Definition of *bba* and quality of fusion in C2 systems

Ksawery Krenc
C4I Research and Development Department
R&D Marine Technology Centre
Gdynia, Poland
ksawery.krenc@ctm.gdynia.pl

Adam Kawalec, Tadeusz Pietkiewicz
The Institute of Radioelectronics
WAT Military University of Technology
Warsaw, Poland
adam.kawalec@wat.edu.pl
tpietkiewicz@wat.edu.pl

***Abstract* – This paper presents the results of numerical experiments devoted to examination of influence of the target attribute hypotheses definitions on quality of information fusion. The main goal of the research works presented herein was to find answers to the subsequent questions: 'In the context of attribute information fusion in C2 systems, is it reasonable to extend the information scope of a sensor?' and 'Does the extension of the sensor information scope always provide tangible benefits in quality of fusion?'**

In order to achieve that there have been defined two measures: decision robustness and decision bias.

In the experimentation Dezert-Smarandache Theory has been used as the main fusion engine.

In this paper, both Bayesian and non-Bayesian basic belief assignments have been considered.

Keywords: Primary hypotheses, posterior hypotheses, target threat, *DSmT*, Bayesian *bba,* non-Bayesian *bba*.

1 Introduction

In C2 systems, fusion algorithms should be prepared for integrating information obtained from sensors, which are ontologically different [1]. One of the factors that influence the quality of the fusion is an exact definition of *basic belief assignments*, based on sensor data [9], [10].

In this paper, the subject of study is a correlation between the definitions of posterior hypotheses and the quality of target attribute fusion. In order to find it out a number of numerical experiments has been made.

During the experimentation, the *target threat* attribute was taken into consideration, due to its high ability to create multiple posterior hypotheses [4].

There have been defined two categories of sensors, used for experimentation, namely: narrow information scope sensors (NISS) and extended information scope sensors (EISS). The former enables to assess two basic classes of the target (i.e. FRIEND and HOSTILE), while the latter enables also to classify FAKER.

In the next sections, examples of posterior hypotheses, descriptions of the numerical experiments, the results of these experiments, and the conclusions will be given.

2 Attribute information fusion for Bayesian *bba*

Consider the following case of the target threat information fusion. It is assumed that *DSm* free model holds and Bayesian *bba* is defined as follows:

$m_1(F) = 0.2,\ m_1(H) = 0.8;$

$m_2(F) = 0.9,\ m_2(H) = 0.1;$

Figure 1 shows the Venn's diagram, related to this case. According to [5], [6], [7], [8], and [10] $F \cap H$ hypothesis describes a training target, called FAKER (F_K), comprehended as a type of FRIEND (F), acting as HOSTILE (H) for exercise purposes. That means the target possesses features of both friendly and hostile target.

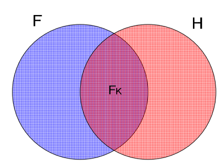

Figure 1 Venn's diagram for the *Threat* attribute

The corresponding evidence table has been presented at Table 1.

Table 1 Evidence table for two sensors discerning FRIEND and HOSTILE targets

m_2 \ m1	F [0.2]	H [0.8]
F [0.9]	F [0.18]	$F \cap H$ [0.72]
H [0.1]	$F \cap H$ [0.02]	H [0.08]

Application of the classical *DSm* rule of combination results in the following *bba*:

$m(F) = 0.18,\ m(H) = 0.08,\ m(F \cap H) = 0.74$

which in the consequence leads to subsequent values of belief functions:

$Bel(F) = m(F) + m(F \cap H) = 0.92$

$Bel(H) = m(H) + m(F \cap H) = 0.82$

$Bel(F \cap H) = m(F \cap H) = 0.74$

In that case the problem resides in the expression of $F \cap H$ which does not fully reflect the nature of FAKER. On one hand FAKER represents the target that encompasses the features of friendly and hostile targets. On the other hand it performs a specific type of FRIEND. Underlying the calculation of the belief

978-0-9824438-2-8/11/$26.00 ©2011 IEEE

functions is the assumption that FAKER supports both hypotheses: FRIEND and HOSTILE.

A simple correction of the calculation, by omitting the $F \cap H$ hypothesis support for HOSTILE, however incompatible with *DST* and *DSmT*, may be regarded as a kind of instant solution. Thus:

$Bel(F) = m(F) + m(F \cap H) = 0.92$

$Bel(H) = m(H) = 0.08$

$Bel(F \cap H) = m(F \cap H) = 0.74$

Notice that this treatment may have some serious repercussions, since FAKER, with the biggest mass assigned in this case, is a posterior hypothesis, which means it has no direct support from the sensor data.

