The Problem of Entropy Production in the Classic Rule of Combination in the Dezert-Smarandache Theory

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Abstract—In this paper, the classic rule of combination in the Dezert-Smarandache theory is found to be not convergent with the number increase of evidential sources since it leaves out the denominator in the Dempster's rule. That is, it is a process of entropy productions. This means the final result of combination is more uncertain, and can not give a good decision. Several illustrative examples are given to explain and testify this problem. Finally, a conclusion is given, in order to point out the necessity of developing some simple and convergent combinational rules in the Dezert-Smarandache theory.

Keywords-evidence reasoning; Dezert-Smarandache theory; combinational rule; belief function theory;

I. INTRODUCTION

With the development of computer science and technology, the belief function theory as one of important intelligent information processing technologies is more and more popular. Many experts and scholars made great achievements in this field[1]-[13]. In the Shafer model, an ultimate refinement of the problem was possible so that singleton focal elements were supposed to be exclusive and exhaustive in the closed-world. Therefore, Dempster [6] proposed a well-known combinational rule on the basis of the Shafer model in (1) according to the Dempster-Shafer theory (DST). This rule was widely applied in different fields. However, since it can not deal with the highly conflictive sources of evidence and its computational amount exponentially increases with more and more focal elements, its application strongly suffers from limitations.

$$\begin{cases} m(\varphi) = 0 \\ m(A) = \dfrac{\sum\limits_{\substack{X,Y\in 2^{\Theta},\\ X\cap Y = A}} m_1(X)m_2(Y)}{1-\sum\limits_{\substack{X,Y\in 2^{\Theta},\\ X\cap Y=\varphi}} m_1(X)m_2(Y)} & \forall(A \neq \varphi)\in 2^{\Theta} \end{cases} \quad (1)$$

Murphy [7] proposed a convex combination rule. This rule consists actually in a simple arithmetic average of belief functions associated with m_1 and m_2. That is,

$$Bel_m(A) = \frac{1}{2}[Bel_1(A) + Bel_2(A)] \quad A \in 2^{\Theta} \quad (2)$$

Dubois and Prade [8] in 1986 proposed a disjunctive rule of combination, that is,

$$\begin{cases} m_{\cup}(\varphi) = 0 \\ m_{\cup}(A) = \sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cup Y = A}} m_1(X)m_2(Y) & \forall(A \neq \varphi)\in 2^{\Theta} \end{cases} \quad (3)$$

In addition, according to the opinion of Dubois and Prade [9], the two sources were reliable when they were not in conflicts, but one of them was right when a conflict occurs. Their combinational rule was given as follows:

$$\begin{cases} m(\varphi) = 0 \\ m(A) = \sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y = A\\ X\cap Y \neq \varphi}} m_1(X)m_2(Y) \\ \quad + \sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y = A\\ X\cap Y = \varphi}} m_1(X)m_2(Y) & \forall A\in 2^{\Theta}, A\neq\varphi \end{cases} \quad (4)$$

According to the opinion of Smets [10], the power-set space was an open-world, and the positive mass may be on the null/empty set, and the division is eliminated by $1-\sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y=\varphi}} m_1(X)m_2(Y)$ like Dempsters rule. His combinational rule for two independent (equally reliable) sources of evidence was given as follows:

$$\begin{cases} m(\varphi) = k_{12} = \sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y=\varphi}} m_1(X)m_2(Y) \\ m(A) = \sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y = A}} m_1(X)m_2(Y) & \forall(A\neq\varphi)\in 2^{\Theta} \end{cases} \quad (5)$$

