The corpuscular structure of matter, the interaction of material particles, and quantum phenomena as a consequence of Selfvariations

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Abstract

In this article we present the basic investigation of the law of selfvariations. We arrive at the central conclusion that the interaction of material particles, the corpuscular structure of matter, and the quantum phenomena can be justified by the law of Selfvariations. We predict a unified interaction between material particles with a unified mechanism (Unified Selfvariations Interaction, USVI). Every interaction is the result of three clearly distinct terms with clearly distinct consequences in the USVI. We predict a wave equation, whose special cases are the Maxwell equations, the Schrödinger equation, and the related wave equations. We determine a mathematical expression for the total of the conservable physical quantities, and we calculate the curent density 4-vector. The corpuscular structure and wave behaviour of matter and their relation emerge clearly, and we give a calculation method for the rest masses of material particles. We prove the «internal symmetry» theorem which justifies the cosmological data, without a presentation of the corresponding analytical calculations. From the study we present, the method for the further investigation of the Selfvariations and their consequences also emerges.

Keywords: Particles and Fields, Quantum Physics.

1. Introduction

The law of Selfvariations describes quantitatively a slight increase of the rest masses of material particles and of the electric charge of particles of matter. It is consistent with the principles of conservation of energy, momentum, angular momentum and electric charge. It is also invariant under the Lorentz-Einstein transformations.

With its formulation, the law of Selfvariations imposes further constraints on the physical laws than those imposed by Special Relativity [1-4]. If by L we denote the set of equations that remain invariant under the Lorentz-Einstein transformations, and by S the set of equations compatible with the law of Selfvariations, it is $S \neq L$ with $S \subset L$.

The most immediate consequence of the law of Selfvariations is that the energy, the momentum, the angular momentum, and the electric charge of material particles are distributed in the surrounding spacetime. This energy distribution in the surrounding spacetime of the material particle is expressed by the "generalized photon" [5]. Generally, a generalized photon has zero rest energy. But the study of Selfvariations showed that the sum of the generalized photons emitted spontaneously by a material particle due to the Selfvariations has rest energy $E_0 \neq 0$. This also holds for the case where each of the individual generalized photons has zero rest energy.

The material particle and the generalized photons with which it interacts, comprise a dynamic system which we called “generalized particle”. We study this continuous interaction in the present article. For the formulation of the equations the following notation is used:

$W =$ the energy of the material particle

$\mathbf{J} =$ the momentum of the material particle

$m_0 =$ the rest mass of the material particle

$E_s =$ the energy of the totality of the generalized photons interacting with the material particle

$\mathbf{P}_s =$ the momentum of the totality of the generalized photons interacting with the material particle

$E_0 =$ the rest energy of the totality of the generalized photons interacting with the material particle

With the above symbolism, the law of Selfvariations for the rest mass is given by equations

$$\begin{aligned} \frac{\partial m_0}{\partial t} &= -\frac{b}{\hbar} E_s m_0 \\ \nabla m_0 &= \frac{b}{\hbar} \mathbf{P}_s m_0 \end{aligned} \qquad (1)$$

in every system of reference $O(\mathrm{t}, x, y, z)$. $\hbar$ is Planck’s constant h divided by 2π, $\hbar = \dfrac{h}{2\pi}, b \in \mathbb{C}, b \neq 0$ and $\nabla = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z} \right)$.

In the study we present, it is proven that the interaction of material particles, the corpuscular structure of matter, and the quantum phenomena can be justified as a consequence of the law of selfvariations. It is easily proven that the cosmological data are predicted and justified by the internal symmetry theorem. We have not included in the present article the analytical mathematical calculations about the consequences of the internal symmetry theorem.

The TSV predicts a unified interaction of material particles (USVI) as given by equation (86). The USVI predicts a common mechanism for all interactions. Every interaction is resolved into three individual terms, clearly distinct from each other, as they appear in the right part of equation (86), and with clearly distinct consequences in the USVI. Equation (86) gives the rate of change of energy and momentum, as well as the orbits of material particles.

We prove the wave equation (160) of the TSV, special cases of which are the Maxwell equations, the Schrödinger equation, and the related wave equations. We determine a single mathematical expression for the conservable physical quantities, and calculate the 4-vector j of the current density. The energy and momentum of a material particle are calculated by solving the wave equation (160) of the TSV.

From the study of the law of selfvariations, equation (128) emerges as central for the theoretical prediction of the corpuscular structure of matter. The combination of equation (128) with the wave equation (160) clearly showcases the corpuscular structure and the wave behaviour of matter, as well as the relation between them. From this combination, a method for the calculation of the rest masses of material particles emerges.

The TSV has two degrees of freedom, since there are two parameters $\lambda, \mu \in \mathbb{C}$, $(\lambda, \mu) \neq (0,0)$ in equation (146), which can have arbitrary values within the web of equations and theorems of the TSV.

The investigation of physical reality is reduced to the determination of the parameters λ and μ in every application of the TSV. The only exception is the case of the «generalized photon», where the system of differential equations of the TSV does not require the determination of parameters λ and μ for its solution.

2. The law of Selfvariations in the macrocosm

In the macrocosm, the energy W and momentum $\mathbf{J}$ of the material particle, the energy E_s and the momentum $\mathbf{P}_s$ of the totality of the generalized photons emitted simultaneously by the material particle are given [5] by equations

$$\begin{aligned} W &= \frac{m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}} \\ \mathbf{J} &= \frac{m_0 \mathbf{u}}{\sqrt{1-\frac{u^2}{c^2}}} \\ E_s &= \frac{E_0}{\sqrt{1-\frac{u^2}{c^2}}} \\ \mathbf{P}_s &= \frac{E_0}{\sqrt{1-\frac{u^2}{c^2}}}\frac{\mathbf{u}}{c^2} \end{aligned} \qquad (2)$$

where $\mathbf{u}$ is the velocity of the material particle.

For the Selfvariation of the rest mass $\frac{E_0}{c^2}$ we accept the symmetric equations of (1), as expressed by equations

$$\begin{aligned} \frac{\partial E_0}{\partial t} &= \frac{b}{\hbar} W E_0 \\ \nabla E_0 &= -\frac{b}{\hbar}\mathbf{J} E_0 \end{aligned} \quad . \quad (3)$$

As we will see in the next paragraphs, equations (3) stem from the law of Selfvariations, that is from equations (1). Also, we note that the energy E_s and the momentum $\mathbf{P}_s$ in equations (2) emerge from the sum of the generalized photons emitted simultaneously by the material particle in all directions. Equations (1) and (3) describe the interaction of the material particle with all of the generalized photons.

Combining equations (1) and (3) with equations (2) we obtain

$$\begin{aligned}
&\frac{\partial m_0}{\partial t} = -\frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}} && (a)\\
&\nabla m_0 = \frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}}\frac{\mathbf{u}}{c^2} && (b)\\
&\frac{\partial E_0}{\partial t} = \frac{b}{\hbar}\frac{E_0 m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}} && (c)\\
&\nabla E_0 = -\frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}}\mathbf{u}. && (d)
\end{aligned} \qquad (4)$$

Symbolizing $d\mathbf{r}$ the displacement of the material particle during a time interval dt we get for the change dm_0 of the rest mass m_0

$$dm_0 = \frac{\partial m_0}{\partial t}dt + \nabla m_0 \cdot d\mathbf{r}$$

and with equations (4; a, b) we obtain

$$dm_0 = -\frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}}dt + \frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}}\frac{\mathbf{u}}{c^2}\cdot d\mathbf{r}$$

and since

$$d\mathbf{r} = \mathbf{u}dt$$

we get

$$dm_0 = -\frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}}dt + \frac{b}{\hbar}\frac{E_0 m_0}{\sqrt{1-\frac{u^2}{c^2}}}\frac{u^2}{c^2}dt$$

$$dm_0 = -\frac{b}{\hbar}E_0 m_0\sqrt{1-\frac{u^2}{c^2}}dt$$

and symbolizing with dS the four-dimensional arc length

$$dS = c\sqrt{1-\frac{u^2}{c^2}}dt$$

we obtain

$$dm_0 = -\frac{b}{c\hbar} E_0 m_0 dS . \quad (5)$$

Similarly, starting from equations (4; c, d) we obtain equation

$$dE_0 = \frac{cb}{\hbar} E_0 m_0 dS . \quad (6)$$

From equations (5) and (6) we get

$$d\left(m_0 c^2 + E_0\right) = 0$$

and we finally get

$$m_0 c^2 + E_0 = \text{constant} . \ (7)$$

The above equations, together with the corresponding ones for the electric charge, justify the totality of the cosmological data [6-12].

In the law of Selfvariations, apart from the rest mass, the physical quantities of energy and momentum are introduced. In the macroscopic consideration of the law [5] we have introduced in equations (1) the velocity of the material particle. We have done the same for the generalized particle. In the following study we will not use of notion of velocity, with few exceptions in order to derive conclusions about the macroscopic consequences of the law.

3. The basic study of the internal structure of the generalized particle

Equations (1) describe the continuous interaction between the material particles and the generalized photons. We study the basic characteristics of this interaction in this paragraph.

We consider a material particle with rest mass $m_0 \neq 0$ and we denote E_0 the rest energy of the whole of the generalized photons interacting with the particle. That is, we consider a generalized particle.

The rest mass m_0 and the rest energy E_0 are given by equations (8) and (9) respectively according to special relativity

$$m_0^2 c^4 = W^2 - c^2 \mathbf{J}^2 \quad (8)$$

$$E_0{}^2 = E_s^2 - c^2 \mathbf{P}_s^2 . \quad (9)$$

We now denote the four-vectors

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ict \\ x \\ y \\ z \end{bmatrix} \quad (10)$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{iW}{c} \\ J_x \\ J_y \\ J_z \end{bmatrix} \quad (11)$$

$$P=\begin{bmatrix}P_0\\ \\ P_1\\ P_2\\ P_3\end{bmatrix}=\begin{bmatrix}\dfrac{iE_s}{c}\\ P_{sx}\\ P_{sy}\\ P_{sz}\end{bmatrix} \qquad (12)$$

where i is the imaginary unit, $i^2=-1$.

Using this notation, equations (1), (8) and (9) are written in the form of equations (13), (14) and (15), respectively

$$\frac{\partial m_0}{\partial x_k}=\frac{b}{\hbar}P_k m_0, k=0,1,2,3 \qquad (13)$$

$$J_0^2+J_1^2+J_2^2+J_3^2+m_0{}^2c^2=0 \quad (14)$$

$$P_0^2+P_1^2+P_2^2+P_3^2+\frac{E_0^2}{c^2}=0. \quad (15)$$

After differentiating equation (14) with respect to $x_k, k=0,1,2,3$ we obtain

$$J_0\frac{\partial J_0}{\partial x_k}+J_1\frac{\partial J_1}{\partial x_k}+J_2\frac{\partial J_2}{\partial x_k}+J_3\frac{\partial J_3}{\partial x_k}+m_0c^2\frac{\partial m_0}{\partial x_k}=0$$

and with equation (13) we obtain

$$J_0\frac{\partial J_0}{\partial x_k}+J_1\frac{\partial J_1}{\partial x_k}+J_2\frac{\partial J_2}{\partial x_k}+J_3\frac{\partial J_3}{\partial x_k}+\frac{b}{\hbar}P_k m_0^2c^2=0$$

and with equation (14) we obtain

$$J_0\frac{\partial J_0}{\partial x_k}+J_1\frac{\partial J_1}{\partial x_k}+J_2\frac{\partial J_2}{\partial x_k}+J_3\frac{\partial J_3}{\partial x_k}-\frac{b}{\hbar}P_k\left(J_0^2+J_1^2+J_2^2+J_3^2\right)=0$$

and we finally arrive at

$$\begin{aligned}&J_0\left(\frac{\partial J_0}{\partial x_k}-\frac{b}{\hbar}P_kJ_0\right)+J_1\left(\frac{\partial J_1}{\partial x_k}-\frac{b}{\hbar}P_kJ_1\right)\\&+J_2\left(\frac{\partial J_2}{\partial x_k}-\frac{b}{\hbar}P_kJ_2\right)+J_3\left(\frac{\partial J_3}{\partial x_k}-\frac{b}{\hbar}P_kJ_3\right)=0, k=0,1,2,3\end{aligned}. \qquad (16)$$

We now symbolize

$$\frac{\partial J_i}{\partial x_k}-\frac{b}{\hbar}P_kJ_i=\lambda_{ki},\ k,i=0,1,2,3. \qquad (17)$$

With this notation, equation (16) can be written in the form

$$J_0\lambda_{k0}+J_1\lambda_{k1}+J_2\lambda_{k2}+J_3\lambda_{k3}=0,\ k=0,1,2,3. \qquad (18)$$

Also, from equation (17) we see that the physical quantities $\lambda_{ki}, k, \mathrm{i}=0,1,2,3$ have units of $\dfrac{kgr}{s}$.

We now need the 4×4 matrix T as given by equation

$$T = \begin{bmatrix} \lambda_{00} & \lambda_{01} & \lambda_{02} & \lambda_{03} \\ \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix}. \qquad (19)$$

With this notation, equation (18) can be written in the form

$$TJ = 0. \qquad (20)$$

We now prove the following theorem:

" For $m_0 \neq 0$, and for every $k, \mathrm{i} = 0,1,2,3$ equation (21) holds

$$\frac{\partial P_i}{\partial x_k} = \frac{\partial P_k}{\partial x_i}. \qquad (21)\text{"}$$

Indeed, by differentiating equation (13) with respect to x_i, $i = 0,1,2,3$, we get

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right) = \frac{b}{\hbar}\frac{\partial}{\partial x_i}\left(P_k m_0\right)$$

and using the identity

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right) = \frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right)$$

we get

$$\frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right) = \frac{b}{\hbar}\frac{\partial}{\partial x_i}\left(P_k m_0\right)$$

and with equation (13) we have

$$\frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_i m_0\right) = \frac{b}{\hbar}\frac{\partial}{\partial x_i}\left(P_k m_0\right)$$

$$P_i \frac{\partial m_0}{\partial x_k} + m_0 \frac{\partial P_i}{\partial x_k} = P_k \frac{\partial m_0}{\partial x_i} + m_0 \frac{\partial P_k}{\partial x_i}$$

and with equation (13) we also have

$$P_i \frac{b}{\hbar} P_k m_0 + m_0 \frac{\partial P_i}{\partial x_k} = P_k \frac{b}{\hbar} P_i m_0 + m_0 \frac{\partial P_k}{\partial x_i}$$

$$m_0\left(\frac{\partial P_i}{\partial x_k} - \frac{\partial P_k}{\partial x_i}\right) = 0$$

and since $m_0 \neq 0$, we obtain equation (21).

We now prove the following theorem:

"For $m_0 \neq 0$, and for every $k, i, v = 0,1,2,3$, the following equation holds

$$\frac{\partial \lambda_{ki}}{\partial x_v} - \frac{b}{\hbar} P_v \lambda_{ki} = \frac{\partial \lambda_{vi}}{\partial x_k} - \frac{b}{\hbar} P_k \lambda_{vi} \;.(22)\text{"}$$

Indeed, by differentiating equation (17)

$$\lambda_{ki} = \frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar} P_k J_i, \;\; k, i = 0,1,2,3$$

with respect to $x_v,\ v=0,1,2,3$ we get

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial}{\partial x_v}\left(\frac{\partial J_i}{\partial x_k}\right)-\frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

and with identity

$$\frac{\partial}{\partial x_v}\left(\frac{\partial J_i}{\partial x_k}\right)=\frac{\partial}{\partial x_k}\left(\frac{\partial J_i}{\partial x_v}\right)$$

we get

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial}{\partial x_k}\left(\frac{\partial J_i}{\partial x_v}\right)-\frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

and with equation (17) we have

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_v J_i+\lambda_{vi}\right)-\frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial \lambda_{vi}}{\partial x_k}+\frac{b}{\hbar}\frac{\partial}{\partial x_k}\left(P_v J_i\right)-\frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial \lambda_{vi}}{\partial x_k}+\frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k}+\frac{b}{\hbar}J_i\frac{\partial P_v}{\partial x_k}-\frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v}-\frac{b}{\hbar}J_i\frac{\partial P_k}{\partial x_v}$$

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial \lambda_{vi}}{\partial x_k}+\frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k}-\frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v}+\frac{b}{\hbar}J_i\left(\frac{\partial P_v}{\partial x_k}-\frac{\partial P_k}{\partial x_v}\right)$$

and with equation (21) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial \lambda_{vi}}{\partial x_k}+\frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k}-\frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v}$$

and with equation (17) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial \lambda_{vi}}{\partial x_k}+\frac{b}{\hbar}P_v\left(\frac{b}{\hbar}P_k J_i+\lambda_{ki}\right)-\frac{b}{\hbar}P_k\left(\frac{b}{\hbar}P_v J_i+\lambda_{vi}\right)$$

and we finally have

$$\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{\partial \lambda_{vi}}{\partial x_k}+\frac{b}{\hbar}P_v\lambda_{ki}-\frac{b}{\hbar}P_k\lambda_{vi}$$

which is equation (22).

In the following paragraphs the physical meaning of the quantities $\lambda_{ki},\ k,i=0,1,2,3$ will emerge.

4. The Lorentz-Einstein-Selfvariations Symmetry

In this paragraph we calculate the Lorentz-Einstein transformations of the physical quantities λ_{ki} , $k,i=0,1,2,3$. A result that emerges is that the elements of matrix T of equation (19) are not independent of each other. Matrix T has internal symmetries that emerge from the Lorentz-Einstein transformations. These symmetries have to do with the interchange of indices k and i in the physical quantities $\lambda_{ki},\ k,i=0,1,2,3$.

We consider an inertial frame of reference $O'(t',\mathrm{x}',\mathrm{y}',\mathrm{z}')$ moving with velocity $(u,0,0)$ with respect to another inertial frame of reference $O(t,\mathrm{x},\mathrm{y},\mathrm{z})$, with their origins O' and O coinciding at $t'=t=0$. We will calculate the Lorentz-Einstein [1-4] transformations for the physical quantities $\lambda_{ki},\ k,i=0,1,2,3$. We begin with transformations (23) and (24)

$$\begin{aligned}
\frac{\partial}{\partial t'} &= \gamma\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\\
\frac{\partial}{\partial x'} &= \gamma\left(\frac{\partial}{\partial x}+\frac{u}{c^2}\frac{\partial}{\partial t}\right)\\
\frac{\partial}{\partial y'} &= \frac{\partial}{\partial y}\\
\frac{\partial}{\partial z'} &= \frac{\partial}{\partial z}
\end{aligned} \tag{23}$$

$$\begin{aligned}
W' &= \gamma\left(W-uJ_x\right) & E_s{}' &= \gamma\left(E_s-uP_{sx}\right)\\
J_x{}' &= \gamma\left(J_x-\frac{u}{c^2}W\right) & P_{sx}{}' &= \gamma\left(P_{sx}-\frac{u}{c^2}E_s\right)\\
J_y{}' &= J_y & P_{sy}{}' &= P_{sy}\\
J_z{}' &= J_z & P_{sz}{}' &= P_{sz}
\end{aligned} \tag{24}$$

where $\gamma=\left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}}$.