The described fusion case may be regarded as a follow-up of the one of two possible events:

- integration of information, originated from two sensors, which detect different features of the target or
- integration of conflicting information, originated from two sensors one of which is reliable, and another is corrupted.

In the first event FAKER hypothesis seems to be highly appropriate. As a result of combination of evidence the biggest mass is assigned to it. Finally, due to the hypotheses hierarchy, according to which FAKER supports FRIEND, the corresponding belief function for a friendly target reaches the highest value. The lowest mass value is assigned to HOSTILE (0.08 after the correction).

In the second event the posterior hypothesis $F \cap H$, previously defined as FAKER should be regarded specifically. According to *DST* and *DSmT* the mass corresponding to that hypothesis is conflicting. Therefore omitting it in the belief function calculation may provide a serious corruption of the final decision, due to the fact the friendly target hypothesis would be fostered regularly.

3 Attribute information fusion for non- Bayesian *bba*

For comparison, consider another target threat attribute fusion case, where every sensor provides additional information related to the training value (FAKER). That means the acquired *bba* is non-Bayesian. Similarly as in the previous example it has been assumed that *DSm* free model holds. The gathered evidence has been summarized with the following table:

Table 2 Evidence table for two sensors discerning FRIEND, HOSTILE and FAKER targets

m_2 \ m1	F [0.2]	H [0.4]	$F \cap H$ [0.4]
F [0.5]	F [0.1]	$F \cap H$ [0.2]	$F \cap H$ [0.2]
H [0.1]	$F \cap H$ [0.02]	H [0.04]	$F \cap H$ [0.04]
$F \cap H$ [0.4]	$F \cap H$ [0.08]	$F \cap H$ [0.16]	$F \cap H$ [0.16]

Application of the classical *DSm* rule of combination results in the following *bba*:

$m(F) = 0.1$, $m(H) = 0.04$, $m(F \cap H) = 0.86$

which in the consequence leads to subsequent values of belief functions:

$Bel(F) = m(F) + m(F \cap H) = 0.96$

$Bel(H) = m(H) + m(F \cap H) = 0.9$

$Bel(F \cap H) = m(F \cap H) = 0.86$

Applying the analogical correction as in the previous example leads to the following belief function values:.

$Bel(F) = m(F) + m(F \cap H) = 0.96$

$Bel(H) = m(H) = 0.04$

$Bel(F \cap H) = m(F \cap H) = 0.86$

The fundamental difference between these two examples resides in the quantitative support for FAKER hypothesis. In the second case it is supported in three ways:

- directly: when both sensors identify the target as FAKER (Table 2: the black-colored mass);
- when only one sensor identifies the target as FAKER (Table 2: the green-colored mass);
- indirectly: as a combination of FRIEND and HOSTILE hypotheses, similarly as in the first example (Table 2: the purple-colored mass);

This means that (in the second example) the risk of allocating the FAKER hypothesis inappropriately high mass value, while one of the sensors delivers false information, was significantly reduced.

Discerning these two cases underlies an alternative correction method, which may be regarded as less 'invasive'. Namely, in the second case it is possible to apply a decomposition of FAKER for particle hypotheses: SPECIFIC_FAKER and CONFLICTING_FAKER, as follows:

$$m(F_K) = m(F_{SK}) + m(F_{CK}) \quad (1)$$

where:

$$\begin{aligned} m(F_{SK}) &= m_1(F \cap H)\cdot m_2(F) + m_1(F \cap H)\cdot m_2(H) + \\ &+ m_1(F)\cdot m_2(F \cap H)) + m_1(H)\cdot m_2(F \cap H) + \\ &+ m_1(F \cap H)\cdot m_2(F \cap H) = 0.2 + 0.04 + 0.08 + \\ &+ 0.16 + 0.16 = 0.64 \end{aligned}$$

$$\begin{aligned} m(F_{CK}) &= m_1(H)\cdot m_2(F) + m_1(F)\cdot m_2(H) \\ &= 0.2 + 0.02 = 0.22 \end{aligned}$$

Such decomposition enables to calculate the belief function values as follows:

$$\begin{aligned} Bel(F) &= m(F) + m(F_{SK}) + m(F_{CK}) \\ &= 0.1 + 0.64 + 0.22 = 0.96 \end{aligned}$$

$Bel(H) = m(H) + m(F_{CK}) = 0.04 + 0.22 = 0.26$

$Bel(F_K) = m(F_{SK}) + m(F_{CK}) = 0.86$

The correction above enables to utilize the complete information that resides in the descriptive definition of FAKER, while maintaining compliance of the belief function calculation with *DST* and *DSmT*.

