The Extended VIKOR Method for Multiple Criteria Decision Making Problem Based on Neutrosophic Hesitant Fuzzy Set

Peide Liu [a,b,*], Lili Zhang [b]
[a,] *School of Economics and Management, Civil Aviation University of China, Tianjin 300300, China*
[b]*School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan Shandong 250014, China*
**The corresponding author: peide.liu@gmail.com*

Abstract: Neutrosophic hesitant fuzzy set is the generalization of neutrosophic set and the hesitant fuzzy set, and the VIKOR method is an effective decision making method which select the optimal alternative by the maximum ''group utility'' and minimum of an ''individual regret''. In this paper, we firstly introduced some operational laws, comparison rules and the Hamming distance measure of neutrosophic hesitant fuzzy set, and detailedly described the decision making steps of VIKOR method. Then we extended the VIKOR method to process the Neutrosophic hesitant fuzzy information, and proposed a multiple criteria decision making method based on neutrosophic hesitant fuzzy VIKOR method, and an illustrative example shows the effectiveness and feasibility of the proposed approach.

Key words: Neutrosophic hesitant fuzzy set; VIKOR method; Multiple criteria decision making (MCDM)

1. Introduction

Decision making has been widely used in the politics, economic, military, management and the other fields. But in real life, the decision-making information is often inconsistent, incomplete and indeterminate, and how to express the decision-making information is the primary task. Since the fuzzy set (FS) theory was proposed by Zadeh [1], fuzzy multiple criteria decision-making problems have been widely researched. But FS only has one membership, and it cannot denote some complex fuzzy information. For example, during voting, there are ten persons voting for an issue, three of them give the "agree'', four of them give the "disagree", and the others abstain from voting. Obviously, FS cannot fully express the polling information. Atanassov [2,3] defined the intuitionistic fuzzy set (IFS) by adding a non-membership function based on FS, i.e., IFS consists of truth-membership $T_A(x)$ and falsity-membership $F_A(x)$. The above example can be expressed by membership 0.3 and non-membership 0.4. However, IFSs can only handle incomplete information, and cannot deal with the inconsistent and indeterminate information. And the indeterminacy degree $1\text{-}T_A(x)\text{-}F_A(x)$ in IFSs has always been ignored. In some complicated decision making environment, IFS also has some limitations in some complex decision-making situation. For instance, when an expert is called to make an opinion about a statement, he/she may give the possibility of right is 0.5 and the possibility of false is 0.6 and the uncertain possibility is 0.2 [4]. On this occasion, IFS doesn't cope with this type of information. To handle this type of decision-making problems, Smarandache [5] proposed the neutrosophic set (NS) by adding an indeterminacy-membership function based on IFS. In NS, the truth-membership, false-membership and indeterminacy-membership are totally independent. To simplify neutrosophic set and applied it to practical problems, Wang et al. [6] defined a single valued neutrosophic set (SVNS) with some examples. Ye [7,8] defined the cross-entropy and the correlation coefficient of SVNS which was applied to single valued neutrosophic decision-making problems. At the same time, FS only has one membership which will limit some decision making problems. As a generalization of fuzzy set, Torra and Narukawa [9], Torra [10] put forward the hesitant fuzzy sets (HFSs) which use several possible values to instead of the membership degree. Then, Chen et al. [11] defined interval valued hesitant fuzzy sets (IVHFSs) which each membership degree is extended to interval numbers. Zhao et al. [12] developed hesitant triangular fuzzy set and series of aggregation operators for the hesitant triangular fuzzy sets based on the Einstein operations. Meng et al. [13] gave the linguistic hesitant fuzzy sets (LHFSs) and developed a series of linguistic hesitant fuzzy hybrid weighted operators. Farhadinia [14] and Ye [15] proposed the dual hesitant fuzzy sets and dual interval hesitant fuzzy sets. Peng et al. [16] represented the hesitant interval-valued intuitionistic fuzzy sets (HIVIFSs), and developed some hesitant interval intuitionistic fuzzy number weighted averaging operators based on t-conorms and t-norms. Ye [17] offered a neutrosophic hesitant fuzzy set by combining the hesitant fuzzy sets with single-valued neutrosophic sets (SVNHFS), and then some weighted averaging and weighted geometric operators for SVNHFS are developed.