We then use the notation (10), (11), (12) and obtain the transformations (25) and (26)

$$\begin{aligned}
\frac{\partial}{\partial x_0{}'} &= \gamma\left(\frac{\partial}{\partial x_0}-i\frac{u}{c}\frac{\partial}{\partial x_1}\right)\\
\frac{\partial}{\partial x_1{}'} &= \gamma\left(\frac{\partial}{\partial x_1}+i\frac{u}{c}\frac{\partial}{\partial x_0}\right)\\
\frac{\partial}{\partial x_2{}'} &= \frac{\partial}{\partial x_2}\\
\frac{\partial}{\partial x_3{}'} &= \frac{\partial}{\partial x_3}
\end{aligned} \tag{25}$$

$$
\begin{aligned}
&J_0{}' = \gamma\left(J_0 - i\frac{u}{c}J_1\right) \quad && P_0' = \gamma\left(P_0 - i\frac{u}{c}P_1\right) \\
&J_1{}' = \gamma\left(J_1 + i\frac{u}{c}J_0\right) \quad && P_1' = \gamma\left(P_1 + i\frac{u}{c}P_0\right) \qquad (26) \\
&J_2{}' = J_2 && P_2' = P_2 \\
&J_3{}' = J_3 && P_3' = P_3
\end{aligned}
$$

We now derive the transformation of the physical quantity λ_{00}. From equation (17) for $k = i = 0$ we get for the inertial reference frame $O'(t', \mathrm{x}', \mathrm{y}', \mathrm{z}')$

$$
\lambda_{00}{}' = \frac{\partial J_0{}'}{\partial x_0{}'} - \frac{b}{\hbar} P_0' J_0{}'
$$

and with transformations (25) and (26) we obtain

$$
\lambda_{00}{}' = \gamma^2\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right)\left(J_0 - i\frac{u}{c}J_1\right) - \frac{b}{\hbar}\gamma^2\left(P_0 - i\frac{u}{c}P_1\right)\left(J_0 - i\frac{u}{c}J_1\right)
$$

$$
\lambda_{00}{}' = \gamma^2\left(\frac{\partial J_0}{\partial x_0} - i\frac{u}{c}\frac{\partial J_1}{\partial x_0} - i\frac{u}{c}\frac{\partial J_0}{\partial x_1} - \frac{u^2}{c^2}\frac{\partial J_1}{\partial x_1} - \frac{b}{\hbar}P_0 J_0 + i\frac{u}{c}\frac{b}{\hbar}P_0 J_1 + i\frac{u}{c}\frac{b}{\hbar}P_1 J_0 + \frac{u^2}{c^2}\frac{b}{\hbar}P_1 J_1\right)
$$

and replacing physical quantities $\frac{\partial J_0}{\partial x_0}, \frac{\partial J_1}{\partial x_0}, \frac{\partial J_0}{\partial x_1}, \frac{\partial J_1}{\partial x_1}$ from equation (17) we get

$$
\begin{aligned}
\lambda_{00}{}' = \gamma^2 (&\frac{b}{\hbar}P_0 J_0 + \lambda_{00} - i\frac{u}{c}\frac{b}{\hbar}P_0 J_1 - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\frac{b}{\hbar}P_1 J_0 - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\frac{b}{\hbar}P_1 J_1 \\
&- \frac{u^2}{c^2}\lambda_{11} - \frac{b}{\hbar}P_0 J_0 + i\frac{u}{c}\frac{b}{\hbar}P_0 J_1 + i\frac{u}{c}\frac{b}{\hbar}P_1 J_0 + \frac{u^2}{c^2}\frac{b}{\hbar}P_1 J_1)
\end{aligned}
$$

and we finally obtain equation

$$
\lambda_{00}{}' = \gamma^2\left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11}\right).
$$

Following the same procedure for $k, \mathrm{i} = 0,1,2,3$ we obtain the following 16 equations (27) for the Lorentz-Einstein transformations of the physical quantities λ_{ki} :

$$
\begin{aligned}
\lambda_{00}' &= \gamma^2\left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11}\right)\\
\lambda_{01}' &= \gamma^2\left(\lambda_{01} + i\frac{u}{c}\lambda_{00} - i\frac{u}{c}\lambda_{11} + \frac{u^2}{c^2}\lambda_{10}\right)\\
\lambda_{02}' &= \gamma\left(\lambda_{02} - i\frac{u}{c}\lambda_{12}\right)\\
\lambda_{03}' &= \gamma\left(\lambda_{03} - i\frac{u}{c}\lambda_{13}\right)\\
\lambda_{10}' &= \gamma^2\left(\lambda_{10} - i\frac{u}{c}\lambda_{11} + i\frac{u}{c}\lambda_{00} + \frac{u^2}{c^2}\lambda_{01}\right)\\
\lambda_{11}' &= \gamma^2\left(\lambda_{11} + i\frac{u}{c}\lambda_{10} + i\frac{u}{c}\lambda_{01} - \frac{u^2}{c^2}\lambda_{00}\right)\\
\lambda_{12}' &= \gamma\left(\lambda_{12} + i\frac{u}{c}\lambda_{02}\right)\\
\lambda_{13}' &= \gamma\left(\lambda_{13} + i\frac{u}{c}\lambda_{03}\right)\\
\lambda_{20}' &= \gamma\left(\lambda_{20} - i\frac{u}{c}\lambda_{21}\right)\\
\lambda_{21}' &= \gamma\left(\lambda_{21} + i\frac{u}{c}\lambda_{20}\right)\\
\lambda_{22}' &= \lambda_{22}\\
\lambda_{23}' &= \lambda_{23}\\
\lambda_{30}' &= \gamma\left(\lambda_{30} - i\frac{u}{c}\lambda_{31}\right)\\
\lambda_{31}' &= \gamma\left(\lambda_{31} + i\frac{u}{c}\lambda_{30}\right)\\
\lambda_{32}' &= \lambda_{32}\\
\lambda_{33}' &= \lambda_{33}
\end{aligned}
\qquad (27)
$$

Inspecting equations (27) we see that they are divided into five individual groups of transformations, independent of each other. We rearrange the order of equations (27) to highlight these groups, which we numbered from I to V in equations (28).

$$\left.\begin{aligned}
\lambda_{00}{}' &= \gamma^2\left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11}\right)\\
\lambda_{01}{}' &= \gamma^2\left(\lambda_{01} + i\frac{u}{c}\lambda_{00} - i\frac{u}{c}\lambda_{11} + \frac{u^2}{c^2}\lambda_{10}\right)\\
\lambda_{10}{}' &= \gamma^2\left(\lambda_{10} - i\frac{u}{c}\lambda_{11} + i\frac{u}{c}\lambda_{00} + \frac{u^2}{c^2}\lambda_{01}\right)\\
\lambda_{11}{}' &= \gamma^2\left(\lambda_{11} + i\frac{u}{c}\lambda_{10} + i\frac{u}{c}\lambda_{01} - \frac{u^2}{c^2}\lambda_{00}\right)
\end{aligned}\right\} I$$

$$\left.\begin{aligned}
\lambda_{22}{}' &= \lambda_{22}\\
\lambda_{33}{}' &= \lambda_{33}
\end{aligned}\right\} II$$

$$\left.\begin{aligned}
\lambda_{02}{}' &= \gamma\left(\lambda_{02} - i\frac{u}{c}\lambda_{12}\right)\\
\lambda_{12}{}' &= \gamma\left(\lambda_{12} + i\frac{u}{c}\lambda_{02}\right)\\
\lambda_{20}{}' &= \gamma\left(\lambda_{20} - i\frac{u}{c}\lambda_{21}\right)\\
\lambda_{21}{}' &= \gamma\left(\lambda_{21} + i\frac{u}{c}\lambda_{20}\right)
\end{aligned}\right\} III \qquad (28)$$

$$\left.\begin{aligned}
\lambda_{03}{}' &= \gamma\left(\lambda_{03} - i\frac{u}{c}\lambda_{13}\right)\\
\lambda_{13}{}' &= \gamma\left(\lambda_{13} + i\frac{u}{c}\lambda_{03}\right)\\
\lambda_{30}{}' &= \gamma\left(\lambda_{30} - i\frac{u}{c}\lambda_{31}\right)\\
\lambda_{31}{}' &= \gamma\left(\lambda_{31} + i\frac{u}{c}\lambda_{30}\right)
\end{aligned}\right\} IV$$

$$\left.\begin{aligned}
\lambda_{23}{}' &= \lambda_{23}\\
\lambda_{32}{}' &= \lambda_{32}
\end{aligned}\right\} V$$

Group I of equations (28) is self-consistent when equations (29) and (30) hold

$$\lambda_{00} = \lambda_{11} \qquad (29)$$

$$\lambda_{10} = -\lambda_{01} \qquad (30)$$

With equations (29) and (30) the transformations (28; I, II) can be written in the form of equations (31) and (32)

$$\begin{aligned} \lambda_{00}' &= \lambda_{00} \\ \lambda_{11}' &= \lambda_{11} \\ \lambda_{22}' &= \lambda_{22} \\ \lambda_{33}' &= \lambda_{33} \end{aligned} \qquad (31)$$

$$\lambda_{01}' = \lambda_{01} \qquad (32)$$

Transformations (27) allow for a wide spectrum of relations between the physical quantities λ_{ii}, $i = 0,1,2,3.$ The correlation of physical quantities λ_{ii} can vary all the way from their being non-correlated to being equal, that is

$$\lambda_{00} = \lambda_{11} = \lambda_{22} = \lambda_{33}$$

according to the Lorentz-Einstein transformations.

Group III of equations (28) has the following characteristic property: if we assume that $\lambda_{20} = -\lambda_{02}$, then $\lambda_{21} = -\lambda_{12}$, and vice versa. Indeed, assuming that $\lambda_{20} = -\lambda_{02}$, from the third of equations $(28, III)$ we get

$$-\lambda_{02}{}' = \gamma\left(-\lambda_{02} - i\frac{u}{c}\lambda_{21}\right)$$

$$\lambda_{02}{}' = \gamma\left(\lambda_{02} + i\frac{u}{c}\lambda_{21}\right)$$

and comparing with the first of equations $(28, III)$ we see that

$$\lambda_{21} = -\lambda_{12}.$$

If we now consider that $\lambda_{20} = \lambda_{02}$, we similarly obtain $\lambda_{21} = \lambda_{12}$, and vice versa. Indeed, from the third of equations $(28, III)$ for $\lambda_{20} = \lambda_{02}$, we get

$$\lambda_{02}{}' = \gamma\left(\lambda_{20} - i\frac{u}{c}\lambda_{21}\right)$$

and comparing with the third of equations $(28, III)$ we obtain

$$\lambda_{21} = \lambda_{12}.$$

Similar conclusions are derived for group IV of equations (28). Following the same procedure it can be proved that for $\lambda_{30} = -\lambda_{03}$ it is also $\lambda_{31} = -\lambda_{13}$, and vice versa. Also, for $\lambda_{30} = \lambda_{03}$ it is $\lambda_{31} = \lambda_{13}$, and the other way around.

In group V of equations (28) we can have either $\lambda_{32} = -\lambda_{23}$, or $\lambda_{32} = \lambda_{23}$, or the physical quantities λ_{32} and λ_{23} can be independent during the interchange of the indices 2 and 3. That is, they behave like the physical quantities λ_{ii}, $i = 0,1,2,3$, at least according to the Lorentz-Einstein transformations. Thus, we end up with the following four sets of equations (33), (34), (35) and (36)

$$\begin{aligned}
\lambda_{10} &= -\lambda_{01} \\
\lambda_{20} &= -\lambda_{02} \\
\lambda_{30} &= -\lambda_{03} \\
\lambda_{21} &= -\lambda_{12} \\
\lambda_{31} &= -\lambda_{13}
\end{aligned} \qquad (33)$$

$$\begin{aligned}
\lambda_{10} &= -\lambda_{01} \\
\lambda_{20} &= \lambda_{02} \\
\lambda_{30} &= \lambda_{03} \\
\lambda_{21} &= \lambda_{12} \\
\lambda_{31} &= \lambda_{13}
\end{aligned} \qquad (34)$$

$$\begin{aligned}
\lambda_{10} &= -\lambda_{01} \\
\lambda_{20} &= -\lambda_{02} \\
\lambda_{30} &= \lambda_{03} \\
\lambda_{21} &= -\lambda_{12} \\
\lambda_{31} &= \lambda_{13}
\end{aligned} \qquad (35)$$

$$\begin{aligned}
\lambda_{10} &= -\lambda_{01} \\
\lambda_{20} &= \lambda_{02} \\
\lambda_{30} &= -\lambda_{03}\,. \\
\lambda_{21} &= \lambda_{12} \\
\lambda_{31} &= -\lambda_{13}
\end{aligned} \qquad (36)$$

In every case, transformations (27) obtain the form

$$\begin{aligned}
&\lambda'_{ii} = \lambda_{ii},\ \mathrm{i} = 0,1,2,3 \\
&\lambda'_{01} = \lambda_{01} && \lambda'_{10} = \lambda_{10} \\
&\lambda'_{02} = \gamma\left(\lambda_{02} - i\frac{u}{c}\lambda_{12}\right) && \lambda'_{20} = \gamma\left(\lambda_{20} - i\frac{u}{c}\lambda_{21}\right) \\
&\lambda'_{12} = \gamma\left(\lambda_{12} + i\frac{u}{c}\lambda_{02}\right) && \lambda'_{21} = \gamma\left(\lambda_{21} + i\frac{u}{c}\lambda_{20}\right). \qquad (37) \\
&\lambda'_{03} = \gamma\left(\lambda_{03} - i\frac{u}{c}\lambda_{13}\right) && \lambda'_{30} = \gamma\left(\lambda_{30} - i\frac{u}{c}\lambda_{31}\right) \\
&\lambda'_{13} = \gamma\left(\lambda_{13} + i\frac{u}{c}\lambda_{03}\right) && \lambda'_{31} = \gamma\left(\lambda_{31} + i\frac{u}{c}\lambda_{30}\right) \\
&\lambda'_{23} = \lambda_{23} && \lambda'_{32} = \lambda_{32}
\end{aligned}$$

Transformations (37) apply only at flat spacetime.

5. Physical quantities λ_{ki}, $k,i = 0,1,2,3$ and the conservation principles of energy and momentum

In the case of the spontaneous emission of generalized photons by the material particles due to the Selfvariations, we have proven that the conservation of momentum, energy and electric charge holds [5] (paragraphs 4.4 and 4.5 by direct calculation of the total energetic content of a finite part of spacetime, in the same paragraphs through the continuity equation, and in paragraphs 4.7 and 4.8 through the energy-momentum tensor). In this paragraph we correlate the conservation of energy and momentum of the generalized particle with the physical quantities λ_{ki}, $k,i = 0,1,2,3$.

Firstly, we prove the following theorem:

“ For $m_0 \neq 0$ the following propositions are equivalent:

A. The generalized particle conserves its momentum $J_i + P_i$ along the axis x_i, $i = 0,1,2,3$, i.e.

$$J_i + P_i = c_i = \text{constant}. \qquad (38)$$

B.
$$\frac{\partial P_i}{\partial x_k} = -\frac{b}{\hbar} P_k J_i - \lambda_{ki} = \frac{\partial P_k}{\partial x_i} \qquad (39)$$

for every $k = 0,1,2,3$. ”

Indeed, if equation (38) holds, then we differentiate with respect to x_k, $k = 0,1,2,3$ obtaining

$$\frac{\partial J_i}{\partial x_k} + \frac{\partial P_i}{\partial x_k} = 0$$

$$\frac{\partial P_i}{\partial x_k} = -\frac{\partial J_i}{\partial x_k}$$

and with equation (17) we obtain

$$\frac{\partial P_i}{\partial x_k} = -\frac{b}{\hbar} P_k J_i - \lambda_{ki}$$

and with equation (21) we obtain

$$\frac{\partial P_k}{\partial x_i} = \frac{\partial P_i}{\partial x_k} = -\frac{b}{\hbar} P_k J_i - \lambda_{ki},$$

which is equation (39).

Conversely, if equation (39) holds for every $k = 0,1,2,3$, we obtain

$$\frac{\partial P_i}{\partial x_k} = -\frac{b}{\hbar} P_k J_i - \lambda_{ki}$$

and with equation (17) we get

$$\frac{\partial P_i}{\partial x_k} = -\frac{\partial J_i}{\partial x_k}$$

$$\frac{\partial}{\partial x_\kappa}\left(J_i + P_i\right) = 0$$

and since this equation holds for every $k = 0,1,2,3$, we obtain equation (38).

From the previous theorem we conclude that equation (39) gives the rates of change $\frac{\partial P_i}{\partial x_k}, \frac{\partial P_k}{\partial x_i}$, $\mathrm{k} = 0,1,2,3$ when the generalized particle conserves its momentum along the axis x_i, $i = 0,1,2,3$. When the generalized particle conserves its momentum for every axis x_i then equation (39) holds for every $k, i = 0,1,2,3$.

We now prove the following theorem:

"If the generalized particle conserves its momentum along the axes x_i and x_k with $k \neq i$, then:

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{2\hbar}\left(c_k P_i - c_i P_k\right) \qquad (40)$$

$k, i = 0,1,2,3,\ k \neq i$."

Indeed, since the generalized photon conserves its momentum along the axes x_i and x_k, equations (41) hold:

$$\begin{aligned} P_i &= c_i - J_i \\ P_k &= c_k - J_k \end{aligned} . \qquad (41)$$

Combining equations (21) and (41) we obtain

$$\frac{\partial}{\partial x_k}\left(c_i - J_i\right) = \frac{\partial}{\partial x_i}\left(c_k - J_k\right)$$

$$\frac{\partial J_i}{\partial x_k} = \frac{\partial J_k}{\partial x_i}$$

and with equation (17) we get

$$\frac{b}{\hbar} P_k J_i + \lambda_{ki} = \frac{b}{\hbar} P_i J_k + \lambda_{ik}$$

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right)$$

which is equation (40). The remaining equalities in equation (40) are derived by considering equations (41). Equation (40) holds for $k \neq i,\ k, \mathrm{i} = 0,1,2,3$, since equation (21), from which equation (41) results, is an identity for $k = i$ and gives no information in this case.

An immediate consequence of the preceding theorem is that if the generalized particle conserves its momentum in every axis, then equation (40) holds for every $k, \mathrm{i} = 0,1,2,3$.

From equation (40) we obtain the following theorem:

TSV theorem for the symmetry of indices

“For $m_0 \neq 0$ and if the generalized particle conserves its momentum along the axes x_i and x_k with $k \neq i$, the following equivalences hold:

A. $$\lambda_{ik} = \lambda_{ki} \Leftrightarrow J_k P_i = J_i P_k \Leftrightarrow c_i J_k = c_k J_i \Leftrightarrow c_k P_i = c_i P_k \tag{42}$$

B. $\lambda_{ik} = -\lambda_{ki} \Leftrightarrow$
$$\lambda_{ki} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{2\hbar}\left(c_k P_i - c_i P_k\right) \tag{43}$$
$k, i = 0,1,2,3,\ k \neq i$.”

The theorem is an immediate consequence of equation (40). Furthermore, if the generalized particle conserves its momentum along every axis $x_i,\ i = 0,1,2,3$, then the equalities (42) and (43) hold for every $k \neq i, k, i = 0,1,2,3$.

We now consider the four-vector $C,$ as given by equation

$$C = J + P = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}. \tag{44}$$

When the generalized particle conserves its momentum along every axis, then the four-vector C is constant. Also, we denote M_0 the total rest mass of the generalized particle, as given by equation

$$C^T C = c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2\, \mathrm{c}^2, \tag{45}$$

where C^T is the adjoint of the column vector C.

For reasons that will become apparent later in our study, we give the following definitions: We name the symmetry $\lambda_{ik} = \lambda_{ki},\ k \neq i,\ k, i = 0,1,2,3$ internal symmetry, and the symmetry $\lambda_{ik} = -\lambda_{ki},\ k \neq i,\ k, i = 0,1,2,3$ external symmetry.

We now prove the following theorem:

First Theorem of the TSV (Internal Symmetry Theorem)

“If the generalized particle conserves its momentum in every axis, the following hold:

A. $\lambda_{ik} = \lambda_{ki}$ for every $k, i = 0,1,2,3 \Leftrightarrow$
the four-vectors J, P and C are parallel $\Leftrightarrow$ $P = \Phi J$ (46)
where $\Phi \in \mathbb{C},\ \Phi \neq 0$.

B. For $\Phi = -1$ the following equation holds:
$$E_0 = \pm m_0 c^2 \tag{47}$$

C. For $\Phi \neq -1$ the following equations hold:
$$\Phi = K \exp\left[-\frac{b}{\hbar}\left(c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3\right)\right] \tag{48}$$

$$m_0c^2 = \pm \frac{M_0c^2}{1+K\exp\left[-\frac{b}{\hbar}\left(c_0X_0+c_1X_1+c_2X_2+c_3X_3\right)\right]} \qquad (49)$$

$$E_0 = \pm \frac{M_0c^2K\exp\left[-\frac{b}{\hbar}\left(c_0X_0+c_1X_1+c_2X_2+c_3X_3\right)\right]}{1+K\exp\left[-\frac{b}{\hbar}\left(c_0X_0+c_1X_1+c_2X_2+c_3X_3\right)\right]} \qquad (50)$$

where K is a dimensionless constant physical quantity.

D. $\lambda_{ik} = \lambda_{ki}$ for every $k, \mathrm{i} = 0,1,2,3 \Leftrightarrow$

$\lambda_{ki} = 0$ for every $k, \mathrm{i} = 0,1,2,3.$ (51)"

Equivalence (46) results immediately from equivalence (42). For $\Phi = 0$, from equation (46) we have that $P = 0$ which is impossible, since in this case the Selfvariations of the rest mass $m_0 \neq 0$, do not exist, as seen from equation (13). Therefore, $\Phi \neq 0$.

For $\Phi = -1$, from equation (46) we get $P = -J$, and from equations (14) and (15) we see that $E_0^2 = m_0^2c^4$, which is equation (47).