It is worth of notice, that in both of the cases, independently of the introduced correction, FAKER hypothesis is never going to be accepted. This is due to the hypotheses hierarchy, according to which FAKER, as a subtype of a friendly target, supports FRIEND hypothesis. Thus: $Bel(F) \geq Bel(F_K)$. Then, the posterior hypothesis of FAKER performs a kind of auxiliary hypothesis to resolve the problem of *basic* target threat classification (i.e. FRIEND/HOSTILE).

If more precise classification is necessary, the respective decomposition of FRIEND (e.g. SPECIFIC_FRIEND and FAKER) and hypotheses refinement are required, just as shown at Figure 2 .

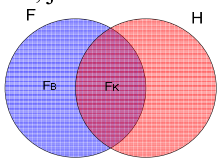

Figure 2 Venn's diagram for the *Threat* attribute with distinguished battle target

In such case the respective belief functions should be calculated as follows:

$Bel(F_B) = m(F) + m(F_{CK}) = 0.1 + 0.22 = 0.33$
$Bel(H) = m(H) + m(F_{CK}) = 0.04 + 0.22 = 0.26$
$Bel(F_K) = m(F_{SK}) + m(F_{CK}) = 0.86$

4 Examination of fusion quality

Within the numerical examination work there have been realized:

- *Decision robustness* examination and
- Changeability of the belief functions.

The *decision robustness* performs a kind of decision stability margin. That is the degree, to which extent the decision is resistant to obstacles caused by both types of uncertainty (random and deterministic).

In the examination, presented herein the decision robustness is based on the quantitative and qualitative analysis of the fusion cases, where slight modification of the input (sensor) data determines the decision change.

The *decision robustness* may be expressed as follows:

$$R_D = 1 - \frac{n_C}{N} \quad (2)$$

where:

n_C – the number of conflicting theses[1];

N – the number of possible theses (measurement scenarios). $R_D = 1$ means the best system.

Another measure, very useful while examination, is a *decision bias*. The *decision bias* defines the tendency of the preference of one prior hypothesis, regarding the other one, under the assumption of symmetric mass distribution.

The *decision bias* may be expressed as follows:

$$b_D = 1 - \frac{\min\{n_H, n_F\}}{\max\{n_H, n_F\}} \quad (3)$$

$$\forall m_1(F|F) = m_2(F|F) = m_1(H|H) = m_2(H|H)$$

where:

n_H – the number of HOSTILE theses;

n_F – the number of FRIEND theses;

$m_x(F|F)$ – the mass (originated from the x-th sensor) of the FRIEND hypothesis, on condition the data from the x-th sensor indicate the target is FRIEND.

The belief functions changeability examination performs a complement to the *decision robustness* study. Its general idea is to define the pace of the belief function change, while modification of the selected sensor data.

4.1 Examination of robustness for NISS

In the experimentation three possible interpretations of sensor data have been considered, namely: FRIEND (F), HOSTILE (H), and UNKNOWN (U). With assumption of two existing sensors, that makes nine possible measurement-decision scenarios. Additionally, the simulated *bba* was symmetric, as follows:

- for the reliable hypotheses: $m_1(F|F) = m_2(F|F) = m_1(H|H) = m_2(H|H) = 0.9$;
- for the unreliable hypotheses: $m_1(F|H) = m_2(F|H) = m_1(H|F) = m_2(H|F) = 0.1$;
- for hypotheses equally reliable:

$m_1(F|F \vee H) = m_1(F) = m_1(H) = m_2(F) = m_2(H) = 0.5$;

Table 3 shows an example evidence table for a combination of two conflicting *bbas*:

$m_1(F|F) = 0.9$, $m_1(H|F) = 0.1$;
$m_2(F|H) = 0.1$, $m_2(H|H) = 0.9$;

Table 3 Evidence table for two NISSes

	m1	0,9	0,1
m2		F	H
0,1	F	0,09	0,01
0,9	H	0,81	0,09

Calculation of the respective belief functions leads to the subsequent results:

$Bel(F) = m(F) + m(F \cap H) = 0.91$
$Bel(H) = m(H) + m(F \cap H) = 0.91$
$Bel(F \cap H) = m(F \cap H) = 0.82$

Introducing UNKNOW thesis, which is not directly related to any of the existing hypotheses, requires a brief comment. Namely, should a combination of the gathered evidence result in equal *bba* (i.e. the target may be equally interpreted as FRIEND or HOSTILE), UNKNOW thesis is applicable. Thus:

$$U = F \vee H \quad (4)$$

Table 4 summarizes the results of the experiment. The case of combination of conflicting *bba* mentioned above was colored in red.