As mentioned above, HFS and NS are extended in two directions based on FS, the HFS assigns the membership function a set of possible values, which is a good method to deal with uncertain information in practical decision making; however, it cannot process indeterminate and inconsistent information, while the NS can easily denote uncertainty, incomplete and inconsistent information. Obviously, each of them has its advantages and disadvantages. So, we further propose the neutrosophic hesitant fuzzy sets (NHFSs) by combining the HFS and NS, which extend truth-membership degree, indeterminacy-membership degree, and falsity-membership degree of an element to a fixed set to HFS, i.e., which may have a set of different values. Moreover, the VIKOR method is an important tools to process the fuzzy decision-making problems, which is based on the particular measure of "closeness" to the ‘‘ideal" solution, using linear programing method during the process of decision-making, and order the hesitant fuzzy numbers by index of attitude and choose the alternatives under the acceptable advantage and the stability of the decision-making process to get a compromise solution, which achieving the maximum ‘‘group utility’’ and minimum of an ‘‘individual regret’’.

In order to achieve the above purposes, this paper’s organization structure is as follows. In the next section, we represent the single valued neutrosophic set, HFSs, NHFSs, the traditional VIKOR method. In section 3, we extend the traditional VIKOR method based on neutrosophic hesitant fuzzy set, and an approach is given. In Section 4, we give a numerical example to elaborate the effectiveness and feasibility of our approach. In Section 5, we give the main concluding remarks of this paper.

2. Preliminaries

2.1 The single valued neutrosophic set

Definition 1 [18]. Let X be a universe of discourse, with a generic element in X denoted by x . A single valued neutrosophic set A in X is characterized by:

$$A = \{x(T_A(x), I_A(x), F_A(x)) | x \in X\} \tag{1}$$

where the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership, the indeterminacy-membership and the falsity-membership of the element $x \in X$ to the set A respectively. For each point x in X , we have $T_A(x), I_A(x), F_A(x) \in [0,1]$, and

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3 .$$

For convenience, we can use $x = (T_x, I_x, F_x)$ to denote an element x in SVNS, and the element x is called a single valued neutrosophic number (SVNN).

To compare two SVNNs, Smarandache and Vlâdâreanu offered the partial order relation between two neutrosophic numbers as follows.

Definition 2 [19]. For two SVNNs $x=(T_1,I_1,F_1)$ and $y=(T_2,I_2,F_2)$, iff (if and only if) $T_1 \le T_2$, $I_1 \ge I_2$, $F_1 \ge F_2$, then $x \le y$.

Obviously, in practical applications, many cases can't satisfy the above conditions. With respect to these, Ye [20] presented a comparison method based on the cosine similarity measure of a SVNN $x=(T, I, F)$ to ideal solution (1,0,0), and offered the definition of the cosine similarity:

$$S(x)=\frac{T}{\sqrt{T^2+I^2+F^2}}.$$

Definition 3 [20]. Suppose $x=(T_1,I_1,F_1)$ and $y=(T_2,I_2,F_2)$ are two SVNNs, if $S(x) \le S(y)$, then $x \le y$.

Definition 4. Let $x=(T_1,I_1,F_1)$ and $y=(T_2,I_2,F_2)$ are two SVNNs, then the normalized Hamming distance between x and y is defined as follows:

$$d(x,y)=\frac{1}{3}\left(|T_1-T_2|+|I_1-I_2|+|F_1-F_2|\right)$$

2.2 The hesitant fuzzy set (HFS)

Definition 4 [21]. Let X be a non-empty fixed set, a HFS A on X is in terms of a function $h_A(x)$ that when applied to X returns a subset of [0,1], which can be denoted by the following mathematical symbol:

$$A=\left\{\langle x,h_A(x)\rangle \middle| x\in X\right\} \tag{2}$$

where $h_A(x)$ is a set of some values in $[0,1]$, representing the possible membership degrees of the element $x \in X$ to A.For convenience, we call $h_A(x)$ a hesitant fuzzy element (HFE), denoted by h, which reads $h=\{\gamma | \gamma \in h\}$.