According to the opinion of Yager [11][12], in case of conflict, the result was not reliable, so that the conflict factor $1-\sum\limits_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y=\varphi}} m_1(X)m_2(Y)$ played the role of an absolute discounting term added to the weight of ignorance. The commutative (but not associative) Yager rule was given as follows:

$$
\begin{cases}
m(\varphi) = 0 \\
m(A) = \sum\limits_{\substack{X,Y \in 2^{\Theta} \\ X \cap Y = A}} m_1(X)\, m_2(Y) \quad \forall \begin{pmatrix} A \neq \varphi, \\ A \neq \Theta \end{pmatrix} \in 2^{\Theta} \\
m(\Theta) = m_1(\Theta)\, m_2(\Theta) \\
\quad + \sum\limits_{\substack{X,Y \in 2^{\Theta} \\ X \cap Y = \varphi}} m_1(X)\, m_2(Y) \qquad \text{when } A = \Theta
\end{cases}
\tag{6}
$$

Dezert and Smarandche have recently proposed a DSm combinational rule in more refined framework[13], in order to focus on the fusion of uncertain, highly conflictive and imprecise sources of evidence.

The differences between the Dempster-Shafer theory (DST) [6] and the Dezert-Smarandache theory (DSmT) [13] are:

In the Shafer model, one considers a finite frame of possible exhaustive solutions $\Theta = \{\theta_1, \ldots, \theta_n\}$, and assumes the exclusivity of θ_i and defines belief masses on the classic power set $2^{\Theta} \triangleq (\Theta, \cup)$. In DSmT, the belief masses can be directly defined on the Dedekind's lattice/hyper-power set $D^{\Theta} \triangleq (\Theta, \cup, \cap)$ and even on the super-power set $S^{\Theta} \triangleq (\Theta, \cup, \cap, c(.))$. In the sequel, the generic notation G^{Θ} is used for denoting either 2^{Θ}, D^{Θ} or S^{Θ}. A quantitative basic belief assignment (bba) is a mapping $m(.) : G^{\Theta} \to [0, 1]$ associated to a given body of evidence B, it satisfies $m(\emptyset) = 0$ and $\sum_{A \in G^{\Theta}} m(A) = 1$.

In the free or static DSmT model $M^f(\Theta)$, for two reliable evidence sources, i.e. S_1 and S_2 over the same frame Θ, their belief functions $Bel_1(.)$ and $Bel_2(.)$ are associated with gbba $m_1(.)$ and $m_2(.)$. The classic DSmT rule of combination (DSmC) in (7) is given in [13]. This rule submits to the conjunctive consensus of sources.

$$
\forall C \in D^{\Theta},\; m_{M^f(\Theta)}(C) = \sum_{\substack{A,B \in D^{\Theta} \\ A \cap B = C}} m_1(A)\, m_2(B) \tag{7}
$$

Seen from (7), the classic DSmT rule of combination (DSmC) eliminates $1 - \sum_{\substack{X,Y \in 2^{\Theta} \\ X \cap Y = \varphi}} m_1(X)\, m_2(Y)$ from the denominator in the DST rule of combination [6]. It overcomes one of fatal deficiencies of the DST.

However, although the DSmC can deal with highly conflictive sources of evidence, etc., it has a fatal deficiency, i.e. entropy production. In the section II, we will explain it in detail.

II. The entropy production of the DSmC

The classic DSm rule of combination is difficultly convergent, the combination is a process of entropy production.

Example 1. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2\}$. Their gbbas are given in (8), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

$$
\begin{array}{ccc}
S & m(\theta_1) & m(\theta_2) \\
s_1 & a & 1-a \\
s_2 & a & 1-a \\
\vdots & \vdots & \vdots \\
s_n & a & 1-a
\end{array}
\tag{8}
$$

therefore, we get $m(\theta_1) = a^n$, $m(\theta_2) = (1-a)^n$, $m(\theta_1 \cap \theta_2) = a(1-a)\left[\frac{1-a^{n-1}}{1-a} + \frac{1-(1-a)^{n-1}}{a}\right]$. When $n \to \infty$, $m(\theta_1) \to 0$, $m(\theta_2) \to 0$, $m(\theta_1 \cap \theta_2) \to 1$. With the increase of $m(\theta_1 \cap \theta_2)$, the conflict value becomes greater and greater, and the result becomes more and more uncertain.