Table 4 Hypotheses table for two NISSes

	1	2	3
	HOS	FRD	FRD/HOS
HOS	H	U (F:0.91, H:0.91)	H
FRD	U (F:0.91, H:0.91)	F	F
FRD/HOS	H	F	U (F:0.75, H:0.75)

The decisions presented in Table 4 have been made upon the calculated belief functions, related

[1] A thesis, by its definition, is a confirmed hypothesis. In this paper the thesis is comprehended as an interpretation of hypotheses fusion.

to the subsequent hypotheses. Yellow filling denotes the acceptance of UNKNOWN thesis, which was due to the fact the maximum value of the belief function referred equally to FRIEND and HOSTILE. This is an undesired phenomenon, which decreases the *decision robustness* significantly. The *decision robustness*, in this case, amounts to: $R_D = 0.(6)$[2].

One of the assumptions of the numerical experiments was a symmetric distribution of mass for both sensors. That means that every sensor provides *bba*, which may be interpreted as indicating definite presence of FRIEND or HOSTILE target, or indefinite presence of any of those. However, the degrees of belief for each of the definite cases were equal. Underlying this, an analysis of the fusion results enables to assess the *decision bias*, which amounts to: $b_D=0$.

4.2 Changeability of belief function for selected input data (for NISS)

This section presents an analysis of changeability of the belief functions for the case, where data obtained from one of the sensors indicate the presence of the hostile target. Assume:

$$m_1(F) = 0.2,\ m_1(H) = 0.8;$$

Starting with $m_2(F) = 0.1$ the FRIEND mass was being increased by $\Delta m = 0.1$, with simultaneous decrease of $m_2(H)$, which was presented at Figure 3.

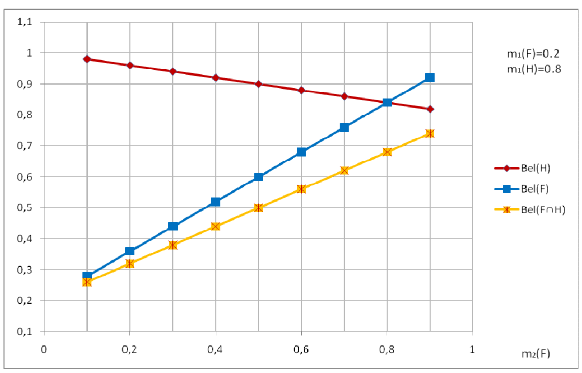

Figure 3 Changeability of belief functions for NISS. Sensor 1: HOSTILE

Values of the belief functions for: HOSTILE, FRIEND, and FAKER are marked with colors: red, blue, and yellow, respectively. The elaborated decision has been made by acceptance the hypothesis, of which the belief function was maximal. Thus it is not difficult to notice that FAKER hypothesis was never going to be accepted, due to the hypotheses hierarchy, mentioned in the previous section.

Figure 3 also presents the decision 'turn', which occurs while: $m_2(F) = 0.8$. This is quite intuitive, remembering the assumption, related to *bba* for one of the sensors. Then, UNKNOWN thesis is going to be accepted, due to the fact the maximum values of the belief functions refer equally to FRIEND and HOSTILE. When $m_2(F) > 0.8$, FRIEND thesis will be accepted as the final decision.

4.3 Examination of robustness for EISS

Similarly, as in the previous experiment, the same three possible interpretations of sensor data have been considered. Simulated mass distribution was also symmetric.

Table 5 presents possible interpretations of data, originated from one sensor. The first three columns contain binary assignment of the considered hypotheses. Successive three columns contain the same assignment, expressed in terms of the obtained *bba*[3]. The next column contains ordinal numbers of the hypotheses, and the last one the respective interpretation.