For any three HFEs h, h_1 and h_2, Torra [21] defined some operations as follows:

(1) $$h^c=\bigcup_{\gamma\in h}\{1-\gamma\} \tag{3}$$

(2) $$h_1\cup h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2} max\{\gamma_1,\gamma_2\}. \tag{4}$$

(3) $$h_1\cap h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2} \min\{\gamma_1,\gamma_2\}. \tag{5}$$

After that, Xia and Xu [22] gave four operations about the HFEs h, h_1, h_2 with a positive scale n:

(1) $$h^n=\bigcup_{\gamma\in h}\{\gamma^n\}, \tag{6}$$

(2) $$nh=\bigcup_{\gamma\in h}\{1-(-\gamma)^n\}, \tag{7}$$

(3) $$h_1\oplus h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1+\gamma_2-\gamma_1\gamma_2\}, \tag{8}$$

(4) $$h_1\otimes h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1\gamma_2\}. \tag{9}$$

Definition 5 [23]. Let h_1 and h_2 be two HFSs on $X=\{x_1,x_2,\cdots,x_n\}$, then the hesitant normalized Hamming distance measure between h_1 and h_2 is defined as:

$$\left\|h_1-h_2\right\|=\frac{1}{l}\sum_{j=1}^{l}\left|h_{1\sigma(j)}-h_{2\sigma(j)}\right|,$$

where $l(h)$ is the number of the elements in the h, in most cases, $l(h_1)\neq l(h_2)$, and for convenience, let $l=\max\{l(h_1),l(h_2)\}$. For operability, we should extend the shorter ones until both of them have the same length when compared. The best way to extend the shorter one is to add the same value in it. In fact, we can extend the shorter one by adding any value in it. The selection of the value mainly depends on the decision makers' risk preferences. Optimists anticipate desirable outcomes and may add the maximum value, while pessimists expect unfavorable outcomes and may add the minimum value.

For example, let $h_1=\{0.1,\ 0.2,\ 0.3\}$, $h_2=\{0.4,\ 0.5\}$, and $l(h_1)>l(h_2)$. For operability, we extend h_2 to $h_2=\{0.4,\ 0.4,\ 0.5\}$ until it has the same length of h_1, the optimist may extend h_2 as $h_2=\{0.4, 0.5, 0.5\}$ and the pessimist may extend it as $h_2=\{0.4, 0.4, 0.5\}$. Although the results may be different when we extend the shorter one by adding different values, this is reasonable because the decision makers' risk preferences can directly influence the final decision. The same situation can also be found in many existing Refs. [24–26]. In this paper, we assume that the decision makers are all pessimistic (other situations can be studied similarly).

2.3 The neutrosophic hesitant fuzzy set

In this section, we will propose the neutrosophic hesitant fuzzy set by combining neutrosophic set with hesitant fuzzy set.

Definition 5 [27,28]. Let X be a non-empty fixed set, a neutrosophic hesitant fuzzy set (NHFS) on X is expressed by:

$$N=\left\{\left\langle x,\tilde{t}(x),\tilde{i}(x),\tilde{f}(x)\right\rangle \middle| x\in X\right\}, \tag{10}$$

where $\tilde{t}(x)=\left\{\tilde{\gamma}\middle|\tilde{\gamma}\in\tilde{t}(x)\right\}$, $\tilde{i}(x)=\left\{\tilde{\delta}\middle|\tilde{\delta}\in\tilde{i}(x)\right\}$, and $\tilde{f}(x)=\left\{\tilde{\eta}\middle|\tilde{\eta}\in\tilde{f}(x)\right\}$ are three sets with some values in interval $[0,1]$, which represents the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of the element $x\in X$ to the set N, and satisfies these limits :

$\tilde{\gamma}\in[0,1]$, $\tilde{\delta}\in[0,1]$, $\tilde{\eta}\in[0,1]$ and $0\leq\sup\tilde{\gamma}^{+}+\sup\tilde{\delta}^{+}+\sup\tilde{\eta}^{+}\leq 3$, where

$\tilde{\gamma}^{+}=\bigcup_{\tilde{\gamma}\in\tilde{t}(x)}\max\{\tilde{\gamma}\}$, $\tilde{\delta}^{+}=\bigcup_{\tilde{\delta}\in\tilde{i}(x)}\max\{\tilde{\delta}\}$, and $\tilde{\eta}^{+}=\bigcup_{\tilde{\eta}\in\tilde{f}(x)}\max\{\tilde{\eta}\}$ for $x\in X$.

The $\tilde{n}=\{\tilde{t}(x),\tilde{i}(x),\tilde{f}(x)\}$ is called a neutrosophic hesitant fuzzy element (NHFE) which is the basic unit of the NHFS and is denoted by the symbol $\tilde{n}=\left\{\tilde{t},\tilde{i},\tilde{f}\right\}$.