Example 2. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2\}$. Their gbbas are given in (9), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

$$
\begin{array}{ccc}
S & m(\theta_1) & m(\theta_2) \\
s_1 & 1-a & a \\
s_2 & a & 1-a \\
\vdots & \vdots & \vdots \\
s_n & a & 1-a
\end{array}
\tag{9}
$$

when $a \geq \frac{1}{2}$, we get $m(\theta_1) = a^{n-1}(1-a)$, $m(\theta_2) = a(1-a)^{n-1}$, $m(\theta_1 \cap \theta_2) = a^n + (1-a)^n + a(1-a)\left[\frac{1-a^{n-2}}{1-a} + \frac{1-(1-a)^{n-2}}{a}\right]$, $m(\theta_1) \geq m(\theta_2)$. When $n \to \infty$, $m(\theta_1) \to 0$, $m(\theta_2) \to 0$ and $m(\theta_1 \cap \theta_2) \to 1$.

Example 3. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2\}$. Their gbbas are given in (10), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

$$
\begin{array}{ccc}
S & m(\theta_1) & m(\theta_2) \\
s_1 & a & 1-a \\
s_2 & 1-a & a \\
\vdots & \vdots & \vdots \\
s_{i+1} & 1-a & a \\
\vdots & \vdots & \vdots \\
s_n & a & 1-a
\end{array}
\tag{10}
$$

we get $m(\theta_1) = a^{n-i}(1-a)^i$, $m(\theta_2) = a^i(1-a)^{n-i}$, $m(\theta_1 \cap \theta_2) = \left[a^{n-i}\frac{1-a^i}{1-a} + (1-a)^{n-i}\frac{1-(1-a)^i}{a}\right] + a(1-a)\left[\frac{1-a^{n-i-2}}{1-a} + \frac{1-(1-a)^{n-i-2}}{a}\right]$. When $n \to \infty$, $m(\theta_1) \to 0$, $m(\theta_2) \to 0$ and $m(\theta_1 \cap \theta_2) \to 1$.

Example 4. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Their gbbas are given in (11), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

S	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_3)$
s_1	a	b	$1-a-b$
s_2	c	$1-c$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_n	c	$1-c$	0

(11)

we get $m(\theta_1) = \lim\limits_{n\to\infty} ac^{n-1} = 0$,

$m(\theta_2) = \lim\limits_{n\to\infty} b(1-c)^{n-1} = 0$,

$$\begin{aligned} &m(\theta_1 \cap \theta_2) \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} a(1-c) + bc + ac(1-c) + bc(1-c) \\ + ac^2(1-c) + bc(1-c)^2 + \cdots + \\ ac^{n-2}(1-c) + bc(1-c)^{n-2} \end{array} \right\}, \\ &= \lim_{n\to\infty} \left[bc\frac{1-(1-c)^{n-1}}{c} + a(1-c)\frac{1-c^{n-1}}{1-c} \right] \\ &= a + b \end{aligned}$$

$m(\theta_1 \cap \theta_3) = \lim\limits_{n\to\infty} c^{n-1}(1-a-b) = 0$,

$m(\theta_2 \cap \theta_3) = \lim\limits_{n\to\infty} (1-c)^{n-1}(1-a-b) = 0$,

$$\begin{aligned} &m(\theta_1 \cap \theta_2 \cap \theta_3) \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} c(1-a-b)(1-c) + c(1-a-b)(1-c) + \\ c^2(1-a-b)(1-c) + c(1-a-b)(1-c)^2 \\ + \cdots + c^{n-2}(1-a-b)(1-c) + \\ c(1-a-b)(1-c)^{n-2} \end{array} \right\} \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} (1-a-b)(1-c)\left[\dfrac{c\left(1-c^{n-2}\right)}{1-c}\right] + \\ c(1-a-b)\left[\dfrac{(1-c)\left(1-(1-c)^{n-2}\right)}{c}\right] \end{array} \right\} \\ &= 1-a-b \end{aligned}$$