Table 5 Hypotheses table with interpretation

F	H	F∩H	m(F)	m(H)	m(F∩H)		
0	0	0	-	-	-		
0	0	1	0,1	0,1	0,8	1	FAKER
0	1	0	0,1	0,8	0,1	2	HOSTILE
0	1	1	0,1	0,45	0,45	3	HOS/FAK
1	0	0	0,8	0,1	0,1	4	FRIEND
1	0	1	0,45	0,1	0,45	5	FRD/FAK
1	1	0	0,45	0,45	0,1	6	FRD/HOS
1	1	1	0,(3)	0,(3)	0,(3)	7	faulty sensor

Table 6 performs one of the obtained evidence tables for a case of combination of two conflicting *bba*:

$$m_1(F|F) = 0.8,\ m_1(H|F) = 0.1,\ m_1(F \cap H|F) = 0.1;$$
$$m_2(F|H) = 0.1,\ m_2(H|H) = 0.8,\ m_2(F \cap H|H) = 0.1;$$

Hypotheses of FRIEND and FAKER have been distinguished with colors: green and pink, respectively. The rest of the colors refers to FAKER hypothesis, whereas its actual tints correspond to the respective particle hypotheses, mentioned in section 3.

Table 6 Evidence table for two EISSes

	m1	0,8	0,1	0,1
m2		F	H	F∩H
0,1	F	0,08	0,01	0,01
0,8	H	0,64	0,08	0,08
0,1	F∩H	0,08	0,01	0,01

The numerical experiments have examined two cases of FAKER decompositions:

- Two subtypes decomposition, according to (1), where:

$$Bel(F) = m(F) + m(F_{SK}) + m(F_{CK}) \tag{5}$$
$$Bel(H) = m(H) + m(F_{CK}) \tag{6}$$
$$Bel(F_K) = m(F_{SK}) + m(F_{CK}) \tag{7}$$

- Three-element decomposition[4], according to (8):

[2] Notation 0.(X) means a repeating decimal that is 0.XX…

[3] The first and the last case simplify to the same situation. Therefore one of them has been omitted.

[4] Since F_{FK} supports only FRIEND, and F_{HK} supports only HOSTILE, such decomposition is called 'three-element'.

$$m(F_K) = m(F_{FK}) + m(F_{HK}) + m(F_{KK}) + m(F_{CK}) \quad (8)$$

where:

$$Bel(F) = m(F) + m(F_{FK}) + m(F_{KK}) + m(F_{CK}) \quad (9)$$

$$Bel(H) = m(H) + m(F_{HK}) + m(F_{CK}) \quad (10)$$

$$Bel(F_K) = m(F_{FK}) + m(F_{HK}) + m(F_{KK}) + m(F_{CK}) \quad (11)$$

FK: friend-faker, *HK:* hostile-faker, *KK:* faker-faker

Table 7 summarizes the results of the numerical experiments performed with application of two-element FAKER decomposition, where the described above fusion case comprises one of the elements of this table (marked with red).

Table 7 Hypotheses table for two-element FAKER decomposition case

	FAK	HOS	HOS/FAK	FRD	FRD/FAK	FRD/HOS	FAIL
Dec 2	1	2	3	4	5	6	7
1	F	F	F	F	F	F	F
2	F	H	F	F	F	H	F
3	F	F	F	F	F	F	F
4	F	F	F	F	F	F	F
5	F	F	F	F	F	F	F
6	F	H	F	F	F	F	F
7	F	F	F	F	F	F	F

Analogical results for three-element FAKER decomposition have been summarized in Table 8.

Table 8 Hypotheses table for three-element FAKER decomposition case

	FAK	HOS	HOS/FAK	FRD	FRD/FAK	FRD/HOS	FAIL
Dec 3	1	2	3	4	5	6	7
1	F∩H	F∩H	F∩H	F∩H	F∩H	F∩H	F∩H
2	F∩H	H	H	F∩H	F∩H	H	H
3	F∩H	H	F∩H	F∩H	F∩H	F∩H	F∩H
4	F∩H	F∩H	F∩H	F	F	F	F
5	F∩H	F∩H	F∩H	F	F	F∩H	F∩H
6	F∩H	H	F∩H	F	F∩H	F	F∩H
7	F∩H	H	F∩H	F	F∩H	F∩H	F∩H

Comparing the results obtained for both cases of decomposition, it is easy to notice the significant disparity in the number of decisions, indicating FRIEND and HOSTILE. Calculating the respective values of *decision bias* results in:

$$b_D(2F) = 0.935,\ b_D(3F) = 0.(2);$$

The respective *decision robustness* values amount to:

$$R_D(2F) = 1,\ R_D(3F) = 1;$$

This means, that in both cases, with symmetric *bba* , it is possible to determine which hypothesis will be accepted.