Then, some basic operations of NHFEs are defined as follows:

Definition 6. Let $\tilde{n}_1=\left\{\tilde{t}_1,\tilde{i}_1,\tilde{f}_1\right\}$ and $\tilde{n}_2=\left\{\tilde{t}_2,\tilde{i}_2,\tilde{f}_2\right\}$ be two NHFEs in a non-empty fixed set X, then

(1) $\tilde{n}_1\cup\tilde{n}_2=\left\{\tilde{t}_1\cup\tilde{t}_2,\tilde{i}_1\cap\tilde{i}_2,\tilde{f}_1\cap\tilde{f}_2\right\}$ (11)

(2) $\tilde{n}_1\cap\tilde{n}_2=\left\{\tilde{t}_1\cap\tilde{t}_2,\tilde{i}_1\cup\tilde{i}_2,\tilde{f}_1\cap\tilde{f}_2\right\}$ (12)

Therefore, for two NHFEs $\tilde{n}_1, \tilde{n}_2$ and a positive scale $k > 0$, the operations can be defined as follows:

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \{\tilde{t}_1 \oplus \tilde{t}_2, \tilde{i}_1 \otimes \tilde{i}_2, \tilde{f}_1 \otimes \tilde{f}_2\} = \bigcup_{\tilde{\gamma}_2 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1, \tilde{\gamma}_2 \in \tilde{t}_2, \tilde{\delta}_2 \in \tilde{i}_2, \tilde{\eta}_2 \in \tilde{f}_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2, \delta_1\delta_2, \eta_1\eta_2\}$ (13)

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \{\tilde{t}_1 \otimes \tilde{t}_2, \tilde{i}_1 \oplus \tilde{i}_2, \tilde{f}_1 \oplus \tilde{f}_2\} = \bigcup_{\tilde{\gamma}_2 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1, \tilde{\gamma}_2 \in \tilde{t}_2, \tilde{\delta}_2 \in \tilde{i}_2, \tilde{\eta}_2 \in \tilde{f}_2} \{\gamma_1\gamma_2, \delta_1 + \delta_2 - \delta_1\delta_2, \eta_1 + \eta_2 - \eta_1\eta_2\}$ (14)

(3) $k\tilde{n}_1 = \bigcup_{\tilde{\gamma}_2 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1} \{1 - (1-\gamma_1)^k, \delta_1^k, \eta_1^k\}$ (15)

(4) $\tilde{n}_1^k = \bigcup_{\tilde{\gamma}_2 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1} \{\gamma_1^k, 1 - (1-\delta_1)^k, 1 - (1-\eta_1)^k\}$ (16)

Theorem 1. [29]. Let $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ be two NHFEs in a non-empty fixed set X, and $\eta, \eta_1, \eta_2 > 0$, then we have

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \tilde{n}_2 \oplus \tilde{n}_1$; (17)

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \tilde{n}_2 \otimes \tilde{n}_1$; (18)

(3) $\eta(\tilde{n}_1 \oplus \tilde{n}_2) = \eta\tilde{n}_1 \oplus \eta\tilde{n}_2$; (19)

(4) $\eta_1\tilde{n}_1 \oplus \eta_2\tilde{n}_1 = (\eta_1 + \eta_2)\tilde{n}_1$; (20)

(5) $\tilde{n}_1^{\eta} \otimes \tilde{n}_2^{\eta} = (\tilde{n}_2 \otimes \tilde{n}_1)^{\eta}$; (21)

(6) $\tilde{n}_1^{\eta_1} \otimes \tilde{n}_1^{\eta_2} = \tilde{n}_1^{\eta_1 + \eta_2}$; (22)

Definition 7. For an NHFE $\tilde{n}$,

$$s(\tilde{n}) = \left[\frac{1}{l}\sum_{i=1}^{l}\tilde{\gamma}_i + \frac{1}{p}\sum_{i=1}^{p}(1-\tilde{\delta}_i) + \frac{1}{q}\sum_{i=1}^{q}(1-\tilde{\eta}_i)\right] \Big/ 3 \quad (23)$$

is called the score function of $\tilde{n}$, where l, p, q are the number of the values in $\tilde{\gamma}$, $\tilde{\delta}$, $\tilde{\eta}$, respectively. Obviously, $s(\tilde{n})$ is a value belonged to $[0,1]$.