From equation (46) we have $P_i = \Phi J_i$ for every $i = 0,1,2,3$ and in combination with equation $J_i + P_i = c_i$ we get for $\Phi \neq -1$ equations (52) and (53)

$$J_i = \frac{c_i}{\Phi+1},\ i = 0,1,2,3 \qquad (52)$$

$$P_i = \frac{\Phi c_i}{\Phi+1},\ i = 0,1,2,3\,. \qquad (53)$$

Combining equations (14) and (52) we get

$$m_0^2c^2 + \frac{1}{(\Phi+1)^2}\left(c_0^2+c_1^2+c_2^2+c_3^2\right) = 0$$

and with equation (45) we obtain equation

$$m_0^2c^2 - \frac{M_0^2c^2}{(\Phi+1)^2} = 0\,. \quad (54)$$

Differentiating equation (54) with respect to $x_v,\ v = 0,1,2,3$ and considering equation (13) we obtain

$$\frac{2b}{\hbar}P_v m_0^2c^2 + \frac{2M_0^2c^2}{(\Phi+1)^3}\frac{\partial\Phi}{\partial x_v} = 0$$

and with equation (54) we have

$$\frac{b}{\hbar}P_v\frac{M_0^2c^2}{(\Phi+1)^2}+\frac{M_0^2c^2}{(\Phi+1)^3}\frac{\partial\Phi}{\partial x_v}=0$$

$$\frac{\partial\Phi}{\partial x_v}=-\frac{b}{\hbar}P_v(\Phi+1)$$

and with equation (53) for $i=v$ we arrive at equation

$$\frac{\partial\Phi}{\partial x_v}=-\frac{b}{\hbar}c_v\Phi,\ v=0,1,2,3. \quad (55)$$

By integration of equation (55) we obtain

$$\Phi=K\exp\left[-\frac{b}{\hbar}\left(c_0X_0+c_1X_1+c_2X_2+c_3X_3\right)\right]$$

where K is the integration constant, which is equation (48).

Combining equations (54) and (48) we obtain

$$m_0c^2=\pm\frac{M_0c^2}{1+K\exp\left[-\frac{b}{\hbar}\left(c_0X_0+c_1X_1+c_2X_2+c_3X_3\right)\right]}$$

which is equation (49). Combining equations (15), (53) and (45) we obtain

$$\frac{E_0^2}{c^2}-\frac{\Phi^2M_0^2c^2}{(\Phi+1)^2}=0$$

$$E_0=\pm\frac{M_0c^2\Phi}{\Phi+1}$$

and with equation (48) we get equation (50).

Equations (49) and (50) are equivalent with the equations of TSV which justify the cosmological data. Indeed, after combining them we obtain equation

$$m_0c^2+E_0=\pm M_0c^2$$

which is equation (7). Furthermore, we observe from equations (4), which refer to the case of generalized photons spontaneously emitted by the material particle, that the four-vectors J and P are parallel. Thus, in the case of equations (4) the above fundamental theorem holds.

In order to prove equation (7) in paragraph 2 we used equations (3), which we can now prove. Differentiating equation (50) with respect to x_k, $k=0,1,2,3$ we arrive at equation

$$\frac{\partial E_0}{\partial x_k}=-\frac{b}{\hbar}J_kE_0,\ k=0,1,2,3. \ (56)$$

Considering equation (11), equation (56) is equivalent to equations (3). Of course, we now know that equations (3) hold when the four-vectors J and P are parallel to each other, for the case of the internal symmetry.

We studied the case of a material particle with rest mass $m_0 \neq 0$. Therefore, from equation (54) we see that $M_0 \neq 0$. Furthermore, as we already observed during the proof of equivalence (46), it is also $\Phi \neq 0$, hence from equation (48) we obtain $K \neq 0$ So from equations (49) and (50) we obtain $m_0 \neq 0$ and $E_0 \neq 0$ in the case of the symmetry $\lambda_{ik} = \lambda_{ki}$, $k, \mathrm{i} = 0,1,2,3$.

Combining equations (52) and (53) with equation (48) we get respectively equations (57) and (58) for the case of the internal symmetry

$$J_i = \frac{c_i}{1 + K \exp\left[-\frac{b}{\hbar}\left(c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3\right)\right]} \qquad (57)$$

$$i = 0,1,2,3$$

$$P_i = \frac{c_i K \exp\left[-\frac{b}{\hbar}\left(c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3\right)\right] d}{1 + K \exp\left[-\frac{b}{\hbar}\left(c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3\right)\right]}. \qquad (58)$$

$$i = 0,1,2,3$$

We now prove equivalence (51). For $\lambda_{ki} = 0$ for every $k, \mathrm{i} = 0,1,2,3$ it obviously is $\lambda_{ik} = \lambda_{ki}$. In order to prove the inverse of equivalence (51), we differentiate equation (57) with respect to $x_k, k = 0,1,2,3$ and get

$$\frac{\partial J_i}{\partial x_k} = \frac{c_i c_k K \frac{b}{\hbar} \exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]}{\left(1 + K \exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]\right)^2}$$

and with equation (57), as well as equation (58) for $i = k$ we get

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i$$

and with equation (17) we get $\lambda_{ki} = 0$, which completes equivalence (51).

With the proof of equivalence (51) we can see that equation (43), initially proven for the symmetry $\lambda_{ik} = -\lambda_{ki}, k = 0,1,2,3$, is of general validity. That is, equation (59) generally holds:

$$\lambda_{ki} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{2\hbar}\left(c_k P_i - c_i P_k\right). \qquad (59)$$

$$k \neq i, \;\; k, i = 0,1,2,3$$

We begin the study of the external symmetry by proving the following theorem:

“In the external symmetry, the 4-vector C of the total energy content of the generalized particle cannot vanish:

$$C \neq 0. \qquad (60)”$$

Indeed, for $C=0$ we obtain $J=-P$ from equation (44). Therefore, the four-vectors J and P are parallel. According to equivalence (46) the parallelism of the four-vectors J and P is equivalent to the internal symmetry. Therefore, in the external symmetry it is $C \neq 0$.

We now prove the following theorem:

Second Theorem of the TSV

“ If the generalized particle conserves its momentum along every axis, and the symmetry $\lambda_{ik} = -\lambda_{ki}$ holds for every $\mathrm{k} \neq \mathrm{i}, k, i = 0,1,2,3$, then:

A. $c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = 0 \qquad (61)$

for every $i \neq v, v \neq k, k \neq i,\ k, i, v = 0,1,2,3$.

B. $\dfrac{\partial \lambda_{ki}}{\partial x_v} = \dfrac{b}{\hbar} P_v \lambda_{ki} - \dfrac{bc_v}{2\hbar}\lambda_{ki} = -\dfrac{b}{\hbar} J_v \lambda_{ki} + \dfrac{bc_v}{2\hbar}\lambda_{ki} \qquad (62)$

for every $k \neq i,\ k, i = 0,1,2,3$.

C. $TJ = 0 \qquad (63)$

$J_0\lambda_{k0} + J_1\lambda_{k1} + J_2\lambda_{k2} + J_3\lambda_{k3} = 0, \mathrm{k} = 0,1,2,3$.

D. $TP = TC. \qquad (64)”$

If the generalized particle conserves its momentum along every axis and the index symmetry $\lambda_{ik} = -\lambda_{ki}$ holds for $k \neq i,\ k, i = 0,1,2,3$, from equivalence (40) we obtain

$$\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), \mathrm{k} \neq \mathrm{i}, \mathrm{k}, \mathrm{i} = 0,1,2,3. \qquad (65)$$

Considering equation (65) we get

$$c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = \frac{b}{2\hbar}\left[c_i\left(c_k J_v - c_v J_k\right) + c_k\left(c_v J_i - c_i J_v\right) + c_v\left(c_i J_k - c_k J_i\right)\right] = 0.$$

Thus, we get equation (61).

Differentiating equation (65) with respect to $x_v, v = 0,1,2,3$ we obtain

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left(c_i \frac{\partial J_k}{\partial x_v} - c_k \frac{\partial J_i}{\partial x_v}\right)$$

and with equation (17) we get

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left[c_i\left(\frac{b}{\hbar}P_v J_k + \lambda_{vk}\right) - c_k\left(\frac{b}{\hbar}P_v J_i + \lambda_{vi}\right)\right]$$
$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left[\frac{b}{\hbar}P_v\left(c_i J_k - c_k J_i\right) + c_i\lambda_{vk} - c_k\lambda_{vi}\right]$$
$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{\hbar}P_v\frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) + \frac{b}{2\hbar}\left(c_i\lambda_{vk} - c_k\lambda_{vi}\right)$$

and with equation (65) we obtain

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{\hbar}P_v\lambda_{ki} + \frac{b}{2\hbar}\left(c_i\lambda_{vk} - c_k\lambda_{vi}\right)$$

and with equation (61) in the form

$$c_i\lambda_{vk} - c_k\lambda_{vi} = -c_v\lambda_{ki}$$

we get

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{\hbar}P_v\lambda_{ki} - \frac{bc_v}{2\hbar}\lambda_{ki}$$

which is equation (62). The second equality in equation (62) emerges from the substitution

$$P_v = c_v - J_v, v = 0,1,2,3$$

according to equation (44).

Equation (63) is equation (18). Equation (64) results by combining equations (63) and (44)

$$TP = T\left(C - J\right) = TC - TJ = TC.$$

In the case when, for the external symmetry, besides equation $\lambda_{ik} = -\lambda_{ki}$ it is also $\lambda_{ik} = \lambda_{ki}$ for some indices k and i, with $k \neq i, k, i = 0,1,2,3$, we get for these indices k and i that it is $\lambda_{ik} = 0.$ Therefore, in equations (33)-(36), it either holds that $\lambda_{ik} = -\lambda_{ki} \neq 0,$ or $\lambda_{ik} = \lambda_{ki} = 0$ for $k \neq i, k, i = 0,1,2,3.$ Thus, equations (33)-(36) can be stated in the form of equations (66)-(69):

$$\begin{aligned}
&\lambda_{10} = -\lambda_{01} \neq 0 \vee \lambda_{10} = \lambda_{01} = 0\\
&\lambda_{20} = -\lambda_{02} \neq 0\\
&\lambda_{30} = -\lambda_{03} \neq 0\\
&\lambda_{21} = -\lambda_{12} \neq 0\\
&\lambda_{31} = -\lambda_{13} \neq 0\\
&\lambda_{23} = -\lambda_{32} \neq 0
\end{aligned} \qquad (66)$$

$$
\begin{aligned}
&\lambda_{10} = -\lambda_{01} \neq 0 \vee \lambda_{10} = \lambda_{01} = 0 \\
&\lambda_{20} = \lambda_{02} = 0 \\
&\lambda_{30} = \lambda_{03} = 0 \\
&\lambda_{21} = \lambda_{12} = 0 \\
&\lambda_{31} = \lambda_{13} = 0 \\
&\lambda_{32} = -\lambda_{23} \neq 0 \vee \lambda_{32} = \lambda_{23} = 0
\end{aligned} \qquad (67)
$$

$$
\begin{aligned}
&\lambda_{10} = -\lambda_{01} \neq 0 \vee \lambda_{10} = \lambda_{01} = 0 \\
&\lambda_{20} = -\lambda_{02} \neq 0 \vee \lambda_{20} = \lambda_{02} = 0 \\
&\lambda_{30} = \lambda_{03} = 0 \\
&\lambda_{21} = -\lambda_{12} \neq 0 \vee \lambda_{21} = \lambda_{12} = 0 \\
&\lambda_{31} = \lambda_{13} = 0 \\
&\lambda_{32} = -\lambda_{23} \neq 0 \vee \lambda_{32} = \lambda_{23} = 0
\end{aligned} \qquad (68)
$$

$$
\begin{aligned}
&\lambda_{10} = -\lambda_{01} \neq 0 \vee \lambda_{10} = \lambda_{01} = 0 \\
&\lambda_{20} = \lambda_{02} = 0 \\
&\lambda_{30} = -\lambda_{03} \neq 0 \vee \lambda_{30} = \lambda_{03} = 0 \\
&\lambda_{21} = \lambda_{12} = 0 \\
&\lambda_{31} = -\lambda_{13} \neq 0 \vee \lambda_{31} = \lambda_{13} = 0 \\
&\lambda_{32} = -\lambda_{23} \neq 0 \vee \lambda_{32} = \lambda_{23} = 0
\end{aligned} \; . \qquad (69)
$$

In the following paragraphs, the physical content of the physical quantities $\lambda_{ki}, k \neq i, k, i = 0,1,2,3$, as well as of the theorems we proved in this paragraph, emerge.

6. The Unified Selfvariations Interaction (USVI)

According to the law of selfvariations every material particle interacts both with the generalized photons emitted by itself due to the selfvariations, and with the generalized photons originating from other material particles. In the second case, an indirect interaction emerges between material particles through the generalized photons. Generalized photons emitted by one material particle interact with another material particle. Through this mechanism the TSV predicts a unified interaction between material particles. The individual interactions only emerge from the different, for each particular case, physical quantity Q which selfvariates, resulting in the emission of the corresponding generalized photons.

In this paragraph we study the basic characteristics of the USVI. We suppose that for the generalized particle the conservation of energy-momentum holds, hence the equations of the preceding paragraph also hold.

For the rate of change of the four-vector $\frac{1}{m_0} J$ we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right) = -\frac{J_i}{m_0^2}\frac{\partial m_0}{\partial x_k} + \frac{1}{m_0}\frac{\partial J_i}{\partial x_k}$$

and with equations (13) and (17) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right) = -\frac{J_i}{m_0^2}\frac{b}{\hbar}P_k m_0 + \frac{1}{m_0}\left(\frac{b}{\hbar}P_k J_i + \lambda_{ki}\right)$$

and we finally obtain

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right) = \frac{\lambda_{ki}}{m_0},\ k,i = 0,1,2,3. \qquad (70)$$

According to equation (70), when $\lambda_{ki} \neq 0$ for at least two indices $k,i,\ \ k,i = 0,1,2,3,$ the kinetic state of the material particle is disturbed. According to equivalence (51) in the internal symmetry it is $\lambda_{ki} = 0$ for every $k,i = 0,1,2,3.$ Therefore, in the internal symmetry the material particle maintains its kinetic state. In an isotropic space we expect that the spontaneous emission of generalized photons by the material particle cannot disturb its kinetic state. Consequently, the internal symmetry concerns the spontaneous emission of generalized photons by the material particle in an isotropic space.

In contrast, in the case of the external symmetry it can be $\lambda_{ki} \neq 0$ for some indices $k,i,\ \ k,i = 0,1,2,3$. Therefore, the external symmetry must be due to generalized photons with which the material particle interacts, and which originate from other material particles. The distribution of generalized photons depends on the position in space of the material particle relative to other material particles. This leads to the destruction of the isotropy of space for the material particle. The external symmetry factor will emerge in the study that follows.

The initial study of the Selfvariations [5] concerned the rest mass and the electric charge. The study we have presented up to this point allows us to study the Selfvariations in their most general expression.

We consider a physical quantity Q which we shall call selfvariating "charge Q ", or simply charge Q, unaffected by every change of reference frame, therefore Lorentz-Einstein invariant, and obeys the law of Selfvariations, that is equation

$$\frac{\partial Q}{\partial x_k} = \frac{b}{\hbar}P_k Q,\ k = 0,1,2,3. \qquad (71)$$

In equation (71) the momentum $P_k, \mathrm{k} = 0,1,2,3$, i.e. the four-vector P, depends on the selfvariating charge Q. Two material particles carrying a selfvariating charge of the same nature interact with each other when generalized photons emitted by the charge Q_1 of one of them, interact with the charge Q of the other. In this particular case, we denote Q the charge of the material particle we are studying.

The rest mass m_0 is defined as a quantity of mass or energy divided by c^2, which is invariant according to the Lorentz-Einstein transformations. The 4-vector of the momentum J of the material particle is related to the rest mass m_0 through equation (14). The charge Q contributes to the energy content of the material particle and, therefore, also contributes to its rest mass. Furthermore, the charge Q modifies the 4-vector of momentum J of the material particle and, therefore, contributes to the variation of the rest mass m_0 of the material particle. Consequently, for the change of the four-vector J of the material particle due to the charge Q, the four-vector P of equation (71) enters into equation (17). The rest mass m_0 is due to the energy content given to the material particle by the charge Q, and constitutes part, or even the whole, depending on the situation, of the total rest mass of the material particle. The consequences of this conclusion become evident when we calculate the rate of change of the four-vector $\frac{1}{Q}J$.

Third Theorem of the TSV

"The rate of change of the four-vector $\frac{1}{Q}J$ due to the Selfvariations of the charge Q is given by equation

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q}, \quad k,i=0,1,2,3. \qquad (72)$$

For $k \neq i$ the physical quantities $\frac{\lambda_{ki}}{Q}$ are given by

$$\frac{\lambda_{ki}}{Q}=za_{ki}, \quad \mathrm{k} \neq \mathrm{i}, \mathrm{k}, \mathrm{i}=0,1,2,3, \qquad (73)$$

where z is the function

$$z=\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]. \qquad (74)$$

For the constants a_{ki} the following equations hold

$$\begin{aligned} c_i a_{vk}+c_k a_{iv}+c_v a_{ki}&=0\\ J_i a_{vk}+J_k a_{iv}+J_v a_{ki}&=0\\ P_i a_{vk}+P_k a_{iv}+P_v a_{ki}&=0 \end{aligned} \qquad (75)$$

for every $i \neq v, v \neq k, k \neq i,\ i,k,v=0,1,2,3$."

In order to prove the theorem, we take

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=-\frac{J_i}{Q^2}\frac{\partial Q}{\partial x_k}+\frac{1}{Q}\frac{\partial J_i}{\partial x_k}$$

and with equations (71) and (17) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q},$$

which is equation (72).

Equations (17) and (71) hold for every $k, \mathrm{i}=0,1,2,3.$. Therefore, equation (72) also holds for every $k,i=0,1,2,3.$. For $k \neq i,\ k,i=0,1,2,3$ and $v=0,1,2,3$ equation (62) holds and, since $Q \neq 0$, we obtain

$$Q\frac{\partial \lambda_{ki}}{\partial x_v}=\frac{b}{\hbar}P_v Q\lambda_{ki}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

and with equation (71) we get

$$Q\frac{\partial \lambda_{ki}}{\partial x_v}=\lambda_{ki}\frac{\partial Q}{\partial x_v}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

$$\frac{\partial}{\partial x_v}\left(\frac{\lambda_{ki}}{Q}\right)=-\frac{bc_v}{2\hbar}\frac{\lambda_{ki}}{Q}$$

and integrating we obtain

$$\frac{\lambda_{ki}}{Q}=a_{ki}\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right],$$

where $a_{ki}, k\neq i, k, i=0,1,2,3$ are the integration constants, and with (74) we get equation (73).

The relation $a_{ik}=-a_{ki}$ for $k\neq i, k, i=0,1,2,3$, as well as the first of equations (75), result from the combination of equations (61) and (73). To prove the second and third of equations (75) we consider equation (59).

In the following proofs we presuppose the relations $a_{ik}=-a_{ki}$ and $\lambda_{ik}=-\lambda_{ki}, k\neq i, k, i=0,1,2,3$. We will also use equation

$$\frac{\partial z}{\partial x_k}=-\frac{bc_k}{2\hbar}z, k=0,1,2,3, \qquad (76)$$

which results immediately from equation (74).

For $k=i, k, i=0,1,2,3$ equation (73) does not hold. So we define the physical quantities Φ_k as given by equation

$$\Phi_k=\frac{\lambda_{kk}}{Q}, k=0,1,2,3 \ . \qquad (77)$$

Furthermore, we define the 4×4 diagonal matrix Λ given by

$$\Lambda=\begin{bmatrix}\Phi_0 & 0 & 0 & 0\\ 0 & \Phi_1 & 0 & 0\\ 0 & 0 & \Phi_2 & 0\\ 0 & 0 & 0 & \Phi_3\end{bmatrix}. \qquad (78)$$

The physical quantities $\Phi_k, k=0,1,2,3$ are calculated in the following paragraphs, where we will also see their physical content.

We now define the three-vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, as given by equations (79) and (80) respectively

$$\boldsymbol{\alpha}=\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}=\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_z\end{bmatrix}=\frac{1}{Q}\begin{bmatrix}ic\lambda_{01}\\ ic\lambda_{02}\\ ic\lambda_{03}\end{bmatrix} \qquad (79)$$

$$\boldsymbol{\beta}=\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\end{bmatrix}=\begin{bmatrix}\beta_x\\ \beta_y\\ \beta_z\end{bmatrix}=\frac{1}{Q}\begin{bmatrix}\lambda_{32}\\ \lambda_{13}\\ \lambda_{21}\end{bmatrix}. \qquad (80)$$

Vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ contain all of the physical quantities λ_{ki} for $k \neq i, k, i = 0,1,2,3$ s, ince $\lambda_{ik} = -\lambda_{ki}$. Furthermore, from transformations (37), and given that the charge Q remains invariant under Lorentz-Einstein transformations, it emerges that the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are transformed like the intensities of the electric field $\boldsymbol{\varepsilon}$ and of the magnetic field $\mathbf{B}$, respectively.