Example 5. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2\}$. Their gbbas are given in (12), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

S	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_1 \cap \theta_2)$
s_1	a	b	$1-a-b$
s_2	a	b	$1-a-b$
$\vdots$	$\vdots$	$\vdots$	$1-a-b$
s_n	a	b	$1-a-b$

(12)

when $n \to \infty$, we get $m(\theta_2) = \lim\limits_{n\to\infty} b^n = 0$, $m(\theta_2) = \lim\limits_{n\to\infty} b^n = 0$, $m(\theta_1 \cap \theta_2) = \lim\limits_{n\to\infty} 1 - a^n - b^n = 1$.

Example 6. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2\}$. Their gbbas are given in (13), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

S	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_1 \cup \theta_2)$
s_1	a	b	$1-a-b$
s_2	a	b	$1-a-b$
$\vdots$	$\vdots$	$\vdots$	$1-a-b$
s_n	a	b	$1-a-b$

(13)

when $n \to \infty$, we get

$$\begin{aligned} &m(\theta_1) \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} a(1-b)^{n-1} + a(1-a-b)(1-b)^{n-2} + \\ a(1-a-b)^2(1-b)^{n-3} + \cdots \\ + a(1-a-b)^{n-2}(1-b) + a(1-a-b)^{n-1} \end{array} \right\} \\ &= \lim_{n\to\infty} a \left\{ \begin{array}{l} (1-b)^{n-1} + (1-a-b)(1-b)^{n-2} + \\ (1-a-b)^2(1-b)^{n-3} + \cdots \\ + (1-a-b)^{n-2}(1-b) + (1-a-b)^{n-1} \end{array} \right\} \\ &= \lim_{n\to\infty} a(1-b)^{n-1} \left\{ \begin{array}{l} 1 + \dfrac{1-a-b}{1-b} + \left[\dfrac{1-a-b}{1-b}\right]^2 + \\ \cdots + \left[\dfrac{1-a-b}{1-b}\right]^{n-1} \end{array} \right\} \\ &= \lim_{n\to\infty} a(1-b)^{n-1} \left[\frac{1-\left(\frac{1-a-b}{1-b}\right)^n}{1-\frac{1-a-b}{1-b}} \right] \\ &= 0 \end{aligned}$$

$$\begin{aligned} &m(\theta_2) \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} b(1-a)^{n-1} + b(1-a-b)(1-a)^{n-2} + \\ b(1-a-b)^2(1-a)^{n-3} + \cdots + \\ b(1-a-b)^{n-2}(1-a) + b(1-a-b)^{n-1} \end{array} \right\} \\ &= \lim_{n\to\infty} b(1-a)^{n-1} \left\{ \begin{array}{l} 1 + \dfrac{1-a-b}{1-a} + \left[\dfrac{1-a-b}{1-a}\right]^2 \\ + \cdots + \left[\dfrac{1-a-b}{1-a}\right]^{n-1} \end{array} \right\} \\ &= \lim_{n\to\infty} b(1-a)^{n-1} \left[\frac{1-\left(\frac{1-a-b}{1-a}\right)^n}{1-\frac{1-a-b}{1-a}} \right] \\ &= 0 \end{aligned}$$