Suppose $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ are any two NHFEs, the comparison method of NHFEs is expressed as follows [17, 18]:

(1) If $S(\tilde{n}_1) > S(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$;

(2) If $S(\tilde{n}_1) < S(\tilde{n}_2)$, then $\tilde{n}_1 < \tilde{n}_2$;

(3) If $S(\tilde{n}_1) = S(\tilde{n}_2)$, then $\tilde{n}_1 = \tilde{n}_2$.

Definition 8. Let $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ are any two NHFEs, then the normalized Hamming distance between $\tilde{n}_1$ and $\tilde{n}_2$ is defined as follows:

$$d(\tilde{n}_1, \tilde{n}_2) = \|\tilde{n}_1 - \tilde{n}_2\| = \frac{1}{3}\left(|\tilde{\gamma}_1 - \tilde{\gamma}_2| + |\tilde{\delta}_1 - \tilde{\delta}_2| + |\tilde{\eta}_1 - \tilde{\eta}_2|\right)$$
$$= \frac{1}{3}\left(\left|\frac{1}{l}\sum_{j=1}^{l}\left|\tilde{\gamma}_{1\sigma(j)} - \tilde{\gamma}_{2\sigma(j)}\right|\right| + \left|\frac{1}{l}\sum_{j=1}^{l}\left|\tilde{\delta}_{1\sigma(j)} - \tilde{\delta}_{2\sigma(j)}\right|\right| + \left|\frac{1}{l}\sum_{j=1}^{l}\left|\tilde{\eta}_{1\sigma(j)} - \tilde{\eta}_{2\sigma(j)}\right|\right|\right)$$

2.4 VIKOR method

The VIKOR method was introduced for multi-criteria optimization problem. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria, which can help the decision makers to get a final solution. Here, the compromise solution is a feasible solution which is the closest to the ideal, and a compromise means an agreement established by mutual concessions [30]. It introduces the multi-criteria ranking index on the base of the particular measure of "closeness" to the "ideal" solution [31].The multi-criteria measure for compromise ranking is developed from the L_p-metric used as an aggregating function in a compromise programming method [32]. Development of the VIKOR method is started with the following form of L_p-metric:

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[(f_j^* - f_{ij}) / (f_j^* - f_j^-) \right]^p \right\}^{1/p} \quad 1 \le p \le \infty; i = 1,2,3,..., m.$$

In the VIKOR method $L_{1,i}$ (as S_i)and $L_{\infty,i}$ (as R_i)are used to formulate ranking measure. The solution obtained by $\min S_i$ is with a maximum group utility ("majority" rule), and the solution obtained by $\min R_i$ is with a minimum individual regret of the "opponent".

Assuming that each alternative is evaluated by each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The various m alternatives are denoted as $A_1, A_2, A_3, ..., A_m$. For alternative A_i, the rating of the j th aspect is denoted by f_{ij}, i.e. f_{ij} is the value of j th criterion function for the alternative A_i; n is the number of criteria.

The compromise ranking algorithm of the VIKOR method has the following steps:

(1) Determine the best f_j^* and the worst f_j^- values of all criterion functions $j = 1,2,\ldots,n$. If the jth function represents a benefit then:

$$f_j^* = \max_i f_{ij}, \ f_j^- = \min_i f_{ij}$$

(2) Compute the values S_i and R_i; $i = 1,2,..., m$, by these relations:

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-),$$

$$R_i = \max_j w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-),$$

where w_j are the weights of criteria, expressing their relative importance.

(3) Compute the values $Q_i : i = 1,2,..., m$, by the following relation:

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*)$$

where

$S^* = \min_i S_i, S^- = \max_i S_i$,

$R^* = \min_i R_i, R^- = \max_i R_i$

v is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here suppose that $v = 0.5$.

(4) Rank the alternatives, sorting by the values S , R and Q in decreasing order. The results are three ranking lists.

(5) Propose as a compromise solution the alternative A' , which is ranked the best by the measure Q (Minimum) if the following two conditions are satisfied:

C1. Acceptable advantage: $Q(A'') - Q(A') \geq DQ$, where A'' is the alternative with second position in the ranking list by Q;DQ=1/(m-1); m is the number of alternatives.