Combining equations (79) and (80) with equation (73), the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are written in the form of equations (81) and (82), respectively

$$\boldsymbol{\alpha}=\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}=\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_z\end{bmatrix}=icz\begin{bmatrix}\alpha_{01}\\ \alpha_{02}\\ \alpha_{03}\end{bmatrix} \qquad (81)$$

$$\boldsymbol{\beta}=\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\end{bmatrix}=\begin{bmatrix}\beta_x\\ \beta_y\\ \beta_z\end{bmatrix}=z\begin{bmatrix}\alpha_{32}\\ \alpha_{13}\\ \alpha_{21}\end{bmatrix}. \qquad (82)$$

We write equation (17) in the form

$$\frac{\partial J_i}{\partial x_k}=\frac{b}{\hbar}P_k J_i+\lambda_{ki}, \mathrm{k,i}=0,1,2,3. \qquad (83)$$

The rate of change of the momentum of the material particle equals the sum of the two terms in the right part of equation (83). For $k = 0$, and since $x_0 = ict$, equation (83) gives the rate of change of the particle momentum with respect to time t, i.e. the physical quantity we call “force”. By using the concept of force, as defined by Newton, we also have to use the concept of velocity. For this reason we symbolize $\mathbf{u}$ the velocity of the material particle, as given by equation

$$\mathbf{u}=\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}u_x\\ u_y\\ u_z\end{bmatrix}. \qquad (84)$$

Also, we define the 4-vector of the velocity u, as given by equation

$$u=\begin{bmatrix}u_0\\ u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}ic\\ u_x\\ u_y\\ u_z\end{bmatrix}. \qquad (85)$$

We now prove the following theorem:

Fourth Theorem of the TSV

"The rates of change with respect to time $t\left(x_0 = ict\right)$ of the four-vectors J and P of the momentum of the generalized particle carrying charge Q are given by equations

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J - \frac{i}{c} Q\Lambda u - \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} \qquad (86)$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Qdx_0} J + \frac{i}{c} Q\Lambda u + \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix}. \qquad (87)\text{ "}$$

The matrix Λ is given in equation (78). By $\mathbf{u}\times\boldsymbol{\beta}$ we denote the outer product of vectors $\mathbf{u}$ and $\boldsymbol{\beta}$.

We now prove the first of equations (86):

$$\frac{d}{dt}\left(\frac{J_0}{Q}\right) = \frac{\partial}{\partial t}\left(\frac{J_0}{Q}\right) + u_1 \frac{\partial}{\partial x}\left(\frac{J_0}{Q}\right) + u_2 \frac{\partial}{\partial y}\left(\frac{J_0}{Q}\right) + u_3 \frac{\partial}{\partial z}\left(\frac{J_0}{Q}\right)$$

and using the notation of equation (10) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right) = ic\frac{\partial}{\partial x_0}\left(\frac{J_0}{Q}\right) + u_1 \frac{\partial}{\partial x_1}\left(\frac{J_0}{Q}\right) + u_2 \frac{\partial}{\partial x_2}\left(\frac{J_0}{Q}\right) + u_3 \frac{\partial}{\partial x_3}\left(\frac{J_0}{Q}\right)$$

and with equation (72) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right) = ic\frac{\lambda_{00}}{Q} + u_1\frac{\lambda_{10}}{Q} + u_2\frac{\lambda_{20}}{Q} + u_3\frac{\lambda_{30}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right) = \frac{\lambda_{00}}{Q} - \frac{i}{c}\left(u_1\frac{\lambda_{10}}{Q} + u_2\frac{\lambda_{20}}{Q} + u_3\frac{\lambda_{30}}{Q}\right)$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right) = \frac{\lambda_{00}}{Q} + \frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q} + u_2\frac{\lambda_{02}}{Q} + u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{1}{Q}\frac{dJ_0}{dx_0} - \frac{J_0}{Q^2}\frac{dQ}{dx_0} = \frac{\lambda_{00}}{Q} + \frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q} + u_2\frac{\lambda_{02}}{Q} + u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{dJ_0}{dx_0} = \frac{dQ}{Qdx_0} J_0 + \lambda_{00} + \frac{i}{c}\left(u_1\lambda_{01} + u_2\lambda_{02} + u_3\lambda_{03}\right)$$

and with equations (77) and (79) we have

$$\frac{dJ_0}{dx_0} = \frac{dQ}{Qdx_0} J_0 + Q\Phi_0 - \frac{i}{c} Q\left(\frac{i}{c}u_1\alpha_1 + \frac{i}{c}u_2\alpha_2 + \frac{i}{c}u_3\alpha_3\right)$$

which is the first of equations (86) since

$$-\frac{i}{c}Q\Phi_0 u_0 = -\frac{i}{c}Q\Phi_0 ic = Q\Phi_0 \, .$$

We prove the second of equations (86) and we can similarly prove the third and the fourth:

$$\frac{d}{dt}\left(\frac{J_x}{Q}\right) = \frac{\partial}{\partial t}\left(\frac{J_x}{Q}\right) + u_1 \frac{\partial}{\partial x}\left(\frac{J_x}{Q}\right) + u_2 \frac{\partial}{\partial y}\left(\frac{J_x}{Q}\right) + u_3 \frac{\partial}{\partial z}\left(\frac{J_x}{Q}\right)$$

and using the notation of equations (10) and (11) we obtain

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right) = \frac{ic\partial}{\partial x_0}\left(\frac{J_1}{Q}\right) + u_1 \frac{\partial}{\partial x_1}\left(\frac{J_1}{Q}\right) + u_2 \frac{\partial}{\partial x_2}\left(\frac{J_1}{Q}\right) + u_3 \frac{\partial}{\partial x_3}\left(\frac{J_1}{Q}\right)$$

and with equation (72) we get

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right) = ic\frac{\lambda_{01}}{Q} + u_1\frac{\lambda_{11}}{Q} + u_2\frac{\lambda_{21}}{Q} + u_3\frac{\lambda_{31}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_1}{Q}\right) = -\frac{iu_1}{c}\frac{\lambda_{11}}{Q} + \frac{\lambda_{01}}{Q} - \frac{iu_2}{c}\frac{\lambda_{21}}{Q} + \frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{1}{Q}\frac{dJ_1}{dx_0} - \frac{J_1}{Q^2}\frac{dQ}{dx_0} = -\frac{iu_1}{c}\frac{\lambda_{11}}{Q} + \frac{\lambda_{01}}{Q} - \frac{iu_2}{c}\frac{\lambda_{21}}{Q} + \frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{dJ_1}{dx_0} = \frac{dQ}{Qdx_0}J_1 - \frac{iu_1}{c}\lambda_{11} + \lambda_{01} - \frac{iu_2}{c}\lambda_{21} + \frac{iu_3}{c}\lambda_{13}$$

and with equations (77), (79) and (80), we obtain

$$\frac{dJ_1}{dx_0} = \frac{dQ}{Qdx_0}J_1 - \frac{i}{c}Q\Phi_1 - \frac{i}{c}Q\alpha_1 - \frac{i}{c}Q\left(u_2\beta_3 - u_3\beta_2\right)$$

which is the second of equations (86). Equation (87) results from the combination of equations (44) and (86).

Using the symbol $\mathbf{J}$ for the momentum vector of the material particle

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}$$

and taking into account equations (10) and (11), the set of equations (86) can be written in the form

$$
\begin{aligned}
\frac{dW}{dt} &= \frac{dQ}{Qdt}W + Qc^2\Phi_0 + Q\mathbf{u}\cdot\boldsymbol{\alpha} \\
\frac{d\mathbf{J}}{dt} &= \frac{dQ}{Qdt}\mathbf{J} + Q\begin{bmatrix}\Phi_1 u_1 \\ \Phi_2 u_2 \\ \Phi_3 u_3\end{bmatrix} + Q\left(\boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta}\right)
\end{aligned} \quad . \qquad (88)
$$

Equations (88) give the rate of change of the energy W and momentum $\mathbf{J}$ of the material particle with respect to time t. From equation (8) we can calculate the contribution of charge Q to the rate of change of the rest mass m_0 of the material particle with respect to time t.

The rate of change of the four-vector J of the momentum of the material particle is given by the sum of the three terms in the right part of equation (86). The USVI and its consequences for the material particle depend on which of these terms is the strongest and which is the weakest. This can be studied on the basis of the characteristics of each individual term. The third term on the right of equation (86) is known as the Lorentz force, in the case of electromagnetic fields. We now prove the following theorem about the vector pair $(\boldsymbol{\alpha},\boldsymbol{\beta})$.

Fifth Theorem of the TSV

“ For the vector pair $(\boldsymbol{\alpha},\boldsymbol{\beta})$ the following equations hold:

$$
\nabla\cdot\boldsymbol{\alpha} = -\frac{icbz}{2\hbar}\left(c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03}\right) \qquad \text{(a)}
$$

$$
\nabla\cdot\boldsymbol{\beta} = 0 \qquad \text{(b)}
$$

$$
\nabla\times\boldsymbol{\alpha} = -\frac{\partial\boldsymbol{\beta}}{\partial t} \qquad \text{(c)} \qquad (89)
$$

$$
\nabla\times\boldsymbol{\beta} = -\frac{bz}{2\hbar}\begin{bmatrix} c_0\alpha_{01} + c_2\alpha_{21} + c_3\alpha_{31} \\ c_0\alpha_{02} + c_2\alpha_{12} + c_3\alpha_{32} \\ c_0\alpha_{03} + c_2\alpha_{13} + c_3\alpha_{23}\end{bmatrix} + \frac{\partial\boldsymbol{\alpha}}{c^2\partial t} . \qquad \text{(d)}
$$ **”**

Differentiating equations (81) and (82) with respect to x_k, $k = 0,1,2,3$ and considering equation (76), we obtain equations

$$
\frac{\partial\boldsymbol{\alpha}}{\partial x_k} = -\frac{bc_k}{2\hbar}\boldsymbol{\alpha} \qquad (90)
$$

$$
\frac{\partial\boldsymbol{\beta}}{\partial x_k} = -\frac{bc_k}{2\hbar}\boldsymbol{\beta} \quad . \qquad (91)
$$

From equations (90) and (91) we can easily derive equations (89). Indicatively we prove equation (89,b). From equation (82) we obtain

$$\nabla\cdot\boldsymbol{\beta}=\alpha_{32}\frac{\partial z}{\partial x_1}+\alpha_{13}\frac{\partial z}{\partial x_2}+\alpha_{21}\frac{\partial z}{\partial x_3}$$

and with equation (76) we get

$$\nabla\cdot\boldsymbol{\beta}=-\frac{bz}{2\hbar}\left(c_1\alpha_{32}+c_2\alpha_{13}+c_3\alpha_{21}\right)$$

and with the first of equations (75) for $(i,v,k)=(1,3,2)$ we get

$$\nabla\cdot\boldsymbol{\beta}=0\,.$$

From equations (86) and (89) we conclude that the vector pair $(\boldsymbol{\alpha},\boldsymbol{\beta})$ expresses the intensity of the USVI field according to the paradigm of the classical definition of the field potential. From equation (17) it emerges that the physical quantities $\lambda_{ki},k,\mathrm{i}=0,1,2,3$ have units (dimensions) of $kg\cdot s^{-1}$. Thus, from equation (79) it emerges that, if Q plays the role of the rest mass, the intensity $\boldsymbol{\alpha}$ has units of ms^{-2}. If Q is the electric charge, the intensity $\boldsymbol{\alpha}$ has units of NCb^{-1}. Through equations (81) and (82) we can determine the units of the constants α_{ki}, $k\neq i,k,\mathrm{i}=0,1,2,3$, which depend on the nature of the selfvariating charge Q.

From equations (89 b,c) we conclude that the potential is always defined in the $(\boldsymbol{\alpha},\boldsymbol{\beta})$- field of the USVI. That is, the scalar potential

$$V=V\left(t,x,y,z\right)=V\left(x_0,x_1,x_2,x_3\right)$$

and the vector potential $\mathbf{A}$

$$\mathbf{A}=\mathbf{A}\left(t,x,y,z\right)=\mathbf{A}\left(x_0,x_1,x_2,x_3\right)=\begin{bmatrix}A_1\\A_2\\A_3\end{bmatrix}=\begin{bmatrix}A_x\\A_y\\A_z\end{bmatrix}$$

are defined through the equations

$$\begin{aligned}&\boldsymbol{\beta}=\nabla\times\mathbf{A}\\&\boldsymbol{\alpha}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t}=-\nabla V-\frac{ic\partial\mathbf{A}}{\partial x_0}\,.\end{aligned}$$

We can introduce in the above equations the gauge function f. That is, we can add to the scalar potential V the term

$$-\frac{\partial f}{\partial t}=-\frac{ic\partial f}{\partial x_0}$$

and to the vector potential $\mathbf{A}$ the term

$$\nabla f$$

for an arbitrary function f

$$f = f(t, x, y, z) = f(x_0, x_1, x_2, x_3)$$

without changing the intensity $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of the field. The proof of the above equations is known and trivial [13-17] and we will not repeat it here. For the field potential of the USVI the following theorem holds:

Sixth Theorem of the TSV

"In the $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-field of USVI the pair of scalar-vector potentials $(V, \mathbf{A})$ is always defined through equations

$$\begin{aligned} &\boldsymbol{\beta} = \nabla \times \mathbf{A} \\ &\boldsymbol{\alpha} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = ic\nabla A_0 - \frac{ic\partial \mathbf{A}}{\partial x_0}. \end{aligned} \qquad (92)$$

The four-vector A of the potential

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} \frac{iV}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix} \qquad (93)$$

is given by equation

$$A_i = \begin{cases} \frac{2\hbar}{b} \frac{\alpha_{ki}}{c_k} z + \frac{\partial f_k}{\partial x_i}, \text{for } i \neq k \\ \frac{\partial f_k}{\partial x_i}, \text{for } i = k \end{cases} \qquad (94)$$

where $c_k \neq 0, k, i = 0,1,2,3$ and f_k is the gauge function."

Equations (92) are equivalent to equations (89) as we have already mentioned. The proof of equation (94) can be performed through the first of equations (75)

$$\begin{aligned} &c_i a_{vk} + c_k a_{iv} + c_v a_{ki} = 0 \\ &i \neq v, v \neq k, k \neq i, \ i, k, v = 0,1,2,3 \end{aligned}$$

of the third theorem of the TSV. The mathematical calculations do not contribute anything useful to our study, thus we omit them. You can verify that the potential of equation (94) gives equations (81) and (82) through equations (92) taking also into account the first of equations (75).

According to relation (60) it is $c_k \neq 0$ for at least one of the indices $k = 0,1,2,3$. So, from equation (94) the following four sets of the potentials follow:

$$\begin{aligned} & c_0 \neq 0 \\ & A_0 = \frac{\partial f_0}{\partial x_0} \\ & A_1 = \frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3} \end{aligned} \tag{95}$$

$$\begin{aligned} & c_1 \neq 0 \\ & A_0 = \frac{2\hbar z}{b}\frac{\alpha_{10}}{c_1} + \frac{\partial f_1}{\partial x_0} \\ & A_1 = \frac{\partial f_1}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_1}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{13}}{c_1} + \frac{\partial f_1}{\partial x_3} \end{aligned} \tag{96}$$

$$\begin{aligned} & c_2 \neq 0 \\ & A_0 = \frac{2\hbar z}{b}\frac{\alpha_{20}}{c_2} + \frac{\partial f_2}{\partial x_0} \\ & A_1 = \frac{2\hbar z}{b}\frac{\alpha_{21}}{c_2} + \frac{\partial f_2}{\partial x_1} \\ & A_2 = \frac{\partial f_2}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{23}}{c_2} + \frac{\partial f_2}{\partial x_3} \end{aligned} \tag{97}$$

$$c_3 \neq 0$$

$$\begin{aligned} A_0 &= \frac{2\hbar z}{b}\frac{\alpha_{30}}{c_3} + \frac{\partial f_3}{\partial x_0} \\ A_1 &= \frac{2\hbar z}{b}\frac{\alpha_{31}}{c_3} + \frac{\partial f_3}{\partial x_1} \\ A_2 &= \frac{2\hbar z}{b}\frac{\alpha_{32}}{c_3} + \frac{\partial f_3}{\partial x_2} \\ A_3 &= \frac{\partial f_3}{\partial x_3} \end{aligned} \quad . \qquad (98)$$

Indicatively, we calculate the components α_1 and β_1 of the intensity $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of the USVI field from the potentials (95). From the second of equations (92) we obtain

$$\alpha_1 = ic\left(\frac{\partial A_0}{\partial x_1} - \frac{\partial A_1}{\partial x_0}\right)$$

and with equations (95) we get

$$\alpha_1 = ic\left[\frac{\partial}{\partial x_1}\left(\frac{\partial f_0}{\partial x_0}\right) - \frac{\partial}{\partial x_0}\left(\frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1}\right)\right]$$

$$\alpha_1 = -ic\frac{2\hbar}{b}\frac{\alpha_{01}}{c_0}\frac{\partial z}{\partial x_0}$$

and with equation (76) we get

$$\alpha_1 = icz\alpha_{01}$$

that is we get the intensity α_1 of the field, as given by equation (81).

From the first of equations (92) we have

$$\beta_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}$$

and with equations (95) we get

$$\beta_1 = \frac{\partial}{\partial x_2}\left(\frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3}\right) - \frac{\partial}{\partial x_3}\left(\frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2}\right)$$

$$\beta_1 = \frac{2\hbar}{b}\frac{\alpha_{03}}{c_0}\frac{\partial z}{\partial x_2} - \frac{2\hbar}{b}\frac{\alpha_{02}}{c_0}\frac{\partial z}{\partial x_2}$$

and with equation (76) we get

$$\beta_1 = -\frac{c_2\alpha_{03}}{c_0}z + \frac{c_3\alpha_{02}}{c_0}z$$

and considering that $\alpha_{02} = -\alpha_{20}$, we get

$$\beta_1 = -\frac{z}{c_0}\left(c_2\alpha_{03} + c_3\alpha_{20}\right). \qquad (99)$$

From the first of equations (75) for $(i,v,k)=(2,0,3)$ we obtain

$$c_2a_{03}+c_3a_{20}+c_0a_{32}=0$$
$$c_2a_{03}+c_3a_{20}=-c_0a_{32}$$

and substituting into equation (99), we see that

$$\beta_1=z\alpha_{32}$$

that is, we get the intensity β_1 of the field, as given by equation (82).

The gauge functions $f_k, \mathrm{k}=0,1,2,3$ in equations (95)-(98) are not independent of each other. For $c_k\neq 0$ and $c_i\neq 0$ for $k\neq i, k,i=0,1,2,3$ equation (100) holds

$$f_k = f_i+\frac{4\hbar^2 z}{b^2}\frac{\alpha_{ki}}{c_kc_i}, c_kc_i\neq 0, k\neq i,\ k,i=0,1,2,3. \qquad (100)$$

The proof of equation (100) is through the first of equations (75). The proof is lengthy and we omit it. Indicatively, we will prove the third of equations (95) from the third of equations (96) for $k=1$ and $i=0$ in equation (100).

For $c_0\neq 0$ and $c_1\neq 0$ both equations (95) and equations (96) hold. From equation (100) for $k=1$ and $i=0$ we get equation

$$f_1 = f_0+\frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0c_1} \qquad (101)$$

From the third of equations (96) and equation (101) we get

$$A_2=\frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1}+\frac{\partial}{\partial x_2}\left(f_0+\frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0c_1}\right)$$
$$A_2=\frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1}+\frac{\partial f_0}{\partial x_2}+\frac{4\hbar^2}{b^2}\frac{\alpha_{10}}{c_0c_1}\frac{\partial z}{\partial x_2}$$

and with equation (76) we obtain

$$A_2=\frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1}+\frac{\partial f_0}{\partial x_2}-\frac{2\hbar z}{b}\frac{c_2\alpha_{10}}{c_0c_1}$$
$$A_2=\frac{2\hbar z}{bc_0c_1}\left(c_0\alpha_{12}-c_2\alpha_{10}\right)+\frac{\partial f_0}{\partial x_2}$$

and since $\alpha_{10}=-\alpha_{01}$, we get equation

$$A_2=\frac{2\hbar z}{bc_0c_1}\left(c_0\alpha_{12}+c_2\alpha_{01}\right)+\frac{\partial f_0}{\partial x_2}. \qquad (102)$$

From the first of equations (75) for $(i,v,k)=(0,1,2)$ we obtain

$$c_0a_{12}+c_2a_{01}+c_1a_{20}=0$$
$$c_0a_{12}+c_2a_{01}=-c_1a_{20}$$
$$c_0a_{12}+c_2a_{01}=c_1a_{02}$$

and substituting into equation (102) we obtain equation

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2}. \quad (103)$$

Equation (103) is the third of equations (95).

According to equation (100), if $c_k \neq 0$ for more than one of the constants $c_k, k = 0,1,2,3$, the sets of equations of potential resulting from equation (94) have in the end a gauge function. In the application we presented assuming $c_0 \neq 0$ and $c_1 \neq 0$ for a specific gauge function f_0 in equations (95), the gauge function f_1 in equations (96) is given by equation (101).