$$\begin{aligned} &m(\theta_1 \cap \theta_2) \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} 2ab + b\left[(1-b)^2 - (1-a-b)^2\right] + \\ a\left[(1-a)^2 - (1-a-b)^2\right] + \\ b\left[(1-b)^3 - (1-a-b)^3\right] \\ + a\left[(1-a)^3 - (1-a-b)^3\right] + \cdots + \\ b\left[(1-b)^{n-1} - (1-a-b)^{n-1}\right] + \\ a\left[(1-a)^{n-1} - (1-a-b)^{n-1}\right] \end{array} \right\} \\ &= \lim_{n\to\infty} \left\{ \begin{array}{l} (1-b)^2\left[1-(1-b)^{n-2}\right] - \\ (1-a-b)^2\left[1-(1-a-b)^{n-2}\right] + 2ab + \\ (1-a)^2\left[1-(1-a)^{n-2}\right] \end{array} \right\} \\ &= (1-b)^2 - (1-a-b)^2 + 2ab + (1-a)^2 \\ &= 1 \end{aligned}$$

$m(\theta_1 \cup \theta_2) = \lim\limits_{n\to\infty} (1-a-b)^n = 0$

Example 7. Suppose that there are n evidential sources, they have the same discernment framework $\Theta =$

$\{\theta_1, \theta_2, \theta_3\}$. Their gbbas are given in (14), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

S	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_1 \cap \theta_3)$
s_1	a	b	$1-a-b$
s_2	a	b	$1-a-b$
$\vdots$	$\vdots$	$\vdots$	$1-a-b$
s_n	a	b	$1-a-b$

(14)

when $n \to \infty$, we get $m(\theta_1) = \lim_{n\to\infty} a^n = 0$, $m(\theta_2) = \lim_{n\to\infty} b^n = 0$,