Due to the fact that in the real conditions, measurements are always affected by errors, obtained *bbas* should be regarded as approximate. It is also problematic to achieve the complete symmetry, which was one of the assumptions for numerical experiments. In such case defining decision robustness and decision bias may not be sufficient for decision-making. Therefore it is suggested to calculate the respective differences between the maximal value of the belief function and the second high value of the belief function, for a given measurement-decision scenario. This enables to assess a kind of the 'second-order' margin, that determines the stability of decision.

Table 9 and Table 10 summarize calculated differences of the belief functions for two-element FAKER decomposition and three-element FAKER decomposition, respectively. In both cases the least value amounts to $\Delta\mathrm{Bel} = 0.01$, which means that modification of *bba* with this value (or higher) implies the change of decision. The decision robustness values may also be calculated, taking 0.01 as a given precision of the measurement. Thus:

$$R_D^{0.01}(2F) = 0.837\ ,\ R_D^{0.01}(3F) = 0.816\ ;$$

Table 9 Belief functions difference table for two-element FAKER decomposition case

ΔBel	1	2	3	4	5	6	7
1	0,01	0,01	0,01	0,08	0,045	0,045	0,0(3)
2	0,01	0,44	0,01	0,08	0,045	0,125	0,0(3)
3	0,01	0,01	0,01	0,08	0,045	0,045	0,0(3)
4	0,08	0,08	0,08	0,64	0,36	0,36	0,2(6)
5	0,05	0,045	0,045	0,36	0,2025	0,2025	0,15
6	0,04	0,125	0,045	0,36	0,2025	0,19	0,15
7	0,03	0,033	0,0(3)	0,2(6)	0,149993	0,15	0,(1)

Table 10 Belief functions difference table for three-element FAKER decomposition case

ΔBel	1	2	3	4	5	6	7
1	0,15	0,09	0,395	0,01	0,08	0,325	0,2(6)
2	0,09	0,61	0,26	0,01	0,055	0,295	0,1(6)
3	0,395	0,26	0,09	0,01	0,2025	0,055	0,18(3)
4	0,01	0,01	0,01	0,62	0,305	0,305	0,2
5	0,08	0,055	0,2025	0,305	0,1125	0,01	0,0(3)
6	0,325	0,295	0,055	0,305	0,01	0,01	0,0(3)
7	0,26663	0,1(6)	0,18(3)	0,2	0,0(3)	0,0(3)	0,(1)

It is obvious, that in practice the decision change is very important. However, as important as the change in itself is the type of change. It is intuitive that (in terms of tactics) the change of decision from FAKER to FRIEND is less important than from FRIEND to HOSTILE.

Table 9 and Table 10 should be analyzed together with Table 7 and Table 8. In Table 9, yellow color denotes the decision change from FRIEND to FAKER. In Table 10, green color denotes the decision change from FAKER to FRIEND, and red color denotes the change from FRIEND to HOSTILE. Thus for a given precision, equal to 0.01 the respective critical values of decision robustness may be calculated as follows:

$$R_D^{0.01K}(2F) = 1\,,\; R_D^{0.01K}(3F) = 0.980\,;$$

Applying inductive reasoning it is acceptable that for every given value of precision, decision robustness may be estimated, as:

$$R_D^P \leq R_D^{PE} \leq R_D^{PK} \tag{12}$$

where:

E – estimated *decision robustness* index;
P – assumed measurement precision;
K – critical *decision robustness* index;

4.4 Changeability of belief function for selected input data (for EISS)

Similarly, as for NISS the examination of changeability of belief function have been performed for EISS. It was assumed that data originated from one sensor indicate a presence of the hostile target. Assume:

$m_1(F) = 0.1$, $m_1(H) = 0.8$, $m_1(F \cap H) = 0.1$;

Starting with $m_2(F) = 0.1$ the FRIEND mass was being increased by $\Delta m = 0.1$ with simultaneous decrease of $m_2(H)$, which was presented at Figure 4. The mass of $m_2(F \cap H)$ was constant, amounted to $m_2(F \cap H) = 0.2$.