7. The main diagonal of the T matrix

In this paragraph we study the elements of the main diagonal of the matrix T of equation (19), that is, the elements of the matrix Λ of equation (78). Since $z \neq 0$ we define the physical quantities $T_k, k = 0,1,2,3$

$$T_k = \frac{\Phi_k}{z}$$

and we write the physical quantities Φ_k in the form

$$\Phi_k = zT_k, k = 0,1,2,3. \qquad (104)$$

We expand equation (18) for $k = 0,1,2,3$ and get

$$\begin{aligned} J_0\lambda_{00} + J_1\lambda_{01} + J_2\lambda_{02} + J_3\lambda_{03} &= 0 \\ J_0\lambda_{10} + J_1\lambda_{11} + J_2\lambda_{12} + J_3\lambda_{13} &= 0 \\ J_0\lambda_{20} + J_1\lambda_{21} + J_2\lambda_{22} + J_3\lambda_{23} &= 0 \\ J_0\lambda_{30} + J_1\lambda_{31} + J_2\lambda_{32} + J_3\lambda_{33} &= 0 \end{aligned}.$$

Considering equations (73), (77), and (104), and that $\lambda_{ik} = -\lambda_{ki}$ for every $k \neq i, k, i = 0,1,2,3$, we get

$$\begin{aligned} J_0 zQT_0 + J_1 zQ\alpha_{01} + J_2 zQ\alpha_{02} + J_3 zQ\alpha_{03} &= 0 \\ -J_0 zQ\alpha_{01} + J_1 zQT_1 - J_2 zQ\alpha_{21} + J_3 zQ\alpha_{13} &= 0 \\ -J_0 zQ\alpha_{02} + J_1 zQ\alpha_{21} + J_2 zQT_2 - J_3 zQ\alpha_{32} &= 0 \\ -J_0 zQ\alpha_{03} - J_1 zQ\alpha_{13} + J_2 zQ\alpha_{32} + J_3 zQT_3 &= 0 \end{aligned}$$

and since $zQ \neq 0$, we get

$$\begin{aligned} J_0 T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} &= 0 \\ -J_0\alpha_{01} + J_1 T_1 - J_2\alpha_{21} + J_3\alpha_{13} &= 0 \\ -J_0\alpha_{02} + J_1\alpha_{21} + J_2 T_2 - J_3\alpha_{32} &= 0 \\ -J_0\alpha_{03} - J_1\alpha_{13} + J_2\alpha_{32} + J_3 T_3 &= 0 \end{aligned}. \qquad (105)$$

Equations (105) comprise a 4×4 homogeneous linear system of equations with the momenta J_0, J_1, J_2, J_3. as unknowns. Therefore, it always has the trivial solution

$$(J_0, J_1, J_2, J_3) = (0,0,0,0).$$

In this case, from equation (14) we get $m_0 = 0$.

We study the case $m_0 \neq 0$ and are, therefore, interested in the non-zero solutions

$$(J_0, J_1, J_2, J_3) \neq (0,0,0,0)$$

of the system of equations (105). We prove that one case where the system of equations (105) has non-zero solutions is when equation (106) holds:

$$T_0 = T_1 = T_2 = T_3 = 0. \quad (106)$$

In this case, equations (105) are written in the form of equations

$$\begin{aligned} 0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \quad (a) \\ -J_0\alpha_{01} + 0 - J_2\alpha_{21} + J_3\alpha_{13} = 0 \quad (b) \\ -J_0\alpha_{02} + J_1\alpha_{21} + 0 - J_3\alpha_{32} = 0 \quad (c) \\ -J_0\alpha_{03} - J_1\alpha_{13} + J_2\alpha_{32} + 0 = 0 \quad (d) \end{aligned}. \quad (107)$$

By performing the necessary calculations we get the determinant D of the system of equations (107) in the form

$$D = \left(\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21}\right)^2. \qquad (108)$$

Considering equation (73) and that $zQ \neq 0$, we get

$$\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21} = \frac{1}{zQ}\left(\lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21}\right)$$

and with equation (59) we arrive at

$$\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21} = 0. \quad (109)$$

From equations (108) and (109) we obtain $D = 0$. Therefore, the homogeneous linear system of equations (107) has non-zero solutions.

In the case when equations (106) hold, we obtain from equation (77)

$$\Phi_0 = \Phi_1 = \Phi_2 = \Phi_3 = 0$$

and from equation (78) we get $\Lambda = 0$. Therefore, the second term on the right side of equations (86) and (87) of the USVI vanishes when equations (106) hold.

The elements of the main diagonal of matrix T and, equivalently, the physical quantities $T_k, k=0,1,2,3$, have a specific physical content. As we will see, they are related to the curvature of the part of spacetime occupied by the generalized particle.

Applying the Lorentz-Einstein transformations for the physical quantities $\lambda_{ki}, \mathrm{k}=0,1,2,3$ we derive equation (29), $\lambda_{00}=\lambda_{11}$. The reference frame $\mathrm{O}'(t',x',y',z')$ of paragraph 4 moves with respect to the reference frame $\mathrm{O}(t,x,y,z)$ with constant velocity along the x-axis. If we assume that the motion is along the y- or z-axis, the generalization of equation (29) follows; the Lorentz-Einstein transformations lead to the following equation

$$\lambda_{00}=\lambda_{11}=\lambda_{22}=\lambda_{33}=0.$$

We also arrive at this equation from the Lorentz-Einstein transformations of equations (105). The function z and the charge Q are invariant, therefore from equation (73) we conclude that the physical quantities α_{ki} and $\lambda_{ki}, k\neq i, k,i=0,1,2,3$ transform in the same manner according to Lorentz-Einstein. Applying the transformations (26) and (37) on equations (105) we again arrive at

$$\lambda_{00}=\lambda_{11}=\lambda_{22}=\lambda_{33}=0.$$

This is not a transformation equation of the physical quantities $\lambda_{kk}, k=0,1,2,3$ between two inertial reference frames. It is an equation relating the elements of the main diagonal of matrix T in the same inertial frame of reference. Thus, taking into account equations (77) and (104), we obtain equation (110) when the Lorentz-Einstein transformations hold

$$T_0=T_1=T_2=T_3. \qquad (110)$$

In equations (105) at least one of the momenta $J_k, \mathrm{k}=0,1,2,3$ is non-zero. Let it be $J_0\neq 0$, then from equations (105 b,c,d) we get

$$\alpha_{01}=\frac{1}{J_0}\left(J_1T_1-J_2\alpha_{21}+J_3\alpha_{13}\right)$$
$$\alpha_{02}=\frac{1}{J_0}\left(J_1\alpha_{21}+J_2T_2-J_3\alpha_{32}\right)$$
$$\alpha_{03}=\frac{1}{J_0}\left(-J_1\alpha_{13}+J_2\alpha_{32}+J_3T_3\right)$$

and substituting into equation (105 a) we get

$$J_0^2T_0+J_1\left(J_1T_1-J_2\alpha_{21}+J_3\alpha_{13}\right)+J_2\left(J_1\alpha_{21}+J_2T_2-J_3\alpha_{32}\right)+$$
$$J_3\left(-J_1\alpha_{13}+J_2\alpha_{32}+J_3T_3\right)=0$$

and after the calculations we get

$$J_0^2T_0+J_1^2T_1+J_2^2T_2+J_3^2T_3=0. \qquad (111)$$

We arrive at the same equation no matter which of the momentum components $J_k, k = 0,1,2,3$ we consider different from zero.

The Lorentz-Einstein transformations lead to equation (110)

$$T_0 = T_1 = T_2 = T_3$$

so from equation (111) we obtain

$$\mathrm{T}_0\left(J_0^2 + J_1^2 + J_2^2 + J_3^2\right) = 0$$

and with equation (14) we get

$$-T_0 m_0^2 c^2 = 0$$

and since $m_0 \neq 0$ we have $T_0 = 0$, and finally with equation (110) we get

$$T_0 = T_1 = T_2 = T_3 = 0 .$$

Thus, we arrive at the following two conclusions:

“When the Lorentz-Einstein transformations hold for the physical quantities $\lambda_{ki}, \mathrm{k}, i = 0,1,2,3$, then the physical quantities $T_k, \mathrm{k} = 0,1,2,3$ vanish.

$$T_0 = T_1 = T_2 = T_3 = 0, \quad (112)$$ ”

“When

$$T_k \neq 0 \qquad (113)$$

for at least one of the physical quantities

$$T_k, \mathrm{k} \in \{0,1,2,3\},$$

the Lorentz-Einstein transformations do not hold for the physical quantities $\lambda_{ki}, \mathrm{k}, i = 0,1,2,3$.”

From the above we conclude that if relation (113) is valid, then the part of spacetime occupied by the generalized particle cannot be flat, it is curved. Furthermore, from equation (78) we obtain $\Lambda \neq 0$. Therefore, when relation (113) holds, the second term on the right side of equations (86) and (87) is non-zero. This term of the USVI is related to the curvature of spacetime.

In the present study the rest mass m_0 of the material particle is given by the equivalent equations (8) and (14) of special relativity. This is why the Lorentz-Einstein transformations hold for the physical quantities $\lambda_{ki}, \mathrm{k}, i = 0,1,2,3$, as given in equations (37). Therefore, we expect equation (112) to hold, which we also prove:

Combining equations (59) and (73) we get

$$\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = zQ\alpha_{ki}, k \neq i, k, i = 0,1,2,3$$

and we finally get

$$c_k J_i = c_i J_k - \frac{2\hbar zQ\alpha_{ki}}{b}, k \neq i, k, i = 0,1,2,3. \qquad (114)$$

In equation (114) at least one of the physical quantities $c_k, \mathrm{k} = 0,1,2,3$ is not zero according to relation (60). We prove equation (112) for $c_0 \neq 0$, and the proof is similar for $c_k \neq 0$, with $\mathrm{k} \in \{0,1,2,3\}$. From equation (114) for $k = 0$ and $c_0 \neq 0$, we get

$$J_i = \frac{c_i}{c_0} J_0 - \frac{2\hbar zQ\alpha_{0i}}{bc_0}, i = 1,2,3. \qquad (115)$$

Differentiating equation (115) with respect to x_0, and considering equations (17) and (73), (76), and (71) we get

$$\frac{b}{\hbar} P_0 J_i + zQ\alpha_{0i} = \frac{c_i}{c_0}\left(\frac{b}{\hbar} P_0 J_0 + zQT_0\right) - \frac{2\hbar\alpha_{0i}}{bc_0}\left(-\frac{bc_0}{2\hbar} zQ + \frac{b}{\hbar} P_0 zQ\right)$$

and with equation (115) we have

$$zQ\alpha_{0i} = \frac{c_i}{c_0} zQT_0 - \frac{2\hbar\alpha_{0i}}{bc_0}\left(-\frac{bc_0}{2\hbar} zQ\right)$$

and since $zQ \neq 0$ we get

$$c_i T_0 = 0, i = 1,2,3. \qquad (116)$$

We differentiate equation (115) with respect to x_i, $i = 1,2,3$, and taking into account equations (17) and (73), (76), and (71), we get

$$\frac{b}{\hbar} P_i J_i + zQT_i = \frac{c_i}{c_0}\left(\frac{b}{\hbar} P_i J_0 + zQa_{i0}\right) - \frac{2\hbar\alpha_{0i}}{bc_0}\left(-\frac{bc_i}{2\hbar} zQ + \frac{b}{\hbar} P_i zQ\right)$$

and with equation (115) we get

$$zQT_i = \frac{c_i}{c_0} zQa_{i0} - \frac{2\hbar\alpha_{0i}}{bc_0}\left(-\frac{bc_i}{2\hbar} zQ\right)$$

and since $zQ \neq 0$, we obtain

$$T_i = \frac{c_i}{c_0} a_{i0} + \frac{c_i}{c_0} a_{0i}$$

$$T_i = -\frac{c_i}{c_0} a_{0i} + \frac{c_i}{c_0} a_{0i}$$

so we get

$$T_i = 0,\ i = 1,2,3\,.$$

According to equation (44), the Lorentz-Einstein transformation of the physical quantities $c_k, k = 0,1,2,3$ is given by equations (26). From the transformations given in (26) it is easily verified that if in the inertial frame of reference $\mathrm{O}'(t',x',y',z')$ it is $c_1^{'} = c_2^{'} = c_3^{'} = 0$, then in the frame of reference $\mathrm{O}(t,x,y,z)$ it is $(c_0,c_1,c_2,c_3) = (0,0,0,0)$. Therefore, in equation (116) at least one of the physical quantities $c_i, i = 1,2,3$ is non-zero, thus $T_0 = 0$.

Combining equations (17) and (44) we obtain

$$\lambda_{ki} = \frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar}\left(c_k - J_k\right) J_i, k, i = 0,1,2,3. \qquad (117)$$

In equation (117) the rest mass m_0 does not appear. By defining the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ through equation (117), we bypass the special relativity equation (14). Therefore, starting from equation (117) we can study the consequences of the Selfvariations for any relation between the momenta $J_k, k = 0,1,2,3$ and the rest mass m_0 of the particle, which is not necessarily given by equation (14) of special relativity. In these cases the Lorentz-Einstein transformations do not necessarily hold and, therefore, the same will be true of equation (112).

8. The equation of the TSV regarding the corpuscular structure of matter

In the equations we have presented in the previous paragraphs, some physical quantities behave as “real numbers”, and some as “complex numbers”. By dividing these physical quantities with others of the same dimension, we can introduce complex numbers into the equations of the TSV. For instance, we can state equation (14) in the form

$$\left(\frac{J_0}{m_0 c}\right)^2 + \left(\frac{J_1}{m_0 c}\right)^2 + \left(\frac{J_2}{m_0 c}\right)^2 + \left(\frac{J_3}{m_0 c}\right)^2 + 1 = 0$$

$$\frac{J_k}{m_0 c} \in \mathbb{C}, k = 0,1,2,3\,.$$

The introduction of complex numbers into the equations of the TSV is not necessary as long as we keep in mind that some sums of squares of the TSV are equal to zero.

We give one more example related to the study in this paragraph. From the first of equations (75) for $(i,v,k) = (0,2,1)$ we obtain equation

$$c_0 a_{21} + c_1 a_{02} + c_2 a_{10} = 0$$

and since $a_{10} = -a_{01}$, we get

$$c_0 a_{21} + c_1 a_{02} - c_2 a_{01} = 0. \qquad (118)$$

From equations (11), (12) and (44) we obtain

$$c_0 = \frac{i}{c}\left(W + E_s\right)$$

and if we suppose that $W + E_s$ is a "real number", c_0 is a "complex number". Therefore, in equation (118), the physical quantities $c_1, c_2, a_{21}, a_{02}, a_{01}$ cannot all be "real numbers".

The physical quantities c_k and a_{ki}, $k \neq i$, $k,i = 0,1,2,3$ generally behave as "complex or hypercomplex numbers". For physical quantities a_{ki}, $k \neq i$, $k,i = 0,1,2,3$ there is a sum of squares that equals zero. We determine this sum in this paragraph.

We consider the 4×4 matrices M and N, as given by equations (119) and (120):

$$M = \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix} \qquad (119)$$

$$N = \begin{bmatrix} 0 & \alpha_{32} & \alpha_{13} & \alpha_{21} \\ -\alpha_{32} & 0 & -\alpha_{03} & \alpha_{02} \\ -\alpha_{13} & \alpha_{03} & 0 & -\alpha_{01} \\ -\alpha_{21} & -\alpha_{02} & \alpha_{01} & 0 \end{bmatrix}. \qquad (120)$$

Using matrix N, equations (75) are written in the form

$$NC = NJ = NP = 0. \quad (121)$$

From equations (121) we obtain

$$N^2 C = N^2 J = N^2 P = 0. \qquad (122)$$

Also, performing the calculations and considering equation (109), we obtain equations

$$\begin{aligned} MN &= NM = 0 \\ |M| &= |N| = 0 \end{aligned}. \qquad (123)$$

We now prove the following theorem:

Seventh Theorem of the TSV

"For the matrices M and N the following hold:

$$M^2 + N^2 = -a^2 I \tag{124}$$

$$a^2 = a_{01}^2 + a_{02}^2 + a_{03}^2 + a_{32}^2 + a_{13}^2 + a_{21}^2 \tag{125}$$

where I is the 4×4 unit matrix.

For $\alpha \neq 0$, matrix M has two eigenvalues τ_1 and τ_2 with corresponding eigenvectors v_1 and v_2 given by equations

$$\begin{gathered}\tau_1 = ia \\ v_1 = \frac{1}{a}\begin{pmatrix} 0 \\ a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} - \frac{i}{a^2}\begin{pmatrix} a_{01}^2 + a_{02}^2 + a_{03}^2 \\ a_{03}a_{13} - a_{02}a_{21} \\ a_{01}a_{21} - a_{03}a_{32} \\ a_{02}a_{32} - a_{01}a_{13} \end{pmatrix} \\ \tau_2 = -ia \\ v_2 = \frac{1}{a}\begin{pmatrix} 0 \\ a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} + \frac{i}{a^2}\begin{pmatrix} a_{01}^2 + a_{02}^2 + a_{03}^2 \\ a_{03}a_{13} - a_{02}a_{21} \\ a_{01}a_{21} - a_{03}a_{32} \\ a_{02}a_{32} - a_{01}a_{13} \end{pmatrix}\end{gathered} . \tag{126}$$

For $\alpha \neq 0$, matrix N has the same eigenvalues τ_1 and τ_2 with matrix M, and the corresponding eigenvectors n_1 and n_2 are given by equations

$$\begin{gathered}\tau_1 = ia \\ n_1 = \frac{1}{a}\begin{pmatrix} 0 \\ a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} - \frac{i}{a^2}\begin{pmatrix} a_{32}^2 + a_{13}^2 + a_{21}^2 \\ a_{02}a_{21} - a_{03}a_{13} \\ a_{03}a_{32} - a_{01}a_{21} \\ a_{01}a_{13} - a_{02}a_{32} \end{pmatrix} \\ \tau_2 = -ia \\ n_2 = \frac{1}{a}\begin{pmatrix} 0 \\ a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} + \frac{i}{a^2}\begin{pmatrix} a_{32}^2 + a_{13}^2 + a_{21}^2 \\ a_{02}a_{21} - a_{03}a_{13} \\ a_{03}a_{32} - a_{01}a_{21} \\ a_{01}a_{13} - a_{02}a_{32} \end{pmatrix}\end{gathered} . \tag{127}$$

For the physical quantities a_{ki}, $k \neq i$, $k, i = 0,1,2,3$ equation (128) holds:

$$a^2 = a_{01}^2 + a_{02}^2 + a_{03}^2 + a_{32}^2 + a_{13}^2 + a_{21}^2 = 0 . \qquad (128)$$ ”

Matrices M and N are given by equations (119) and (120). The proof of equations (124), (125), (126) and (127) is done through the proper mathematical calculations and the use of equation (109).

We now multiply equation (124) from the right with the column-matrices C, J and P and obtain equations

$$M^2C + N^2C = -a^2C$$

$$M^2J + N^2J = -a^2J$$

$$M^2P + N^2P = -a^2P$$

and with equation (122) we get

$$\begin{gathered} M^2C = -a^2C \\ M^2J = -a^2J \, . \\ M^2P = -a^2P \end{gathered} \quad (129)$$

From equations (129) we conclude that, for $\alpha \neq 0$, matrix M^2 obtains the eigenvalue $\tau = -\alpha^2$, with the four-vectors C, J and P being parallel to the corresponding eigenvector v of matrix M^2. Therefore, for $\alpha \neq 0$, the four-vectors C, J and P are parallel to each other, which is impossible in the external symmetry according to the internal symmetry theorem. Therefore, $\alpha = 0$, in order for matrix M^2 to not have the eigenvector v. Thus, we obtain equation (128).

Equation (128) highlights the factors on which the rest masses m_0 and $\frac{\mathrm{E}_0}{c^2}$, as well as the total rest mass M_0 of the generalized particle, depend. For the determination of these factors we consider the three-vectors $\mathbf{C}$, $\mathbf{J}$ and $\mathbf{P}$, as given by equations (130), (131), and (132), respectively

$$\mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \qquad (130)$$

$$\mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} \qquad (131)$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} . \qquad (132)$$

Equations (75) for $(i,v,k)=(1,2,3)$ express the orthogonality of vectors $\mathbf{C}$, $\mathbf{J}$ and $\mathbf{P}$ with vector $\boldsymbol{\beta}$. We prove the orthogonality between vectors $\mathbf{C}$ and $\boldsymbol{\beta}$, and one can similarly prove the orthogonality of vectors $\mathbf{J}$ and $\mathbf{P}$ with vector $\boldsymbol{\beta}$.

From the first of equations (75) for $(i,v,k)=(1,2,3)$ we obtain

$$c_1 a_{32}+c_2 a_{13}+c_3 a_{21}=0$$

and from equations (82) and (130), the orthogonality between the vectors $\mathbf{C}$ and $\boldsymbol{\beta}$ emerges.

From equations (81), (82) and (109) we conclude that the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are orthogonal. Therefore, the vectors $\mathbf{C}$, $\mathbf{J}$, $\mathbf{P}$ and $\boldsymbol{\alpha}$ belong to the same plane Π. Plane Π is orthogonal to the vector $\boldsymbol{\beta}$.