C2. Acceptable stability in decision making: Alternative A' must also be the best ranked by S or/and R . This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" $(v < 0.5)$. Here, v is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A' and A'' are compromise solutions if only condition C2 is not satisfied, or
- Alternatives A' , A'' ,…, $A^{(M)}$ are compromise solutions if condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

The best alternative, ranked by Q , is the one with the minimum value of Q . The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the "advantage rate". VIKOR is an effective tool in multi-criteria decision making, particularly in a situation where the decision maker is not able, or does not know how to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" (represented by min S) of the "majority", and a minimum of the "individual regret" (represented by min R) of the "opponent". The compromise solutions could be the basis for negotiations, involving the decision maker's preference by criteria weights.

3. VIKOR method for decision making problem with neutrosophic hesitant fuzzy numbers

According to these facts, when determining the exact values of the criteria is difficult or impossible, the neutrosophic hesitant fuzzy set is a very useful tool to deal with uncertainty in avoiding such issues in which each criteria can be described as a neutrosophic hesitant fuzzy set defined in terms of the opinions of decision makers [33] and permits the membership having a set of possible values. Since, there is more appropriate to consider the values of the criteria as neutrosophic hesitant fuzzy element, where neutrosophic hesitant fuzzy elements are benefit criteria.

Therefore, in the present paper, we extend the VIKOR method to solve MADM problem with the neutrosophic hesitant fuzzy set information. To do this, suppose that a decision matrix, denoted by the neutrosophic hesitant fuzzy elements, has the following form(Table 1):

Table 1 Decision making matrix with the neutrosophic hesitant fuzzy set information

	C_1	C_2	$\cdots$	C_n
A_1	$\tilde{n}_{11}$	$\tilde{n}_{12}$	$\cdots$	$\tilde{n}_{1n}$
A_2	$\tilde{n}_{21}$	$\tilde{n}_{22}$	$\cdots$	$\tilde{n}_{2n}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m	$\tilde{n}_{m1}$	$\tilde{n}_{m2}$	$\cdots$	$\tilde{n}_{mn}$

For a multiple criteria decision making problem, let $A=\{A_1, A_2, \cdots, A_m\}$ be a collection of m alternatives, $C=\{C_1, C_2, \cdots, C_m\}$ be a collection of n criteria, whose weight vector is $w=(w_1, w_2, \cdots w_n)^T$ satisfying $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$. Suppose that $\tilde{n}_{ij} = (\tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij})$ is the evaluation value of the criteria C_j with respect to the alternative A_i which is expressed in the form of the neutrosophic hesitant fuzzy information, where $\tilde{t}_{ij} = \{\tilde{\gamma}_{ij} \mid \tilde{\gamma}_{ij} \in \tilde{t}_{ij}\}$, $\tilde{i}_{ij} = \{\tilde{\delta}_{ij} \mid \tilde{\delta}_{ij} \in \tilde{i}_{ij}\}$ and $\tilde{f}_{ij} = \{\tilde{\eta}_{ij} \mid \tilde{\eta}_{ij} \in \tilde{f}_{ij}\}$ are three collections of some values in interval $[0,1]$, which represent the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees, and satisfies following limits:

$\tilde{\gamma} \in [0,1],\ \ \tilde{\delta} \in [0,1],\ \ \tilde{\eta} \in [0,1]$, and $0 \le \sup\tilde{\gamma}^+ + \sup\tilde{\delta}^+ + \sup\tilde{\eta}^+ \le 3$, where

$\tilde{\gamma}^+ = \bigcup_{\tilde{\gamma}_{ij} \in \tilde{t}_{ij}} \max\{\gamma_{ij}\}, \tilde{\delta}^+ = \bigcup_{\tilde{\delta}_{ij} \in \tilde{i}_{ij}} \max\{\delta_{ij}\}, \tilde{\eta}^+ = \bigcup_{\tilde{\eta}_{ij} \in \tilde{f}_{ij}} \max\{\eta_{ij}\}$ .Then we can rank the order of the alternatives.