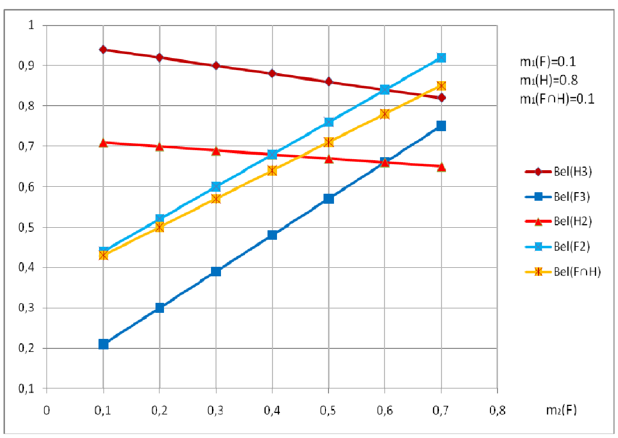

Figure 4 Changeability of belief functions for EISS. Two-element and three-element FAKER decomposition. Sensor 1: HOSTILE

Figure 4 presents the changeability of the belief functions for both cases of the FAKER decomposition. Yellow denotes the belief function referring to FAKER. Dark red and dark blue denote the belief functions referring to HOSTILE and FRIEND, respectively, with the three-element decomposition applied. Whereas the light colors: red and blue denote HOSTILE and FRIEND, respectively, with two-element decomposition applied.

Figure 4 shows the significant predominance of FRIEND hypothesis for two-element FAKER decomposition, which has already been noticed during examination of decision robustness. The respective lines, corresponding to FRIEND and HOSTILE hypotheses for both cases are almost parallel, which is due to the relatively low mass of FAKER, i.e. $m_1(F \cap H)$.

Focusing on the placement of the lines, corresponding to FRIEND and FAKER, it is easy to read the hypotheses hierarchy. Since for two-element FAKER decomposition FAKER completely supports FRIEND (i.e. the complete mass of FAKER is transferred to FRIEND), the respective line corresponding to FRIEND is situated above the one, corresponding to FAKER. On the other hand, since FAKER does not completely support FRIEND in case of three-element decomposition it is reasonable that the respective line, corresponding FRIEND is situated under the one corresponding to FAKER. Thus FRIEND hypothesis in such case will never be accepted.

For three-element FAKER decomposition the decision change occurs close to $m_2(F) = 0.67$. Therefore, for each $m_2(F) > 0.67$ FAKER hypothesis should be accepted. For two-element FAKER decomposition the decision change occurs in the middle of the range. Therefore, for each $m_2(F) > 0.4$ FRIEND hypothesis should be accepted.

The next experiment was to perform the analogical examination of changeability of belief function, however, the *bba* obtained from the first sensor had not precisely identified the *target threat*. Assume:

$m_1(F) = 0.1$, $m_1(H) = 0.45$, $m_1(F \cap H) = 0.45$;
$m_2(F \cap H) = 0.2$

Figure 5, similarly as Figure 4, shows a significant predominance of FRIEND hypothesis for two-element FAKER decomposition. The respective lines, corresponding to FRIEND and HOSTILE hypotheses for both cases are not parallel, which is due to the relatively high mass of FAKER, i.e. $m_1(F \cap H)$.

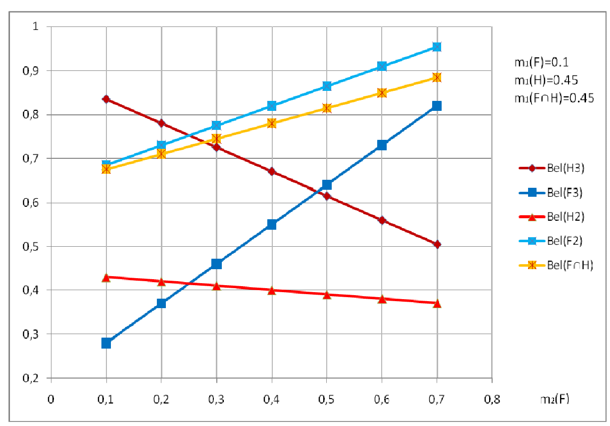

Figure 5 Changeability of belief functions for EISS. Two-element and three-element FAKER decomposition. Sensor 1: FAKER or HOSTILE

Figure 6 presents the changeability of belief function for three-element FAKER decomposition. The line referring to the belief function for FRIEND hypothesis has been omitted intentionally, since it is irrelevant to decision-making.