We now consider the directional angles γ, δ and ν with direction from the first vector to the second, as given by equations

$$\begin{aligned}\gamma &= \left(\mathbf{C}\hat{,}\boldsymbol{\alpha}\right)\\ \delta &= \left(\mathbf{J}\hat{,}\boldsymbol{\alpha}\right). \qquad (133)\\ \nu &= \left(\mathbf{P}\hat{,}\boldsymbol{\alpha}\right)\end{aligned}$$

Considering that the vectors $\mathbf{C}$, $\mathbf{J}$, $\mathbf{P}$ and $\boldsymbol{\alpha}$ belong to the same plane Π, and equations (44) and (130), (131) and (132), we obtain equations

$$\mathbf{J}+\mathbf{P}=\mathbf{C} \qquad (134)$$

$$\frac{\sin(\nu-\gamma)}{\|\mathbf{J}\|}=\frac{\sin(\gamma-\delta)}{\|\mathbf{P}\|}=\frac{\sin(\nu-\delta)}{\|\mathbf{C}\|},$$

for $\|\mathbf{J}\|,\|\mathbf{P}\|,\|\mathbf{C}\|\neq 0,$ where $\|\boldsymbol{\alpha}\|$ is defined as $\|\boldsymbol{\alpha}\|=\left(\boldsymbol{\alpha}^T\boldsymbol{\alpha}\right)^{\frac{1}{2}}$.

From the pairs of equations (44), (130), as well as (14), (131), and (15), (132), we obtain equations

$$\begin{aligned}\|\mathbf{C}\| &= (-c_0^2-M_0^2c^2)^{\frac{1}{2}}\\ \|\mathbf{J}\| &= (-J_0^2-m_0^2c^2)^{\frac{1}{2}}\,|. \quad (135)\\ \|\mathbf{P}\| &= (-P_0^2-\frac{E_0^2}{c^2})^{\frac{1}{2}}\end{aligned}$$

We now prove that for $c_0\neq 0$, the following equations hold

$$\begin{aligned}M_0 c\sin\gamma &= \pm c_0\cos\gamma\\ m_0 c\sin\delta &= \pm J_0\cos\delta. \qquad (136)\end{aligned}$$

$$\mathrm{E}_0 \sin\nu = \pm cP_0 \cos\nu$$

We show the proof of the first of the above equations, since the proof of the other two is along similar lines. From the first of equations (75) for $(i,\nu,k)=(0,2,3)$, $(0,1,3)$, $(0,1,2)$, and $c_0 \neq 0$, we obtain equations

$$a_{32} = \frac{1}{c_0}(c_3\alpha_{02} - c_2 a_{03})$$

$$a_{13} = \frac{1}{c_0}(c_1\alpha_{03} - c_3 a_{01})$$

$$a_{21} = \frac{1}{c_0}(c_2\alpha_{01} - c_1 a_{02})$$

and replacing in equation (128) we get

$$a_{01}^2 + a_{02}^2 + a_{03}^2 + \frac{1}{c_0^2}(c_3\alpha_{02} - c_2 a_{03})^2 + \frac{1}{c_0^2}(c_1\alpha_{03} - c_3 a_{01})^2 + \frac{1}{c_0^2}(c_2\alpha_{01} - c_1 a_{02})^2 = 0$$

$$(a_{01}^2 + a_{02}^2 + a_{03}^2)c_0^2 + c_3^2\alpha_{02}^2 + c_2^2 a_{03}^2 + c_1^2\alpha_{03}^2 + c_3^2\alpha_{01}^2 + c_2^2\alpha_{01}^2 + c_1^2\alpha_{02}^2 - 2c_2c_3\alpha_{02}\alpha_{03} - 2c_1c_3\alpha_{01}\alpha_{03} - 2c_1c_2\alpha_{01}\alpha_{02} = 0$$

$$a_{01}^2\left(c_0^2 + c_2^2 + c_3^2\right) + a_{02}^2\left(c_0^2 + c_1^2 + c_3^2\right) + a_{03}^2\left(c_0^2 + c_1^2 + c_2^2\right) - 2c_2c_3\alpha_{02}\alpha_{03} - 2c_1c_3\alpha_{01}\alpha_{03} - 2c_1c_2\alpha_{01}\alpha_{02} = 0$$

and with equation (45) we get

$$a_{01}^2\left(-M_0^2\,\mathrm{c}^2 - c_1^2\right) + a_{02}^2\left(-M_0^2\,\mathrm{c}^2 - c_2^2\right) + a_{03}^2\left(-M_0^2\,\mathrm{c}^2 - c_3^2\right) - 2c_2c_3\alpha_{02}\alpha_{03} - 2c_1c_3\alpha_{01}\alpha_{03} - 2c_1c_2\alpha_{01}\alpha_{02} = 0$$

and we finally obtain

$$M_0^2\,\mathrm{c}^2\left(a_{01}^2 + a_{02}^2 + a_{03}^2\right) + \left(c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03}\right)^2 = 0. \qquad (137)$$

We now prove relation

$$\left(a_{01}, a_{02}, a_{03}\right) \neq \left(0,0,0\right). \ (138)$$

From the first of equations (75) for $(i,\nu,k)=(0,3,2)$, $(0,1,3)$, $(0,2,1)$ we obtain equations

$$c_0 a_{32} + c_2 a_{03} - c_3 a_{02} = 0$$

$$c_0 a_{13} + c_3 a_{01} - c_1 a_{03} = 0$$

$$c_0 a_{21} + c_1 a_{02} - c_2 a_{01} = 0$$

and supposing that

$$\left(a_{01}, a_{02}, a_{03}\right) = (0,0,0)$$

we get

$$c_0 a_{32} = 0$$

$$c_0 a_{13} = 0$$

$$c_0 a_{21} = 0$$

and because $c_0 \neq 0$, it is also

$$\left(a_{32}, a_{13}, a_{21}\right) = (0,0,0)$$

that is, $a_{ki} = 0$ for every $k \neq i, \quad k, i = 0,1,2,3$, which is impossible in the external symmetry. Hence, for $c_0 \neq 0$, relation (138) holds.

Considering now the first of equations (133), we obtain

$$c_1 a_{01} + c_2 a_{02} + c_3 a_{03} = \left(c_1^2 + c_2^2 + c_3^2\right)^{\frac{1}{2}} \left(a_{01}^2 + a_{02}^2 + a_{03}^2\right)^{\frac{1}{2}} \cos\gamma$$

and substituting this expression into (137) we get

$$M_0^2 \mathrm{c}^2 \left(a_{01}^2 + a_{02}^2 + a_{03}^2\right) + \left(c_1^2 + c_2^2 + c_3^2\right)\left(a_{01}^2 + a_{02}^2 + a_{03}^2\right) \cos^2\gamma = 0$$

and since

$$a_{01}^2 + a_{02}^2 + a_{03}^2 \neq 0$$

because of relation (138) we get

$$M_0^2 \mathrm{c}^2 + \left(c_1^2 + c_2^2 + c_3^2\right) \cos^2\gamma = 0$$

and with equation (45) we get

$$M_0^2 \mathrm{c}^2 - \left(M_0^2 \mathrm{c}^2 + c_0^2\right) \cos^2\gamma = 0$$
$$M_0^2 \mathrm{c}^2 \left(1 - \cos^2\gamma\right) - c_0^2 \cos^2\gamma = 0$$
$$M_0^2 \mathrm{c}^2 \sin^2\gamma = c_0^2 \cos^2\gamma$$
$$M_0 \mathrm{c} \ \sin\gamma = \pm c_0 \cos\gamma$$

which is the first of equations (136).

Because of the extremely large amount of information contained within equations (134) and (136), we will confine ourselves to only one application. We will determine the case for which the total rest mass M_0 of the generalized particle vanishes.

In the case when vectors $\boldsymbol{\alpha}$ and $\mathbf{C}$ are orthogonal, that is for $\gamma = \dfrac{\pi}{2}$, we obtain from the first of equations (136):

$$M_0 = 0 .$$

Then, from the second of equations (134) for $\gamma = \dfrac{\pi}{2}$, we get

$$\begin{gathered} \| \mathbf{J} \| \cos\delta = - \| \mathbf{P} \| \cos\nu \\ \| \mathbf{J} \|^2 \cos^2\delta = \| \mathbf{P} \|^2 \cos^2\nu \end{gathered}$$

and with the second and third of equations (135) we obtain equation

$$\left(J_0^2 + m_0^2 c^2 \right) \cos^2\delta = \left(P_0^2 + \frac{E_0^2}{c^2} \right) \cos^2\nu .$$

From the second and third of equations (136) we get equations

$$\begin{gathered} m_0^2 c^2 = \left(m_0^2 c^2 + J_0^2 \right) \cos^2\delta \\ \frac{E_0^2}{c^2} = \left(\frac{E_0^2}{c^2} + P_0^2 \right) \cos^2\nu \end{gathered}$$

and substituting into the previous equation we get

$$m_0^2 c^2 = \frac{E_0^2}{c^2} .$$

Thus, we get the following set of equations

$$\begin{gathered} \boldsymbol{\alpha} \cdot C = 0 \\ M_0 = 0 \\ E_0 = \pm m_0 c^2 \end{gathered} \quad . \qquad (139)$$

Equation (128), which we used to prove equations (136), is the basic equation of the TSV giving us information about the corpuscular structure of matter. In order to fully comprehend this structure, we also need the eighth theorem of the TSV. We present this theorem, along with its consequences, in the following paragraph.

9. The conserved physical quantities of the generalized particle and the wave equation of the TSV

The generalized particle has a set of conserved physical quantities which we determine in this paragraph. The determination is initially made through equation (89). The proof procedure we follow is identical to the one followed to prove the conservation of electric charge from Maxwell's equations.

Considering equations (89) we define the scalar quantity ρ and the vector quantity $\mathbf{j}$, as given by equations

$$\rho = \sigma \nabla \cdot \boldsymbol{\alpha} = -\sigma \frac{icbz}{2\hbar}\left(c_1 a_{01} + c_2 a_{02} + c_3 a_{03}\right)$$
$$\mathbf{j} = \sigma \frac{c^2 bz}{2\hbar}\begin{pmatrix} -c_0 a_{01} - c_2\, a_{21} + c_3 a_{13} \\ -c_0 a_{02} + c_1\, a_{21} - c_3 a_{32} \\ -c_0 a_{03} - c_1\, a_{13} + c_2 a_{32} \end{pmatrix} \qquad (140)$$

where $\sigma \neq 0$ is a constant. We now prove that for the physical quantities ρ and $\mathbf{j}$ equation (141) holds:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0\,. \qquad (141)$$

From the first of equations (140) we obtain

$$\rho = \sigma \nabla \cdot \boldsymbol{\alpha}$$
$$\frac{\partial \rho}{\partial t} = \sigma \frac{\partial}{\partial t}\left(\nabla \cdot \boldsymbol{\alpha}\right)$$
$$\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\sigma \frac{\partial \boldsymbol{\alpha}}{\partial t}\right)$$

and with the second of equations (140) and equation (89, d) we get

$$\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\sigma c^2 \nabla \times \boldsymbol{\beta} - \mathbf{j}\right)$$
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j}$$

which is equation (141). According to equation (141), the physical quantity ρ is the density of a conserved physical quantity q with current density $\mathbf{j}$.

We now consider the four-vector of the current density j of the conserved physical quantity q, as given by equation

$$j = \begin{pmatrix} j_0 \\ j_1 \\ j_2 \\ j_3 \end{pmatrix} = \begin{pmatrix} i\rho c \\ j_x \\ j_y \\ j_z \end{pmatrix}. \qquad (142)$$

With the use of matrix M, as given by equation (119), equations (140) are written in the form

$$j=\frac{\sigma c^2 bz}{2\hbar}MC. \qquad (143)$$

For an appropriate constant $\sigma=\sigma_0$ in equations (140), the conserved physical quantity q is of the same dimensions (units of measurement) as the selfvariating charge Q. Equivalently, for $\sigma=\sigma_0$ equation (143) gives the current density of charge q of same nature as the selfvariating charge Q. It is easy to realize that if Q is the electric charge, then $\sigma=\varepsilon_0$, where ε_0 is the electric permeability of the vacuum. In the case where Q is the rest mass, then $\sigma=\frac{1}{4\pi G}$, where G is the gravitational constant.

The quantity q, as defined above, is a special case of a conserved physical quantity. We will now determine the general mathematical expression for the conserved physical quantities of the generalized particle. We prove the following theorem:

Eighth Theorem of the TSV

"For the field $(\boldsymbol{\xi},\boldsymbol{\omega})$ of the pair of vectors

$$\boldsymbol{\xi}=ic\Psi\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix} \qquad (144)$$

$$\boldsymbol{\omega}=\Psi\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix} \qquad (145)$$

where $\Psi=\Psi(x_0,x_1,x_2,x_3)$ is a function satisfying equation

$$\frac{\partial\Psi}{\partial x_k}=\frac{b}{\hbar}(\lambda J_k+\mu P_k)\Psi \qquad (146)$$

$k=0,1,2,3,\quad (\lambda,\mu)\neq(0,0),\quad \lambda,\mu\in\mathbb{C}$ are functions of x_0,x_1,x_2,x_3, the following equations hold:

$$\begin{aligned}&\nabla\cdot\boldsymbol{\omega}=0\\&\nabla\cdot\boldsymbol{\xi}=-\frac{\partial\boldsymbol{\omega}}{\partial t}\cdot\end{aligned} \qquad (147)$$

The generalized particle has a set of conserved physical quantities q with density ρ and current density $\mathbf{j}$

$$\rho = \sigma \nabla \cdot \boldsymbol{\xi}$$
$$\mathbf{j} = \sigma c^2 \left(\nabla \times \boldsymbol{\omega} - \frac{\partial \boldsymbol{\xi}}{c^2 \partial t} \right) \tag{148}$$

where $\sigma \neq 0$ are constants, for which conserved physical quantities the following continuity equation holds:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 . \tag{149}$$

The four-vectors of the current density j are given by equation

$$j = -\sigma c^2 M \begin{pmatrix} \frac{\partial \Psi}{\partial x_0} \\ \frac{\partial \Psi}{\partial x_1} \\ \frac{\partial \Psi}{\partial x_2} \\ \frac{\partial \Psi}{\partial x_3} \end{pmatrix} = -\frac{\sigma c^2 b}{\hbar} \Psi M \left(\lambda J + \mu P \right) . \tag{150}$$

The conserved physical quantities q are given by equation

$$icq = \int_V ic \rho \, dV = \int_V j_0 \, dV \tag{151}$$

where V is the volume occupied by the generalized particle.**”**

For the proof of the theorem we first demonstrate the following auxiliary equations (152)-(157)

$$\mathbf{J} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{152}$$

$$\mathbf{P} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{153}$$

$$\mathbf{J} \times \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -J_0 \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{154}$$

$$\mathbf{P}\times\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix}=-P_0\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix} \qquad (155)$$

$$\mathbf{J}\times\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}=\begin{pmatrix}J_2a_{21}-J_3a_{13}\\J_3a_{32}-J_1a_{21}\\J_1a_{13}-J_2a_{32}\end{pmatrix} \qquad (156)$$

$$\mathbf{P}\times\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}=\begin{pmatrix}P_2a_{21}-P_3a_{13}\\P_3a_{32}-P_1a_{21}\\P_1a_{13}-P_2a_{32}\end{pmatrix}. \qquad (157)$$

In order to prove equation (152) we get

$$\mathbf{J}\cdot\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}=J_1a_{32}+J_2a_{13}+J_3a_{21}$$

and with the second of equations (75) for $(i,v,k)=(1,3,2)$, we have

$$\mathbf{J}\cdot\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}=0\,.$$

Similarly, from the third of equations (75) we obtain equation (153). We now get

$$\mathbf{J}\times\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix}=\begin{pmatrix}J_2a_{03}-J_3a_{02}\\J_3a_{01}-J_1a_{03}\\J_1a_{02}-J_2a_{01}\end{pmatrix}=\begin{pmatrix}J_2a_{03}+J_3a_{20}\\J_3a_{01}+J_1a_{30}\\J_1a_{02}+J_2a_{10}\end{pmatrix}$$

and with the second of equations (75) we obtain

$$\mathbf{J}\times\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix}=\begin{pmatrix}-J_0a_{32}\\-J_0a_{13}\\-J_0a_{21}\end{pmatrix}$$

which is equation (154). Similarly, by considering the third of equations (75) we derive equation (155). Equations (156) and (157) are derived by taking into account equations (131) and (132).

Equations (147) are proven with the use of equations (152)-(157). We prove the first as an example. From equation (145) we obtain

$$\nabla \cdot \boldsymbol{\omega} = \nabla \Psi \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix}$$

and with equation (146) we get

$$\nabla \cdot \boldsymbol{\omega} = \frac{b}{\hbar} \lambda \Psi \mathbf{J} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} + \frac{b}{\hbar} \mu \Psi \mathbf{P} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix}$$

and with equations (152) and (153) we obtain

$$\nabla \cdot \boldsymbol{\omega} = 0 .$$

From equations (147) and (148), the continuity equation (149) results. The proof is similar to the one for equation (141). The proof of equation (150) is done with the use of equations (152)-(157), and equation (119). The physical quantities q are conserved, as indicated by the continuity equation (149). Therefore, if the generalized particle occupies volume V , then equation (151) holds.

From equation (17) it emerges that the dimensions of the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ are

$$\left[\lambda_{ki}\right] = kgs^{-1}, k, i = 0,1,2,3 .$$

Thus, from equations (79) and (80), the dimensions of the physical quantities $Q\alpha_{ki}, k, i = 0,1,2$ emerge. Additionally, by combining equations (77) and (104), the dimensions of the physical quantities $T_k, k = 0,1,2,3$ emerge. Thus, we obtain relations

$$\begin{aligned} &\left[Q\alpha_{ki}\right] = kgs^{-1}, k \neq i, k, i = 0,1,2,3 \\ &\left[T_k\right] = kgs^{-1}, k = 0,1,2,3 \end{aligned} . \qquad (158)$$

Using the first of equations (158) we can determine the units of measurement of the $(\boldsymbol{\xi}, \boldsymbol{\omega})$-field for every selfvariating charge Q. When Q is the electric charge, we can verify that the field units are (Vm^{-1},T). When Q is the rest mass, the field units are $\left(ms^{-2}, s^{-1}\right)$. The dimensions of the field depend solely on the units of measurement of the selfvariating charge Q.

From equation (150), and considering that $\lambda, \mu \in \mathbb{C}$, we can, by using the first of equations (158), determine the dimensions of the physical quantities q. If we write the constant σ in the form $\sigma = x\sigma_0$ we obtain relations

$$\begin{aligned} &\sigma = x\sigma_0 \\ &[q] = x[Q] \end{aligned} \qquad (159)$$

where x is a constant. From relations (159) we can determine the set of conserved physical quantities q of the generalized particle by determining the corresponding constant x. We reiterate that for the electric field $\sigma_0 = \varepsilon_0$, and for the gravitational field $\sigma_0 = \frac{1}{4\pi G}$. In the case of the electric field, for $x = \frac{1}{e}$, where e is the charge of the electron, q is a dimensionless conserved physical quantity, that is $q \in \mathbb{C}$. For $x = \frac{\hbar}{e}$, q is a conserved quantity of angular momentum. The eighth theorem of the TSV reveals the conserved physical quantities of the generalized particle.

One of the most important corollaries of the eighth theorem of the TSV is the prediction that the generalized particle has wave-like behaviour. We prove the following corollary:

“For function Ψ the following equation holds

$$\begin{aligned} \sigma c^2 \alpha_{ki} \left(\nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2} \right) &= \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \\ \sigma c^2 \alpha_{ki} \left(\nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} \right) &= \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \end{aligned} \qquad (160)$$

$$k \neq i, \quad k,i = 0,1,2,3 \text{.”}$$

To prove the corollary, considering that $x_0 = ict$, we write equations (147) and (148) in the form

$$\begin{aligned} &\nabla \cdot \boldsymbol{\xi} = -\frac{i}{\sigma c} j_0 \\ &\nabla \cdot \boldsymbol{\omega} = 0 \\ &\nabla \times \boldsymbol{\xi} = -\frac{ic\partial \boldsymbol{\omega}}{\partial x_0} \\ &\nabla \times \boldsymbol{\omega} = \frac{1}{\sigma c^2} \mathbf{j} + \frac{i\partial \boldsymbol{\xi}}{c\partial x_0} \end{aligned} \qquad . \qquad (161)$$

We will also use the identity (162) which is valid for every vector $\boldsymbol{\alpha}$

$$\nabla \times \nabla \times \boldsymbol{\alpha} = \nabla \left(\nabla \cdot \boldsymbol{\alpha} \right) - \nabla^2 \boldsymbol{\alpha} . \qquad (162)$$

From the third of equations (161) we obtain

$$\begin{aligned} \nabla \times \nabla \times \boldsymbol{\xi} &= -\nabla \times \left(\frac{ic\partial \boldsymbol{\omega}}{\partial x_0} \right) \\ \nabla \times \nabla \times \boldsymbol{\xi} &= -\frac{ic\partial}{\partial x_0} \left(\nabla \times \boldsymbol{\omega} \right) \end{aligned}$$

and using the identity (162) we get

$$\nabla\left(\nabla\cdot\boldsymbol{\xi}\right)-\nabla^{2}\boldsymbol{\xi}=-\frac{ic\partial}{\partial x_{0}}\left(\nabla\times\boldsymbol{\omega}\right)$$

and with the first and fourth of equations (161) we get

$$\nabla\left(-\frac{i}{\sigma c}j_{0}\right)-\nabla^{2}\boldsymbol{\xi}=\frac{\partial^{2}\boldsymbol{\xi}}{\partial x_{0}^{2}}-\frac{i}{\sigma c}\frac{\partial\mathbf{j}}{\partial x_{0}}$$

and we finally get

$$\nabla^{2}\boldsymbol{\xi}+\frac{\partial^{2}\boldsymbol{\xi}}{\partial x_{0}^{2}}=\frac{i}{\sigma c}\left(\frac{\partial\mathbf{j}}{\partial x_{0}}-\nabla j_{0}\right). \quad (163)$$

Working similarly from equation (161) we obtain

$$\nabla^{2}\boldsymbol{\omega}+\frac{\partial^{2}\boldsymbol{\omega}}{\partial x_{0}^{2}}=-\frac{1}{\sigma c^{2}}\nabla\times\mathbf{j}. \qquad (164)$$

Combining equations (163) and (164) with equations (144) and (145), we get

$$\alpha_{ki}\left(\nabla^{2}\Psi+\frac{\partial^{2}\Psi}{\partial x_{0}^{2}}\right)=\frac{i}{\sigma c}\left(\frac{\partial j_{i}}{\partial x_{k}}-\frac{\partial j_{k}}{\partial x_{i}}\right),\ k\neq i,\quad k,i=0,1,2,3,$$

which is equation (160).