The procedures of the proposed method as follows:

Step 1. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS):

$A^* = \{\tilde{n}_1^*, \cdots, \tilde{n}_n^*\}$, where $\tilde{n}_j^* = \max\{S(\tilde{n}_{1j}), \cdots, S(\tilde{n}_{mj})\},\ j = 1,2,\cdots,n$

$A^- = \{\tilde{n}_1^-, \cdots, \tilde{n}_n^-\}$, where $\tilde{n}_j^- = \min\{S(\tilde{n}_{1j}), \cdots, S(\tilde{n}_{mj})\},\ j = 1,2,\cdots,n$

Step 2. In this step, compute S_i and R_i as below:

$$S_i = \sum_{j=1}^{n} w_j \left\|\tilde{n}_j^* - \tilde{n}_{ij}\right\| \Big/ \left\|\tilde{n}_j^* - \tilde{n}_j^-\right\|,\ i = 1,2,..., m$$

$$R_i = \max_j\ w_j \left\|\tilde{n}_j^* - \tilde{n}_{ij}\right\| \Big/ \left\|\tilde{n}_j^* - \tilde{n}_j^-\right\|,\ i = 1,2,..., m$$

Step 3. Compute the values $Q_i : i = 1,2,..., m$, by the following relation:

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*)$$

where

$$S^* = \min_i S_i, S^- = \max_i S_i\ ,$$

$R^* = \min_i R_i, R^- = \max_i R_i$

where v is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here suppose that $v = 0.5$

Step 4. Rank the alternatives, sorting by the values S, R and Q in decreasing order. The results are three ranking lists.

Step 5. Propose as a compromise solution the alternative A', which is ranked the best by the measure Q (Minimum) if the following two conditions are satisfied:

C1. Acceptable advantage: $Q(A'') - Q(A') \geq DQ$, where A'' is the alternative with second position in the ranking list by Q;DQ=1/(m-1); *m* is the number of alternatives.

C2. Acceptable stability: Alternative A' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" $(v < 0.5)$. Here, v is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

(i) Alternatives A' and A'' if only condition C2 is not satisfied, or

(ii) Alternatives A', A'', …, $A^{(M)}$ if condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

4. An numerical example

We consider an example [34] where one investment company intends to select an enterprise from the following four alternatives to invest. The four enterprises are marked by $A_i (i = 1,2,3.4)$, and they are measured by three criteria: (1) C_1 (the risk index); (2) C_2 (the growth index); (3) C_3 (environmental impact index), and the evaluation values are denoted by NHFNs and their weight is $w = (0.35,0.25,0.4)^T$.

The decision matrix R is shown in the Table 2.

Table 2The neutrosophic hesitant fuzzy decision matrix

	C_1	C_2	C_3
A_1	{{0.4,0.4,0.5},{0.2}, {0.4}}	{{0.5,0.6 },{0.3},{0.3,0.4}}	{{0.3},{0.2},{0.5,0.6}}
A_2	{{0.7},{0.2},{0.2,0.3}}	{{0.7},{0.1},{0.3}}	{{0.7},{0.2},{0.2}}
A_3	{{0.4,0.6},{0.4},{0.3}}	{{0.6},{0.3},{0.5}}	{{0.6},{0.2,0.3},{0.3}}
A_4	{{0.8},{0.1},{0.2}}	{{0.7},{0.1},{0.2}}	{{0.5},{0.3},{0.2,0.3}}

Step 1. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS):

$A^* = \{\tilde{n}_1^*, \tilde{n}_2^*, \tilde{n}_3^*\} = \{\{\{0.8\},\{0.1\},\{0.2\}\}, \{\{0.7\},\{0.1\},\{0.2\}\}, \{\{0.7\},\{0.2\},\{0.2\}\}\}$

$A^- = \{\tilde{n}_1^-, \tilde{n}_2^-, \tilde{n}_3^-\} = \{\{\{0.4,0.6\},\{0.4\},\{0.3\}\}, \{\{0.6\},\{0.3\},\{0.5\}\}, \{\{0.3\},\{0.2\},\{0.5,0.6\}\}\}$

Step 2. In this step, compute S_i and R_i as below:

$$S_1=\frac{w_1\|n_1^*-n_{11}\|}{\|n_1^*-n_1^-\|}+\frac{w_2\|n_2^*-n_{12}\|}{\|n_2^*-n_2^-\|}+\frac{w_3\|n_3^*-n_{13}\|}{\|n_3^*-n_3^-\|}=0.942$$

$$S_2=\frac{w_1\|n_1^*-n_{21}\|}{\|n_1^*-n_1^-\|}+\frac{w_2\|n_2^*-n_{22}\|}{\|n_2^*-n_2^-\|}+\frac{w_3\|n_3^*-n_{23}\|}{\|n_3^*-n_3^-\|}=0.166$$