$$
\begin{aligned}
m(\theta_1 \cap \theta_3) &= \lim_{n\to\infty} (1-a-b) \left[\begin{array}{l} (1-b)^{n-1} + \\ a(1-b)^{n-2} \\ + a^2(1-b)^{n-3} \\ + \cdots + a^{n-1} \end{array} \right] \\
&= \lim_{n\to\infty} (1-a-b)(1-b)^{n-1} \left[\begin{array}{l} 1 + \frac{a}{1-b} + \left(\frac{a}{1-b}\right)^2 \\ + \cdots \left(\frac{a}{1-b}\right)^{n-1} \end{array} \right] \\
&= \lim_{n\to\infty} (1-a-b)(1-b)^{n-1} \frac{1-\left(\frac{a}{1-b}\right)^n}{1-\frac{a}{1-b}} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
&m(\theta_1 \cap \theta_2) \\
&= \lim_{n\to\infty} ab \left[\begin{array}{l} 3(a+b)^{n-2} + (a^2+b^2)(a+b)^{n-4} \\ + (a^3+b^3)(a+b)^{n-5} + \cdots \\ + (a^{n-3}+b^{n-3})(a+b) + (a^{n-2}+b^{n-2}) \end{array} \right] \\
&= \lim_{n\to\infty} ab \left[\begin{array}{l} (a+b)^{n-2} + a(a+b)^{n-3} + a^2(a+b)^{n-4} \\ + a^3(a+b)^{n-5} + \cdots + a^{n-3}(a+b) + \\ a^{n-2} + (a+b)^{n-2} + b(a+b)^{n-3} + \\ b^2(a+b)^{n-4} + b^3(a+b)^{n-5} + \cdots \\ + b^{n-3}(a+b) + b^{n-2} \end{array} \right] \\
&= \lim_{n\to\infty} ab \left\{ (a+b)^{n-2} \left(\begin{array}{l} 1 + \frac{a}{a+b} + \frac{a^2}{(a+b)^2} + \frac{a^3}{(a+b)^3} \\ + \cdots + \frac{a^{n-2}}{(a+b)^{n-2}} + 1 \\ + \frac{b}{a+b} + \frac{b^2}{(a+b)^2} + \frac{b^3}{(a+b)^3} \\ + \cdots + \frac{b^{n-2}}{(a+b)^{n-2}} \end{array} \right) \right\} \\
&= \lim_{n\to\infty} ab \left[(a+b)^{n-2} \left(\frac{1-\left(\frac{a}{a+b}\right)^{n-1}}{\frac{b}{a+b}} + \frac{1-\left(\frac{b}{a+b}\right)^{n-1}}{\frac{a}{a+b}} \right) \right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
&m(\theta_1 \cap \theta_2 \cap \theta_3) \\
&= \lim_{n\to\infty} \left\{ \begin{array}{l} 2b(1-a-b) + b^2(1-a-b) + \\ b(1-b)(1-a-b) \frac{1-\left(\frac{a}{1-b}\right)^2}{1-\frac{a}{1-b}} + \\ 2ab(1-a-b) + b^3(1-a-b) + \\ b(1-b)^2(1-a-b) \frac{1-\left(\frac{a}{1-b}\right)^3}{1-\frac{a}{1-b}} \\ + \left((a+b)^3 - a^3 - b^3\right)(1-a-b) \\ + b^4(1-a-b) + b(1-b)^3(1-a-b) \frac{1-\left(\frac{a}{1-b}\right)^4}{1-\frac{a}{1-b}} \\ + \left((a+b)^4 - a^4 - b^4\right)(1-a-b) + \cdots + \\ b^{n-1}(1-a-b) + \\ b(1-b)^{n-2}(1-a-b) \frac{1-\left(\frac{a}{1-b}\right)^{n-1}}{1-\frac{a}{1-b}} \\ + \left((a+b)^n - a^n - b^n\right)(1-a-b) \end{array} \right\} \\
&= \lim_{n\to\infty} (1-a-b) \left(\begin{array}{l} b + b^2 + b^3 + \cdots + b^{n-1} + \\ b \left(\begin{array}{l} 1 + (1-b) \frac{1-\left(\frac{a}{1-b}\right)^2}{1-\frac{a}{1-b}} \\ + (1-b)^2 \frac{1-\left(\frac{a}{1-b}\right)^3}{1-\frac{a}{1-b}} + \\ (1-b)^3 \frac{1-\left(\frac{a}{1-b}\right)^4}{1-\frac{a}{1-b}} + \\ \cdots + (1-b)^{n-2} \frac{1-\left(\frac{a}{1-b}\right)^{n-1}}{1-\frac{a}{1-b}} \end{array} \right) \\ + (a+b)^2 + (a+b)^3 + \cdots + (a+b)^n \\ - a^2 - a^3 - \cdots a^n - b^2 - b^3 - \cdots b^n \end{array} \right) \\
&= \lim_{n\to\infty} (1-a-b) \left(\begin{array}{l} b + b^2 + b^3 + \cdots + b^{n-1} + \\ b \left(\begin{array}{l} 1 + (1-b)^2 \frac{1-\left(\frac{a}{1-b}\right)^2}{1-a-b} + \\ (1-b)^3 \frac{1-\left(\frac{a}{1-b}\right)^3}{1-a-b} + \\ (1-b)^4 \frac{1-\left(\frac{a}{1-b}\right)^4}{1-a-b} + \\ \cdots + (1-b)^{n-1} \frac{1-\left(\frac{a}{1-b}\right)^{n-1}}{1-a-b} \end{array} \right) \\ + (a+b)^2 + (a+b)^3 + \cdots + (a+b)^n \\ - a^2 - a^3 - \cdots a^n - b^2 - b^3 - \cdots b^n \end{array} \right) \\
&= \lim_{n\to\infty} (1-a-b) \left(\begin{array}{l} b + b^2 + b^3 + \cdots + b^{n-1} + \frac{b}{1-a-b} \\ \frac{ \left(\begin{array}{l} 1-a-b + \\ (1-b)^2 \left(1-\left(\frac{a}{1-b}\right)^2\right) \\ + (1-b)^3 \left(1-\left(\frac{a}{1-b}\right)^3\right) \\ + (1-b)^4 \left(1-\left(\frac{a}{1-b}\right)^4\right) \\ + \cdots + (1-b)^{n-1} \left(1-\left(\frac{a}{1-b}\right)^{n-1}\right) \end{array} \right) }{1-\frac{a}{1-b}} \\ + (a+b)^2 + (a+b)^3 + \cdots + \\ (a+b)^n - a^2 - a^3 - \cdots a^n - b^2 \\ - b^3 - \cdots b^n \end{array} \right)
\end{aligned}
$$