From equation (160) the following two cases result, as given by equation (165) and equations (166)

$$\nabla^{2}\Psi+\frac{\partial^{2}\Psi}{\partial x_{0}^{2}}=\nabla^{2}\Psi-\frac{\partial^{2}\Psi}{c^{2}\partial t^{2}}=0 \qquad (165)$$

$$\begin{aligned}&\nabla^{2}\Psi+\frac{\partial^{2}\Psi}{\partial x_{0}^{2}}=\nabla^{2}\Psi-\frac{\partial^{2}\Psi}{c^{2}\partial t^{2}}=F\neq 0\\&\alpha_{ki}=\frac{1}{\sigma c^{2}F}\left(\frac{\partial j_{i}}{\partial x_{k}}-\frac{\partial j_{k}}{\partial x_{i}}\right),\ k\neq i,\quad k,i=0,1,2,3\end{aligned}. \qquad (166)$$

In the first case, equation (160) gives the classical wave equation (165). In the second case, i.e. for $F\neq 0$, the physical quantities α_{ki}, $k\neq i,\quad k,i=0,1,2,3$ are expressed as a function of the rate of change of the four-vector of the current density j. The physical quantities α_{ki}, $k\neq i,\quad k,i=0,1,2,3$ enter into a large number of equations of the TSV for the external symmetry. Therefore, through the second of equations (166) we can derive a set of equations, for the function $F\neq 0$, as well as for the rate of change of the four-vector j. Equation (160) can be characterized as **"the wave equation of the TSV"**.

We end the paragraph with the proof of equations (167) for the four-vector j

$$\begin{aligned} Nj &= 0 \\ Mj &= 0 \end{aligned} . \qquad (167)$$

We first combine equations (128) and (129), and obtain

$$\begin{aligned} M^2C &= 0 \\ M^2J &= 0 \\ M^2P &= 0 \end{aligned} . \qquad (168)$$

We now multiply equation (150) with the matrix N from the left and get

$$Nj = -\frac{\sigma c^2 b}{\hbar}\Psi NM\left(\lambda J + \mu P\right)$$

and with the first of equations (123) we obtain the first of equations (167). Multiplying equation (150) with the matrix M from the left we also get

$$Mj = -\frac{\sigma c^2 b}{\hbar}\Psi\left(\lambda M^2 J + \mu M^2 P\right)$$

and with equations (168) we obtain the second of equations (167).

The eighth theorem completes the basic study of the law of Selfvariations. The major part of the study concerns the external symmetry, which is clearly more complicated than the internal one. In the two symmetries we used the same notation for the constant b of the law of Selfvariations, the total constant rest mass M_0 of the generalized particle, as well as the constants c_k, $k = 0,1,2,3$. These constants do not have the same physical content in the two symmetries. The constants that enter into the equations of the TSV in the external symmetry are correlated with the theorems we presented, which determine the values of the above constants. In the internal symmetry, the constants b, M_0 and c_i, $i = 0,1,2,3$ in equations (54), (57) and (58) are not correlated with theorems that correspond to the ones of the external symmetry. Therefore, they should be considered absolute constants.

In the study we presented, we combined equations (14) and (15) with the law of Selfvariations for the rest mass $m_0 \neq 0$, as given in equation (13). We can equally well study the Selfvariations for the rest mass $\frac{E_0}{c^2} \neq 0$ and the symmetric equation

$$\frac{\partial E_0}{\partial x_k} = \frac{b}{\hbar} J_k E_0, \mathrm{k} = 0,1,2,3$$

instead of law (13). We will, of course, not present this second study, since it is clear that the same results emerge and we just have a reversal of roles of the rest masses m_0 and $\frac{E_0}{c^2}$. This remark is made in order to note the fact that the law of Selfvariations holds for

$$m_0 \neq 0 \vee E_0 \neq 0 .$$

In the case where $m_0 = 0$ and $E_0 = 0$, the law of Selfvariations is not defined. Therefore, in the applications of the present study, every case in which it emerges that $m_0 = \frac{E_0}{c^2} = 0$ must be rejected. The study for $m_0 = \frac{E_0}{c^2} = 0$ can be made starting from equations (117) or from their symmetric

$$\lambda_{ki} = \frac{\partial P_i}{\partial x_k} - \frac{b}{\hbar}\left(c_k - P_k\right)P_i, k, i = 0,1,2,3.$$

That is, not starting our investigation from the rest masses masses m_0 or $\frac{E_0}{c^2}$. Of course, we could not arrive at equations (117) without the study we presented, which constitutes the fundamental investigation of the law of Selfvariations.

The law of Selfvariations is connected with all the individual areas of physics, and it is impossible to investigate its consequences in one article. It is for this reason that we have chosen and present four basic applications of the TSV.

10. The generalized photon

From equation (160) there result two states for the generalized particle, as expressed by equations (165) and (166). We shall refer to the state of the generalized particle for which equation (165) is valid, as the "generalized photon". For the generalized particle the following corollary of the eighth theorem of the TSV holds:

"For the generalized particle the following equivalences hold:

$$\nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = 0 \qquad (169)$$

if and only if for each $k \neq i,\ k, i = 0,1,2,3$ it is

$$\frac{\partial j_i}{\partial x_k} = \frac{\partial j_k}{\partial x_i} \qquad (170)$$

if and only if

$$\begin{aligned} \nabla^2\boldsymbol{\xi} - \frac{\partial^2\boldsymbol{\xi}}{c^2\partial t^2} = 0 \\ \nabla^2\boldsymbol{\omega} - \frac{\partial^2\boldsymbol{\omega}}{c^2\partial t^2} = 0 \end{aligned}. \qquad (171)\text{"}$$

In the external symmetry there exists at least one pair of indices $(k,i),\ k \neq i,\ k,i \in \{0,1,2,3\}$, for which $\alpha_{ki} \neq 0$. Therefore, when equation (170) holds, then equation (169) follows from equation (160), and vice versa. Thus, equations (169) and (170) are equivalent. When equation (170) holds, then the right hand sides of equations (163) and (164) vanish, that is, equations (171) hold. The converse also holds, thus equations (170) and (171) are equivalent. Therefore, equations (169), (170), and (171) are equivalent.

For the generalized photon equation (165) holds, which is equation (169). Therefore, equations (170) and (171) also hold. According to equations (171), for the generalized photon the $(\boldsymbol{\xi},\boldsymbol{\omega})$-field is propagating with velocity c in the form of a wave.

We now prove that, for the generalized photon, the four-vector j of the current density of the conserved physical quantities q, varies according to the equations

$$\nabla^2 j_k - \frac{\partial^2 j_k}{c^2 \partial t^2} = 0,\ k = 0,1,2,3\,. \qquad (172)$$

We prove equation (172) for $k=0$, and we can similarly prove it for $k=1,2,3$.

Considering equation (142), we write equation (149) in the form

$$\frac{\partial j_0}{\partial x_0} + \frac{\partial j_1}{\partial x_1} + \frac{\partial j_2}{\partial x_2} + \frac{\partial j_3}{\partial x_3} = 0\,. \qquad (173)$$

Differentiating equation (173) with respect to x_0 we get

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_0}\left(\frac{\partial j_1}{\partial x_1}\right) + \frac{\partial}{\partial x_0}\left(\frac{\partial j_2}{\partial x_2}\right) + \frac{\partial}{\partial x_0}\left(\frac{\partial j_3}{\partial x_3}\right) = 0$$

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_1}\left(\frac{\partial j_1}{\partial x_0}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial j_2}{\partial x_0}\right) + \frac{\partial}{\partial x_3}\left(\frac{\partial j_3}{\partial x_0}\right) = 0$$

and with equation (170) we get

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_1}\left(\frac{\partial j_0}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial j_0}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\frac{\partial j_0}{\partial x_3}\right) = 0$$

$$\frac{\partial^2 j_0}{\partial x_0^2} + \nabla^2 j_0 = 0$$

which is equation (172) for $k=0$, since $x_0 = ict$.

The way in which equations (171) emerge in the TSV is completely different from the way in which the electromagnetic waves emerge in Maxwell's electromagnetic theory. In Maxwell's theory, equations (171) emerge for $j=0$. In the TSV it is $j \neq 0$ due to the Selfvariations. Equations (171) emerge when equation (165) holds, that is, in the first of the two cases of equation (160). Furthermore, according to the TSV, in the electromagnetic waves, the current density j varies according to equation (172).

The solutions of differential equations (169) and (172) are known. Therefore, the functions Ψ and j_k, $k = 0,1,2,3$ are also known for the generalized photon. By knowing function Ψ and the current density j, the theorems of the TSV give a set of data and information about the generalized photon. We reiterate that the four-vector j concerns a set of conserved physical quantities, and not just the electric current density.

11. The generalized particle of the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field

In this paragraph we present the study of the generalized particle of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$. From equations (81), (82) and (144), (145) it follows that the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field is $\Psi = z$, that is

$$\Psi = z = \exp\left[-\frac{b}{2\hbar}\left(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3\right)\right]$$

according to equation (74). Thus, taking into account equation (76) we obtain

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = \left(c_0^2 + c_1^2 + c_2^2 + c_3^2\right)\Psi$$

and with equation (45) we get

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = -\mathrm{M}_0^2c^2\Psi\,. \qquad (174)$$

We first study the generalized photon of the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field. Comparing equations (169) and (174) we conclude that the generalized photon of the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field has a vanishing total rest mass M_0:

$$M_0 = 0\,. \qquad (175)$$

Therefore, from equations (139) we obtain equations

$$\begin{aligned}&\boldsymbol{\alpha}\cdot\mathbf{C} = c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03} = 0\\ &\mathrm{E}_0 = \pm m_0c^2\end{aligned}\,. \qquad (176)$$

The current density j of the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field is given by equations (140). Combining these with the first of equations (176), it is easy to see that for the generalized photon of the $(\boldsymbol{\alpha},\boldsymbol{\beta})$-field the four-vector j of the current density vanishes

$$j = 0\,. (177)$$

Equation (177) does not hold for the generalized photon of every $(\boldsymbol{\xi},\boldsymbol{\omega})$ -field, where we generally have $j \neq 0$. For $\Psi = z$ and with equation (176) we obtain equation

$$\frac{\partial \Psi}{\partial x_k} = \frac{\partial z}{\partial x_k} = -\frac{bc_k}{2\hbar} z = -\frac{bc_k}{2\hbar} \Psi, k = 0,1,2,3 .$$

Comparing this relation with equation (146), and considering equation (44), we conclude that for the $(\boldsymbol{\xi}, \boldsymbol{\omega})$-field equation (178) holds:

$$\lambda = \mu = -\frac{1}{2} . \quad (178)$$

It is easy to verify that equation (177) holds exclusively for the generalized photon of the $(\boldsymbol{\xi}, \boldsymbol{\omega})$-fields for which $\lambda = \mu$ in equation (146). In the cases where $\lambda \neq \mu$ it is also $j \neq 0$.

Let us now suppose that the generalized photon is moving along the axis $x_1 = x$. In this case it is $c_1 \neq 0$ and $c_2 = c_3 = 0$. Taking also into account equations (175) and (45) we get

$$c_0^2 + c_1^2 = 0$$

and we finally get

$$c_1 = \pm i c_0 . \qquad (179)$$

According to equation (179), and since $c_2 = c_3 = 0$ and $x_0 = ict$, we get from equation (74) equation

$$\Psi = z = \exp\left[-\frac{ibc_0}{2\hbar} \left(ct \pm x \right) \right] . \ (180)$$

We now study the generalized particle of field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, i.e. the case where $M_0 \neq 0$ in equation (174). In the $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-field, the functions $\Psi = z$ and $j_k, k = 0,1,2,3$, are already known as given by equation (140). Thus, we can easily study the consequences of equation (151) in the case when the generalized particle occupies a constant volume V.

By combining the first of equations (140) for $\rho \neq 0$ with equation (151), we get

$$q = -\frac{i\sigma cb \left(\boldsymbol{\alpha} \cdot \mathbf{C} \right)}{2\hbar} \exp\left(-\frac{bc_0}{2\hbar} x_0 \right) \int_V \exp\left[-\frac{b}{2\hbar} \left(c_1 x_1 + c_2 x_2 + c_3 x_3 \right) \right] dV \ . \ (181)$$

According to the continuity equation (149), the physical quantity q in equation (181) does not depend on time, that is, it is independent of $x_0 = ict$. Furthermore, the volume $V = V(t)$ of the generalized particle changes with time. In the case where the generalized particle occupies a constant volume V, and given that the physical quantity q does not depend on time, from equation (181) we obtain equations

$$\begin{gathered}\int_V \exp\left[-\frac{b}{2\hbar}\left(c_1x_1+c_2x_2+c_3x_3\right)\right]dV=0\\ q=0\\ (V=\text{constant})\end{gathered} \quad . \qquad (182)$$

Equation (180) holds for the generalized photon of field $(\boldsymbol{\alpha},\boldsymbol{\beta})$. We want to prove the corresponding equation for the generalized particle. Thus, we consider the case where the total momentum of the generalized particle is along the axis $x_1 = x$. In this case, it is $c_1 \neq 0, c_2 = c_3 = 0$, and from equation (45) we get

$$c_0^2 + c_1^2 = -M_0^2c^2 \neq 0. \quad (183)$$

Let now the generalized particle occupy the constant volume V defined by relations (184) in a system of reference $\mathrm{O}(t, x_1, x_2, x_3)$

$$\begin{gathered}\alpha \le x_1 \le \beta\\ 0 \le x_2 \le L_2\\ 0 \le x_3 \le L_3\\ \alpha < \beta\\ L_2, L_3 > 0, L_2, L_3 \text{ constants}\end{gathered} \quad . \qquad (184)$$

For α and β the following relation holds

$$\frac{d\alpha}{dt} = \frac{d\beta}{dt} = u < c\,, \qquad (185)$$

where u the velocity with which volume V is moving in the chosen reference frame.

Combining the first of equations (182) with relations (185), and taking into account that $c_2 = c_3 = 0$, we get

$$\exp\left(-\frac{bc_1\beta}{2\hbar}\right) - \exp\left(-\frac{bc_1\alpha}{2\hbar}\right) = 0$$

and we finally arrive at

$$\begin{gathered}\exp\left(\frac{bc_1L}{2\hbar}\right) = 1\\ L = \beta - \alpha > 0\end{gathered} \quad . \qquad (186)$$

Equation (186) holds only when constant b is an imaginary number, when we get

$$c_1 = n\frac{4\pi\hbar}{L\|b\|}, n = \pm1, \pm2, \pm3, \ldots$$
$$\mathrm{b} \in I \qquad . \qquad (187)$$
$$L = \beta - \alpha > 0$$

Combining equations (183) and (187) we obtain

$$M_0^2c^2 = -c_0^2 - n^2\frac{16\pi^2\hbar^2}{L^2\|b\|^2}, n = 1, 2, 3, \ldots \quad ,(188)$$
$$\mathrm{b} \in \mathrm{I}$$

Therefore, when the generalized particle of the $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-field occupies a constant volume, its total rest mass M_0 is quantized.

Solving equation (188) with respect to c_0 we obtain

$$c_0 = \pm\frac{4n\pi i\hbar}{L\|b\|}\left(1 + \frac{1}{n^2}\frac{L^2\|b\|^2}{16\pi^2\hbar^2}\right)^{\frac{1}{2}}, n = \pm1, \pm2, \pm3, \ldots \qquad (189)$$

Combining equations (173), (187), (189), and considering that $c_2 = c_3 = 0, x_0 = ict$ and $b = \pm i\|b\|$, we obtain equation (190) for the confined $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-field

$$\Psi = \exp\left(\frac{2n\pi i\hbar}{L}\left[\left(1 + \frac{1}{n^2}\frac{L^2\|b\|^2}{16\pi^2\hbar^2}\right)^{\frac{1}{2}} ct \pm x\right]\right), n = \pm1, \pm2, \pm3, \ldots . \qquad (190)$$

Equations (173)-(178) hold generally for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. Equations (179) and (180) hold for the generalized photon of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ in one dimension. Equations (181)-(190) hold for the confined within a stable part of space field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$.

12. The plane Π

In paragraph 8 we defined as Π the plane normal to the vector $\boldsymbol{\beta}$. Taking into account equations (82), the plane Π is defined if and only if the constant vector $\boldsymbol{\tau}$ of equation (191) is not zero.

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \qquad (191)$$

In this paragraph we will study the plane Π for the case where equations (192) hold

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \qquad (192)$$
$$\alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \neq 0$$

As we have already observed regarding the physical quantities in the equations of the TSV, there are sums of squares that are equal to zero. Therefore, equations (191) and (192) are not equivalent.

We now consider a constant vector $\mathbf{n}$ as defined by

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix}. \qquad (193)$$

From equation (128) and the second relation of equations (192), it emerges that relations (194) hold for the vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \qquad (194)$$
$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \neq 0$$

From equations (81), (82), and (109) we get

$$\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21} = 0$$

and with equations (191) and (193) we get

$$\boldsymbol{\tau} \cdot \mathbf{n} = 0. \qquad (195)$$

From this equation we conclude that vector $\mathbf{n}$, being vertical to vector $\boldsymbol{\tau}$, belongs to the plane Π.

We consider a constant vector $\boldsymbol{\mu}$ as given by

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \mathbf{n} \times \boldsymbol{\tau} = \begin{pmatrix} a_{02}\alpha_{21} - a_{03}\alpha_{13} \\ a_{03}\alpha_{32} - a_{01}\alpha_{21} \\ a_{01}\alpha_{13} - a_{02}\alpha_{32} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \qquad (196)$$

Vector $\boldsymbol{\mu}$ lies on plane Π as vertical to vector $\boldsymbol{\tau}$, and is additionally vertical to vector $\mathbf{n}$. Consequently, the vector pair $(\boldsymbol{\mu}, \mathbf{n})$ constitutes an orthogonal vector base on plane Π.

It is easily provable that for vectors $\boldsymbol{\mu}, \mathbf{n}, \boldsymbol{\tau}$, the following equations hold:

$$\begin{aligned} \|\mathbf{n}\|^2 &= -\|\boldsymbol{\tau}\|^2 \\ \|\boldsymbol{\mu}\| &= \pm i\|\mathbf{n}\|^2 = \mp i\|\boldsymbol{\tau}\|^2 \end{aligned}. \qquad (197)$$

In order to prove equations (197) one must take into account equations (109) and (128). The symbol $\|\boldsymbol{\alpha}\|$ for every vector $\boldsymbol{\alpha}$ of the TSV has been defined when we first use it in equations (134).

Combining the pairs of equations (144), (194) and (145), (191), we get

$$\begin{aligned} \boldsymbol{\xi} &= ic\Psi\mathbf{n} \\ \boldsymbol{\omega} &= \Psi\boldsymbol{\tau} \end{aligned} \quad . \qquad (198)$$

Consequently, the plane Π is defined on every point of spacetime to which field $(\boldsymbol{\xi},\boldsymbol{\omega})$ extends and relations (192) hold. Additionally, the orientation of plane Π in space is defined by the field $(\boldsymbol{\xi},\boldsymbol{\omega})$.

From equations (75) for $(i,v,\kappa)=(1,2,3)$ we get equations

$$\begin{aligned} c_1 a_{32} + c_2 a_{13} + c_3 a_{21} &= 0 \\ J_1 a_{32} + J_2 a_{13} + J_3 a_{21} &= 0 \\ P_1 a_{32} + P_2 a_{13} + P_3 a_{21} &= 0 \end{aligned}$$

and with equations (130), (131), (132), and (191) we get

$$\begin{aligned} \boldsymbol{\tau}\cdot\mathbf{C} &= 0 \\ \boldsymbol{\tau}\cdot\mathbf{J} &= 0 \\ \boldsymbol{\tau}\cdot\mathbf{P} &= 0 \end{aligned} \quad . \qquad (199)$$

From equations (199) we conclude that the vectors $\mathbf{C}$, $\mathbf{J}$, and $\mathbf{P}$, as vertical to vector $\boldsymbol{\tau}$, belong to the plane Π.

