, since the internal symmetry expresses exactly the spontaneous realization of the Selfvariations. An analogous situation occurs for the electric charge, and also for every self-variating charge Q. The spontaneous emmision of negative energy (STEM) in spacetime bears two fundamental consequences.

STEM has as a consequence the continuous exhange of information among the material particles. If the Universe is finite, with finite age, there still exist regions which have not exchanged information, as a result of the finite speed of propagation of STEM. This shall be accomplished in the future time. With the passage of time every region of the Universe interacts with an ever increasing part of the rest of the Universe. Now, acccording to equation (11.45), and going back in time, the uncertainty of the position of the material particles tends to infinity. Regions of the Universe, which will interact through the STEM in the future time, have already interacted through the material particles in the past time. The aforementioned argument holds also in the case the Universe is infinite, and has an infinite age. The only diffrence is that all of its points have interacted throught the STEM, as well. Thus, we infer the following fundamental conclusion of the TSV: "The Universe acts as one object".

The second consequence of STEM is the indirect dynamical interaction of the material particles (USVI). When I performed the differentiation for calculating the rate of change of the momentum of a material particle, there were some concrete data: The Law of Selfvariations predicts a cohesive mechanism for all of the interactions. Therefore, after the differentiation somehow the Lorentz force should be derived, and, in some way, the relation between the USVI with the curavature of spacetime, according to the work of Einstein on the theory of the gravitational interaction. Now we know that these two terms are contained within equation (4.19).

In internal symmetry the distribution of the total rest mass M_0 of the generalized particle, between the material particle and STEM, is given by eqs. (3.10) and (3.11). For every selfvariating charge Q there exist $N_J = 40$ matrices of external symmetry with different four-vectors J, P, C, j. therefore there exist $N_J = 40$ ways for the distribution of the rest mass M_0 in external symmetry. In paragraph 8 we proved that in $N_1 = 14$ symmetries of $T = Q\Lambda$ it can hold that $m_0 \neq 0$. The rest $N_T - N_1 = 40$ external symmetries have a rest mass of $m_0 = 0$.

We can also remark some features of the equations (2.10), (2.13) and (4.6). Equation (2.10) cannot be derived without the axiom of Selfvariations. The fundamental physical quantites $\lambda_{ki}, k, i = 0.1, 2, 3$ cannot be derived from the theories of Physics of the former century. That is, if we assumed that the Selfvariations were not manifested, then there would be no possible way for these quantities to be defined. From equation (2.10) comes the whole network of the equations of the TSV, including equations (2.13) and (4.6).

Equations (2.13) and (4.6) express the USVI, and furthermore correlate the corpuscular and the wave behavior of matter. The propeties of the wave function Ψ, as well as of the four-vector j of the conserved physical quantities, are stated exactly by these equations. There exist the four laws of Maxwell exactly because the first equation of the set of equation (4.6) is decomposed into four partial equations. The theorems of paragraph 7, which define the corpuscular structure of matter, are just the consequences of equation (2.10). The same holds also for the theorem of internal symmerty and its consequences, which also result form equation (2.10).

With the $SV - T$ method we can inspect the self-consistency of the whole netwrok of the equations we presented. The TSV is a closed and self-consistent physical theory.

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