$$
\begin{aligned}
&= \lim_{n\to\infty}(1-a-b)\left(\begin{array}{l} b+b^2+b^3+\cdots+b^{n-1}+ \\ \frac{b}{1-a-b}\left(\begin{array}{l}1-b+(1-b)^2\\ +(1-b)^3+(1-b)^4\\ +\cdots+(1-b)^{n-1}\\ -a-a^2-a^3\\ -\cdots-a^{n-1}\end{array}\right) \\ +(a+b)^2+(a+b)^3+ \\ \cdots+(a+b)^n-a^2-a^3 \\ -\cdots a^n-b^2-b^3-\cdots b^n \end{array}\right) \\
&= (1-a-b)\left[\frac{b-b^2}{1-b}+\frac{1-a^2}{1-a}+\frac{(a+b)^2}{1-a-b}\right] \\
&= 1
\end{aligned}
$$

Example 8. Suppose that there are n evidential sources, they have the same discernment framework $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Their gbbas are given in (15), and we sequentially combine these evidential sources according to the classic DSm rule of combination in (7).

S	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_1 \cup \theta_3)$
s_1	a	b	$1-a-b$
s_2	a	b	$1-a-b$
$\vdots$	$\vdots$	$\vdots$	$1-a-b$
s_n	a	b	$1-a-b$

(15)

$$
\begin{aligned}
&m(\theta_1\cap\theta_2) \\
&= \lim_{n\to\infty}\left\{\begin{array}{l}2ab+b\left[(1-b)^2-(1-a-b)^2\right]+ab^2 \\ +b\left[(1-b)^3-(1-a-b)^3\right]+ab^3\end{array}\right. \\
&\left.+\cdots+b\left[(1-b)^{n-1}-(1-a-b)^{n-1}\right]+ab^{n-1}\right\} \\
&= \lim_{n\to\infty}\left\{\begin{array}{l}b(1-b)+b(1-b)^2+\cdots+b(1-b)^{n-1} \\ +ab+ab^2+ab^3+\cdots+ab^{n-1} \\ -b(1-a-b)-b(1-a-b)^2 \\ -\cdots b(1-a-b)^{n-1}\end{array}\right\} \\
&= \lim_{n\to\infty}\left(\begin{array}{l}b(1-b)\frac{\left(1-(1-b)^{n-1}\right)}{b}+ab\frac{1-(b)^{n-1}}{1-b} \\ -b(1-a-b)\frac{1-(1-a-b)^{n-1}}{a+b}\end{array}\right) \\
&= 1-b+\frac{ab}{1-b}-\frac{b(1-a-b)}{a+b}
\end{aligned}
$$

$$
\begin{aligned}
&m(\theta_2\cap(\theta_1\cup\theta_3)) \\
&= \lim_{n\to\infty}(1-a-b)\left[b+b^2+\cdots b^{n-1}\right]+ \\
&b\left[1-a-b+(1-a-b)^2+\cdots(1-a-b)^{n-1}\right] \\
&= \lim_{n\to\infty}\left[b(1-a-b)\frac{1-b^{n-1}}{1-b}+b(1-a-b)\frac{1-(1-a-b)^{n-1}}{a+b}\right] \\
&= \frac{b(1-a-b)}{1-b}-\frac{b(1-a-b)}{a+b}
\end{aligned}
$$