Expanding the first of equations (167) we get equation

$$a_{32} j_1 + a_{13} j_2 + a_{21} j_3 = 0$$

and with equations (142) and (191) we obtain

$$\boldsymbol{\tau}\cdot\mathbf{j} = 0 . \qquad (200)$$

From equation (200) we conclude that the vector $\mathbf{j}$, as vertical to vector $\boldsymbol{\tau}$, lies on plane Π.

From equations (146) and (131, (132) we get

$$\nabla\Psi = \frac{b}{\hbar}(\lambda\mathbf{J} + \mu\mathbf{P})\Psi . \qquad (201)$$

Vectors $\mathbf{J}$ and $\mathbf{P}$ belong to the plane Π. Consequently, vector $\nabla\Psi$, as a linear combination of vectors $\mathbf{J}$ and $\mathbf{P}$, also belongs to the plane Π.

We, thus, come to the conclusion that that the vectors $\mathbf{J}$, $\mathbf{P}$, $\mathbf{C}$, $\mathbf{j}$, and $\nabla\Psi$ belong to the plane Π. These vectors vary according to the theorems of the TSV remaining constantly on plane Π.

We now prove that, when plane Π is defined, the 4-vector j has a rigidly defined internal structure. Expanding the second of equations (167) we get four equations. The first of these is equation

$$\alpha_{01} j_1 + \alpha_{02} j_2 + \alpha_{03} j_3 = 0$$

which, together with equations (142) and (193), is written in the form

$$\mathbf{n}\cdot\mathbf{j}=0. \qquad (202)$$

Vector $\mathbf{j}$ belongs to plane Π and, according to equation (202), is normal to the vector $\mathbf{n}$. Consequently, vector $\mathbf{j}$ is parallel to vector $\boldsymbol{\mu}$. Therefore, vector $\mathbf{j}$ is written in the form

$$\mathbf{j}=\frac{B}{\|\boldsymbol{\mu}\|}\boldsymbol{\mu}, \qquad (203)$$

where B is a function of x_0, x_1, x_2, x_3 with dimensions of the 4-vector j. The remaining three equations from the expansion of the second of equations (167) are equations

$$\begin{aligned}
-a_{01}\, j_0 - a_{21} j_2 + a_{13} j_3 &= 0\\
-a_{02} j_0 + a_{21} j_1 - a_{32} j_3 &= 0\\
-a_{03}\, j_0 - a_{13} j_1 + a_{32} j_2 &= 0
\end{aligned}$$

and with equations (191), (193), and (203) they are written in the form of equation

$$j_0\mathbf{n}=B\boldsymbol{\tau}\times\frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|}$$

and with equation (196) we get

$$j_0\mathbf{n}=\frac{B}{\|\boldsymbol{\mu}\|}\ \boldsymbol{\tau}\times(\mathbf{n}\times\boldsymbol{\tau})$$

and since it is $\mathbf{n}\cdot\boldsymbol{\tau}=0$, we get

$$j_0\mathbf{n}=\frac{B}{\|\boldsymbol{\mu}\|}\ \|\boldsymbol{\tau}\|^2\ \mathbf{n}$$

and because it is $\mathbf{n}\neq 0$, we get

$$j_0 = B\frac{\|\boldsymbol{\tau}\|^2}{\|\boldsymbol{\mu}\|}$$

and with the second of equations (197) we get

$$\begin{aligned}
j_0 &= \pm iB\\
B &= \pm i j_0
\end{aligned}$$

and with equation (203) we get

$$\mathbf{j}=\pm i j_0\,\frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|}. \qquad (204)$$

From equation (204), and taking into account that $j_0 = i\rho c$, we get

$$\mathbf{j} = \pm \rho c \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} . \quad (205)$$

From equations (142) and (204), (205) we get the 4-vector j in the form

$$j = \rho c \begin{bmatrix} i \\ \pm \dfrac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \end{bmatrix} = j_0 \begin{bmatrix} 1 \\ \mp \dfrac{i\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \end{bmatrix} . \quad (206)$$

According to equation (206), the 4-vector j has an extremely complicated structure. This is due to the internal structure of vector $\boldsymbol{\mu}$ as given by equation (196), as well as due to the internal structure of the density ρ as given by the first of equations (150). This structure could not be determined by the physical theories of the last century. Equation (205) is completely different from equation

$$\mathbf{j} = \rho \mathbf{u} ,$$

which is used by last century's theories.

We prove the following corollary:

"The following equations hold in the plane Π

$$\begin{aligned}
&\mathbf{j} = \pm i j_0 \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} = \mp \rho c \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \\
&\frac{\partial j_0}{\partial x_0} \pm \frac{i}{\|\boldsymbol{\mu}\|} \boldsymbol{\mu} \cdot \nabla j_0 = 0 \\
&\nabla j_0 = \pm i \frac{\partial j_0}{\partial x_0} \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} - \sigma c^2 F \mathbf{n} . \qquad (207) \text{ "} \\
&\sigma c^2 F \boldsymbol{\tau} = \pm \frac{i}{\|\boldsymbol{\mu}\|} \boldsymbol{\mu} \times \nabla j_0 \\
&F = \nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2}
\end{aligned}$$

The first of equations (207) is equation (204). The second emerges from the combination of the continuity equation (173) with equation (204). The third and fourth emerge from the combination of the wave equation (160) with equation (204). The last of equations (207) is the first of equations (166).

For the generalized photon, it holds that $F = 0$, according to equation (165). So, taking into account equations (172), we get the following equations for the generalized photon on the plane Π

$$
\begin{aligned}
&\mathbf{j} = \pm i j_0 \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} = \mp \rho c \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \\
&\frac{\partial j_0}{\partial x_0} \pm \frac{i}{\|\boldsymbol{\mu}\|} \boldsymbol{\mu} \cdot \nabla j_0 = 0 \\
&\nabla j_0 = \pm i \frac{\partial j_0}{\partial x_0} \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \qquad . \qquad (208) \\
&\nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2} = 0 \\
&\nabla^2 j_0 + \frac{\partial^2 j_0}{\partial x_0^2} = 0
\end{aligned}
$$

For the generalized photon, the fourth and fifth of equations (208) can be solved and they give the Ψ function and the density $j_0 = i\rho c$, respectively. Then, from the first equation we get the current density $\mathbf{j}$. Also, the second of equations (208) emerges from the third.

The generalized photon is a special case, for $F = 0$, of a generalized particle. For $F \neq 0$, the system of differential equations (207) is not solvable in the simple way that the system (208) is. Additionally, the second, third, and fourth of equations (207) are not independent. Combined in pairs they give the third.

Expanding the first of equations (167) we get four equations. As we have proven, the first of them is equivalent to equation (200), that is, with the fact that vector $\mathbf{j}$ belongs to plane Π. The remaining three are written in the form

$$j_0 \boldsymbol{\tau} = \mathbf{n} \times \mathbf{j} . \qquad (209)$$

Taking into account the first of equations (207), it is easily proven that equation (209) is equivalent to the second of equations (197). Analogous conclusions can be drawn from the second of equations (167), from which equation (202) as well as equation (210) emerge

$$j_0 \mathbf{n} = \boldsymbol{\tau} \times \mathbf{j} . \qquad (210)$$

Regarding the study of the corpuscular structure of the generalized particle in the plane Π, we expand the first of equations (75) for $(i, \mathrm{v}, k) = (0,3,2), (0,1,3), (0,2,1)$ and we obtain equations

$$
\begin{aligned}
c_0 a_{32} + c_2 a_{03} + c_3 a_{20} &= 0 \\
c_0 a_{13} + c_3 a_{01} + c_1 a_{30} &= 0 \\
c_0 a_{21} + c_1 a_{02} + c_2 a_{10} &= 0
\end{aligned}
$$

and since it is $a_{ki} = -a_{ik}$ for every $k \neq i$, $k, i = 0,1,2,3$ we get

$$
\begin{aligned}
c_0 a_{32} + c_2 a_{03} - c_3 a_{02} &= 0 \\
c_0 a_{13} + c_3 a_{01} - c_1 a_{03} &= 0 \quad . \qquad (211) \\
c_0 a_{21} + c_1 a_{02} - c_2 a_{01} &= 0
\end{aligned}
$$

From equations (130), (191), (193), and (211) we get

$$c_0\boldsymbol{\tau} = \mathbf{n}\times\mathbf{C}. \quad (212)$$

From equation (212) we get relation (213) on the plane Π

$$\mathbf{C} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (213)$$

Indeed, if we assume that $\mathbf{C} = 0$ from equation (212) we get $c_0 = 0$ and with equation (130) we get $(c_0, c_1, c_2, c_3) = (0,0,0,0)$, which is impossible due to relation (60).

Because of equation (213), two cases emerge from equation (212) for the corpuscular structure of the generalized particle on the plane Π. In the case where the vectors $\mathbf{n}$ and $\mathbf{C}$ are parallel, from equation (212) we get $c_0 = 0$ and from equation (44) we get

$$M_0^2 c^2 + c_1^2 + c_2^2 + c_3^2 = 0$$

and we finally get

$$M_0^2 c = \pm i\left(c_1^2 + c_2^2 + c_3^2\right)^{\frac{1}{2}} \quad (214)$$
$$(\mathbf{C} \parallel \mathbf{n})$$

for the total rest mass M_0 of the generalized particle.

In the case where the vectors $\mathbf{n}$ and $\mathbf{C}$ are not parallel, from equation (212) we get $c_0 \neq 0$, and, therefore, equations (134), (136), and (139) hold.

We now study the behaviour of the vectors $\mathbf{J}$ and $\mathbf{P}$ on the plane Π. Because of the first of equations (134), it suffices to study the behaviour of vector $\mathbf{J}$. We define the 4×4 matrix H as given by

$$H = \begin{bmatrix} \mathrm{T}_0 & 0 & 0 & 0 \\ 0 & \mathrm{T}_1 & 0 & 0 \\ 0 & 0 & \mathrm{T}_2 & 0 \\ 0 & 0 & 0 & \mathrm{T}_3 \end{bmatrix} = \frac{1}{zQ}\Lambda. \qquad (215)$$

Equation

$$H = \frac{1}{zQ}\Lambda$$

emerges from the combination of equations (78) and (104).

Using matrix H, equations (105) are written in the form

$$MJ = -HJ. \quad (216)$$

The right part of equation (216) is generally not zero (see equation (117) and the concluding remarks of paragraph 7). Taking now into account equations (131) and (193), the first of equations (105) is written in the form

$$\mathbf{n}\cdot\mathbf{J} = -T_0 J_0 . \qquad (217)$$

Taking into account equations (131), (191), and (193), the remaining three equations (105) are written in the form

$$J_0\mathbf{n} = \boldsymbol{\tau}\times\mathbf{J} + \begin{bmatrix} T_1 J_1 \\ T_2 J_2 \\ T_3 J_3 \end{bmatrix} . \qquad (218)$$

Using matrix N as given by equation (120), the second of equations (75) is written in the form

$$NJ = 0 . \qquad (219)$$

This equation is equivalent to the four different equations we get from the second of equations (75). The first of these is equivalent to the second of equations (199), and expresses the fact that the vector $\mathbf{J}$ belongs to the plane Π. The remaining three are written in the form

$$J_0\boldsymbol{\tau} = \mathbf{n}\times\mathbf{J} . \qquad (220)$$

The vector $\mathbf{J}$ belongs to the plane Π and, therefore, is written in the form

$$\mathbf{J} = (\boldsymbol{\mu}\cdot\mathbf{J})\frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|^2} + (\mathbf{n}\cdot\mathbf{J})\frac{\mathbf{n}}{\|\mathbf{n}\|^2}$$

and with equation (217) we get

$$\mathbf{J} = (\boldsymbol{\mu}\cdot\mathbf{J})\frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|^2} - \mathrm{T}_0 J_0 \frac{\mathbf{n}}{\|\mathbf{n}\|^2} . \qquad (221)$$

From the equations (220) and (221) we get

$$J_0\boldsymbol{\tau} = \mathbf{n}\times\left[(\boldsymbol{\mu}\cdot\mathbf{J})\frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|^2} - \mathrm{T}_0 J_0 \frac{\mathbf{n}}{\|\mathbf{n}\|^2}\right]$$

$$J_0\boldsymbol{\tau} = (\boldsymbol{\mu}\cdot\mathbf{J})\frac{\mathbf{n}\times\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|^2}$$

and with equation (196) we get

$$J_0\boldsymbol{\tau} = \frac{(\boldsymbol{\mu}\cdot\mathbf{J})}{\|\boldsymbol{\mu}\|^2}\left[\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\tau})\right]$$

and since $\mathbf{n}\cdot\boldsymbol{\tau} = 0$ we get

$$J_0\boldsymbol{\tau} = -\frac{(\boldsymbol{\mu}\cdot\mathbf{J})}{\|\boldsymbol{\mu}\|^2}\|\mathbf{n}\|^2\,\boldsymbol{\tau}$$

and since $\boldsymbol{\tau} \neq 0$ we get

$$J_0 = -\frac{(\boldsymbol{\mu} \cdot \mathbf{J})}{\|\boldsymbol{\mu}\|^2} \|\mathbf{n}\|^2$$

$$\frac{(\boldsymbol{\mu} \cdot \mathbf{J})}{\|\boldsymbol{\mu}\|^2} = -\frac{J_0}{\|\mathbf{n}\|^2}$$

and with the second of equations (197) we get

$$\frac{\boldsymbol{\mu} \cdot \mathbf{J}}{\|\boldsymbol{\mu}\|} = \pm \frac{iJ_0}{\|\boldsymbol{\mu}\|}$$

and substituting in equation (221) we get

$$\mathbf{J} = \pm J_0 \frac{i\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} - \frac{\mathrm{T}_0 J_0}{\|\mathbf{n}\|} \frac{\mathbf{n}}{\|\mathbf{n}\|}. \qquad (222)$$

Taking into account that $J_0 = \dfrac{iW}{c}$, equation (222) is written as

$$\mathbf{J} = \frac{W}{c}\left(\pm \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} - \frac{i\mathrm{T}_0}{\|\mathbf{n}\|} \frac{\mathbf{n}}{\|\mathbf{n}\|} \right). \qquad (223)$$

From equations (11) and (222), (223) we get the 4-vector J in the form

$$J = \frac{W}{c}\begin{bmatrix} i \\ \pm \dfrac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} - \dfrac{i\mathrm{T}_0}{\|\mathbf{n}\|} \dfrac{\mathbf{n}}{\|\mathbf{n}\|} \end{bmatrix} = J_0 \begin{bmatrix} 1 \\ \mp \dfrac{i\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} - \dfrac{\mathrm{T}_0}{\|\mathbf{n}\|} \dfrac{\mathbf{n}}{\|\mathbf{n}\|} \end{bmatrix}. \qquad (224)$$

The corresponding mathematical expressions for the 4-vector P emerge from the combination of equations (222), (223), and (224) with equation (44). From equation (224) we conclude that J_0 and T_0 are the only variable physical quantities in the 4-vectors J and P.

According to the study we presented, the corpuscular structure of the generalized particle is determined by the 4-vectors C, J, P and their relation. According to equation (207), the wave behaviour of the generalized particle is determined by the 4-vector j. The “connecting element” between the corpuscular structure and the wave behaviour of the generalized particle are equations (146) and (150).

The plane Π is defined when relations (192) hold. Nevertheless, it should be considered the rule in the TSV, while the cases where the plane Π is not defined should be considered as special. These cases can be examined using the theorems of the TSV. For this reason, as well as for reasons of economy of the present article, we will not refer to these special cases.

13. Degrees of freedom of the TSV. The Schrödinger equation

One of the most important conclusions of the eighth theorem is that it gives the degrees of freedom of the equations of the TSV. In equation (146) the parameters $\lambda, \mu \in \mathbb{C}, (\lambda, \mu) \neq (0,0)$ can have arbitrary values or can be arbitrary functions of x_0, x_1, x_2, x_3. Therefore, the investigation of the TSV takes place through the parameters λ and μ of equation (146).

The TSV consists of a closed set of equations. Consequently, every specific choice of the parameters $\lambda, \mu \in \mathbb{C}, (\lambda, \mu) \neq (0,0)$ completely determines the totality of the physical quantities that enter into the equations of the TSV. These include the rest masses M_0, m_0 and $\frac{E_0}{c^2}$, as well as the 4-vectors j of the conservable physical quantities of the generalized particle.

If we set $(\lambda, \mu, b) = (1, 0, i)$ in equation (146), we get equations

$$\begin{aligned} \nabla \Psi &= \frac{i}{\hbar} \mathbf{J} \Psi \\ \frac{\partial \Psi}{\partial x_0} &= \frac{i}{\hbar} J_0 \Psi \end{aligned} \quad . \quad (225)$$

Taking into account that $x_0 = ict$ and $J_0 = \frac{iW}{c}$, we recognize in equations (225) the Schrödinger operators [18-23]. Using the macroscopic mathematical expressions of the momentum $\mathbf{J}$ and energy W of the material particle, we get the Schrödinger equation. The Schrödinger equation is a special case of the wave equation of the TSV.

If we set $(\lambda, \mu, b) = (1, \alpha, i)$ in equation (146), where α the fine structure constant, and take into account equation (44), we get equations

$$\begin{aligned} \nabla \Psi &= \frac{i}{\hbar} \left((1-\alpha) \mathbf{J} + \alpha \mathbf{C} \right) \Psi \\ \frac{\partial \Psi}{\partial x_0} &= \frac{i}{\hbar} \left((1-\alpha) J_0 + \alpha c_0 \right) \Psi \end{aligned} \quad . \quad (226)$$

The fine structure constant in the TSV can have the following three forms

$$\begin{aligned} \alpha &= \frac{e^2}{4\pi\varepsilon_0 c\hbar} \\ \alpha &= \frac{eQ}{4\pi\varepsilon_0 c\hbar} \\ \alpha &= \frac{Q^2}{4\pi\varepsilon_0 c\hbar} \end{aligned} \qquad (227)$$

in the electromagnetic interaction. We denote e the constant value we measure in the lab for the electric charge of the electron [5] (paragraph 4.9). By Q we denote the electron's selfvariating charge.

The combination of equation (226) with each of equations (227), as well as the Schrödinger equation (225), give the exact same results for the hydrogen atom. For the TSV, the investigation of physical reality is put on the following terms: "In the application of the TSV, and in every case except of the generalized photon, the determination of the parameters λ and μ, is sought. This determination can be either theoretical or based on experimental data." The determination of the parameter b of the law of Selfvariations is made from the boundary conditions of the differential equations of the TSV, in the way we did in the application of paragraph 11. Of course, in the solution of that particular problem we cannot rule out the determination of the parameter b by other methods, theoretical or experimental.

In concluding this paragraph, it would be an omission not to refer to the work of Dirac. It is based on the investigation of equation (14) of Special Relativity [24, 25]. The development of the TSV showed that the Dirac equation is a special case of a wave equation when equation (112) holds. Its applicability concerns only flat spacetime. If spacetime is not flat in the hydrogen atom, the Dirac equation is only approximate. In every case, therefore for the hydrogen atom as well, the determination of the parameters λ and μ will give the exact wave equation.

14. Conclusions

In the study we present, it is proven that the interaction of material particles, the corpuscular structure of matter, and the quantum phenomena can be justified as a consequence of the law of Selfvariations. It is easily proven that the cosmological data are predicted and justified by the internal symmetry theorem. We have not included in the present article the analytical mathematical calculations about the consequences of the internal symmetry theorem.

The TSV predicts a unified interaction of material particles (USVI) as given by equation (86). The USVI predicts a common mechanism for all interactions. Every interaction is resolved into three individual terms, clearly distinct from each other, as they appear in the right part of equation (86), and with clearly distinct consequences in the USVI. Equation (86) gives the rate of change of energy and momentum, as well as the orbits of material particles.

We prove the wave equation (160) of the TSV, special cases of which are the Maxwell equations, the Schrödinger equation, and the related wave equations. We determine a single mathematical expression for the conservable physical quantities, and calculate the 4-vector j of the current density. The energy and momentum of a material particle are calculated by solving the wave equation (160) of the TSV.

From the study of the law of Selfvariations, equation (128) emerges as central for the theoretical prediction of the corpuscular structure of matter. The combination of equation (128) with the wave equation (160) clearly showcases the corpuscular structure and the wave behaviour of matter, as well as the relation between them. From this combination, a method for the calculation of the rest masses of material particles emerges.

The TSV has two degrees of freedom, since there are two parameters $\lambda, \mu \in \mathbb{C}$, $(\lambda, \mu) \neq (0,0)$ in equation (146), which can have arbitrary values within the web of equations and theorems of the TSV. The investigation of physical reality is reduced to the determination of the parameters λ and μ in every application of the TSV. The only exception is the case of the «generalized photon», where the system of differential equations of the TSV does not require the determination of parameters λ and μ for its solution.

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