$$S_3=\frac{w_1\|n_1^*-n_{31}\|}{\|n_1^*-n_1^-\|}+\frac{w_2\|n_2^*-n_{32}\|}{\|n_2^*-n_2^-\|}+\frac{w_3\|n_3^*-n_{33}\|}{\|n_3^*-n_3^-\|}=0.733$$

$$S_4=\frac{w_1\|n_1^*-n_{41}\|}{\|n_1^*-n_1^-\|}+\frac{w_2\|n_2^*-n_{42}\|}{\|n_2^*-n_2^-\|}+\frac{w_3\|n_3^*-n_{43}\|}{\|n_3^*-n_3^-\|}=0.187$$

$$R_1=\max_3\left\{\frac{w_1\|n_1^*-n_{11}\|}{\|n_1^*-n_1^-\|},\frac{w_2\|n_2^*-n_{12}\|}{\|n_2^*-n_2^-\|},\frac{w_3\|n_3^*-n_{13}\|}{\|n_3^*-n_3^-\|}\right\}=0.4$$

$$R_2=\max_3\left\{\frac{w_1\|n_1^*-n_{21}\|}{\|n_1^*-n_1^-\|},\frac{w_2\|n_2^*-n_{22}\|}{\|n_2^*-n_2^-\|},\frac{w_3\|n_3^*-n_{23}\|}{\|n_3^*-n_3^-\|}\right\}=0.125$$

$$R_3=\max_3\left\{\frac{w_1\|n_1^*-n_{31}\|}{\|n_1^*-n_1^-\|},\frac{w_2\|n_2^*-n_{32}\|}{\|n_2^*-n_2^-\|},\frac{w_3\|n_3^*-n_{33}\|}{\|n_3^*-n_3^-\|}\right\}=0.35$$

$$R_4=\max_3\left\{\frac{w_1\|n_1^*-n_{41}\|}{\|n_1^*-n_1^-\|},\frac{w_2\|n_2^*-n_{42}\|}{\|n_2^*-n_2^-\|},\frac{w_3\|n_3^*-n_{43}\|}{\|n_3^*-n_3^-\|}\right\}=0.187$$

Step 3. Let $v=0.5$, compute the values $Q_i\,(i=1,2,3,4)$:

$$Q_1=1,\ \ Q_2=0,\ \ Q_3=0.774,\ \ Q_4=0.126$$

Step 4. Rank the alternatives, sorting by the values S , R and Q in decreasing order. The results are three ranking lists, which is depicted in Table 3.

Table 3 The ranking and the compromise solutions.

	A_1	A_2	A_3	A_4	Ranking	Compromise solutions
S	0.942	0.166	0.733	0.187	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2
R	0.4	0.125	0.35	0.187	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2
$Q(v=0.5)$	0.997	0.003	0.774	0.126	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2

Step 5. The ranking of alternatives by Q in decreasing order, the alternative with first position is A_2 with $Q(A_2)=0.003$, and A_4 is the alternative with second position with $Q(A_4)=0.126$. As DQ=1/(m-1)=1/（4-1）=0.333, so

$$Q(A_4)-Q(A_2)=0.123<0.333$$

Which is not satisfied $Q(A_4)-Q(A_2)\ge\frac{1}{4-1}$, but alternative A_2 is the best ranked by S and R ,which satisfies the condition two. By computing, we get:

$$Q(A_4)-Q(A_2)=0.123<0.333$$
$$Q(A_3)-Q(A_2)=0.771>0.333$$

so A_2 , A_4 are both compromise solutions.

5. Conclusion

Neutrosophic hesitant fuzzy set is the generalization of neutrosophic set and the hesitant fuzzy set. Some operational laws, comparison rules of neutrosophic hesitant fuzzy set and the Hamming distance between two neutrosophic hesitant fuzzy numbers are defined. For multiple criteria decision making with neutrosophic hesitant fuzzy sets, the traditional VIKOR method is extended, and an approach is given. In this method, which is based on the particular measure of “closeness” to the ‘‘ideal” solution, using linear programing method during the process of decision-making, and order the hesitant fuzzy numbers by index of attitude and choose the alternatives under the acceptable advantage and the stability of the decision-making process to get a compromise solution, which achieving the maximum ‘‘group utility’’ and minimum of an ‘‘individual regret’’. This method has its own advantages compared with other multiple criteria decision making method based on distance, but it can only solve the decision making problems with criteria is neutrosophic hesitant fuzzy numbers and fixed weights, in the case of uncertain weights is universal in real life, which needs further study.

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