$$
\begin{aligned}
&m(\theta_1) \\
&= \lim_{n\to\infty}\left\{\left(\left(\left[\begin{array}{l}a^2+\\2a(1-a-b)\end{array}\right](1-b)+ a(1-a-b)^2\right)(1-b)+a(1-a-b)^3\right)(1-b)\right. \\
&\quad\left.+\cdots a(1-a-b)^{n-1}\right\} \\
&= \lim_{n\to\infty}\left\{\begin{array}{l}a^2(1-b)^{n-2}+a(1-a-b)(1-b)^{n-2} \\ +a(1-a-b)(1-b)^{n-2}+ \\ a(1-a-b)^2(1-b)^{n-3}+ \\ a(1-a-b)^3(1-b)^{n-4}+ \\ \cdots+a(1-a-b)^{n-1}\end{array}\right\} \\
&= \lim_{n\to\infty} a\left\{\begin{array}{l}(1-b)^{n-1}+(1-a-b)(1-b)^{n-2}+ \\ (1-a-b)^2(1-b)^{n-3}+ \\ (1-a-b)^3(1-b)^{n-4}+ \\ \cdots+(1-a-b)^{n-1}\end{array}\right\} \\
&= \lim_{n\to\infty} a(1-b)^{n-1}\left(\begin{array}{l}1+\frac{1-a-b}{1-b}+\left(\frac{1-a-b}{1-b}\right)^2 \\ +\left(\frac{1-a-b}{1-b}\right)^3+\cdots\left(\frac{1-a-b}{1-b}\right)^{n-1}\end{array}\right) \\
&= \lim_{n\to\infty} a(1-b)^{n-1}\frac{1-\left(\frac{1-a-b}{1-b}\right)^n}{1-\frac{1-a-b}{1-b}} \\
&= \lim_{n\to\infty}(1-b)^n\left[1-\left(\frac{1-a-b}{1-b}\right)^n\right] \\
&= 0
\end{aligned}
$$

$$m(\theta_2) = \lim_{n\to\infty} b^n = 0$$

$$m(\theta_1\cup\theta_3) = \lim_{n\to\infty}(1-a-b)^n = 0$$

III. The Problem and Solution

Through the aforementioned several illustrative examples, we find the DSmC has a fatal deficiency, that is, if evidential sources are combined by using the DSmC, their final combinational result is of entropy production, which means the final result of combination becomes more and more uncertain and is very difficult to give a right decision.

How to solve this problem? In fact, Dezert and Smarandache gave 5 proportional redistribution rules (From PCR1 to PCR5) in the past[14]. For example, the PCR5 rule for two sources is defined by: $m_{PCR5}(\emptyset) = 0$ and $\forall X \in G^{\Theta} \setminus \{\emptyset\}$

$$
m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y\in G^{\Theta}\setminus\{X\}\\ X\cap Y=\emptyset}}\left[\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)}+\frac{m_2(X)^2 m_1(Y)}{m_2(X)+m_1(Y)}\right] \quad (16)
$$

where each element X, and Y, is in the disjunctive normal form. $m_{12}(X) = \sum_{\substack{X_1,X_2\in G^{\Theta}\\ X_1\cap X_2=X}} m_1(X_1)m_2(X_2)$ corresponds to the conjunctive consensus on X between the two sources. All denominators are different from zero. If a denominator is zero, that fraction is discarded. No matter how big or small the conflicting mass is, PCR5 mathematically does a better redistribution of the conflicting mass than the Dempster's rule and other rules. This is because PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the partial conflicting masses only to the sets involved in the conflict by considering the conjunctive normal form of the partial conflict. In addition, Arnaud Martin proposed PCR6[14], it is said that PCR6 is more precise than PCR5.

Whatever for PCR5 or PCR6, both of them seems very complex with the number increase of focal elements, the computation amount will obviously increase. Therefore, for the Dezert-Smarandache theory, to develop some simple and convergent rules will become an urgent need.

IV. Conclusion

In this paper, we give several illustrative examples to explain the problem of entropy production of DSmC, in order to show the necessity of developing some simple and convergent rules and point out our research direction in this field in the future.

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