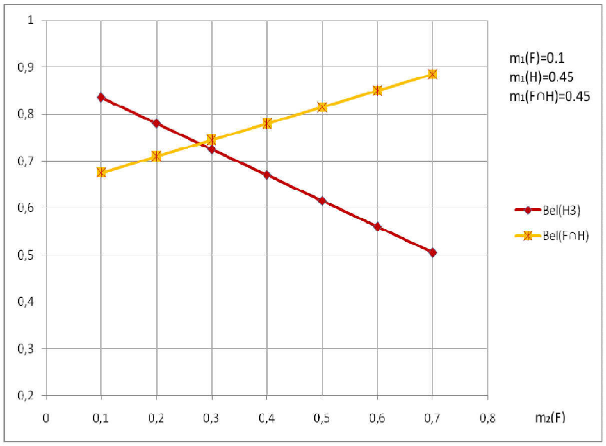

Figure 6 Changeability of belief functions for EISS. Three-element FAKER decomposition. Sensor 1: FAKER or HOSTILE

Comparing the graphs for changeability of belief functions presented at Figure 6 and Figure 4 it is easy to notice, that the belief in the predominant thesis of HOSTILE is being significantly reduced. Increasing the mass corresponding to FAKER results in change of the decision to FAKER close to $m_2(F) = 0.28$, however, similarly as in the previous experiment, FRIEND hypothesis is irrelevant to decision-making. The change of the experiment assumptions also influences the two-element FAKER decomposition case (see Figure 7). Increasing the mass corresponding to FAKER implies that within the entire range of mass m_2 FRIEND is the only relevant hypothesis.

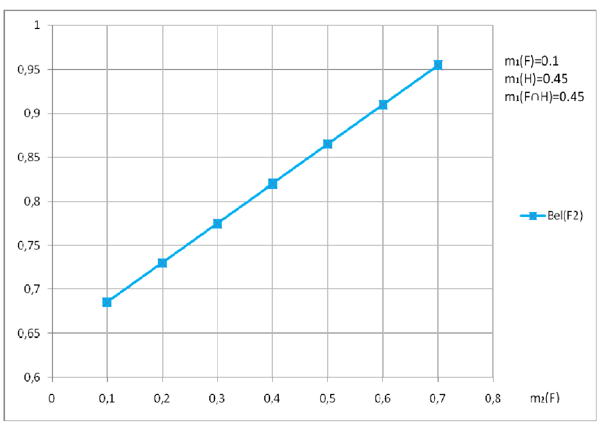

Figure 7 Changeability of belief function for EISS. Two-element FAKER decomposition. Sensor 1: FAKER or HOSTILE

5 Comparison of fusion quality for diverse *bba*

Application of the introduced measures enables to notice that one of the most important disadvantages of NISS information fusion is poor *decision robustness*. In about 30% of cases making decisions whether the target was FRIEND or HOSTILE was impossible. The only way the target could be identified as FAKER, was typically conflicting (by the combination of FRIEND from one sensor and HOSTILE from the other sensor). This conflicting mass is also problematic to manage, due to the fact that, according to *DST* and *DSmT* it should support both FRIEND and HOSTILE, while FAKER is also a subtype of FRIEND.

EISS information fusion enables to deal with the problem of conflicting mass, since in such kind of fusion, the conflicting mass is not the only one indicating FAKER. The discussed cases of two-element decomposition and three-element decomposition differ from each other in terms of the defined measures. Even though, in both cases, the decision robustness values, calculated with the zero error, are equal to unity, which is an undoubted advantage in comparison with NISS fusion, the decision robustness values, calculated with the error of 0.01 differ significantly, providing better results for the two-element decomposition.

However, the *decision bias* is the key measure for the discussed comparison. For the two-element FAKER decomposition case the *decision bias* is relatively high. That means the results obtained with this method are tendentious in the consequence fostering FRIEND thesis. This also affects the *decision robustness* value, mentioned above, making it falsely high. Particularly, the difference in the *decision robustness* values is caused by existence of one case, where the decision change from FRIEND to HOSTILE is possible with error of 0.01. However, it is worth of notice, that this very case holds only if the evidence from both sensors shows no preference of any the types (i.e. FRIEND and HOSTILE are equally probable, according to data from both sensors). Anyway the decrease of the decision robustness is the high price that needs to be paid in three-element decomposition fusion, for this specific case. On the other hand, an arbitral acceptance of FRIEND in two-element decomposition fusion for this very case may seem to be a bit suspicious.

It is very important to notice that in all of the considered cases of fusion, the respective values of belief functions were close. The reader might find it counterintuitive: Since EISS better fits the real world, combination of evidence based on these sensors should probably provide higher values of the respective belief functions. This is not exactly the truth. The reliability of the results obtained for EISS is comparably higher, not the belief functions. This is due to the fact the respective belief functions encompass more focal elements, supported directly by sensor data.

6 Conclusion

The results of the numerical experiments have proven that taking into account the reliability of the elaborated decisions, application of EISS sensors is much more appropriate. Particularly, the best benefits may be achieved by applying three-element decomposition of FAKER hypothesis. This mechanism enables to decrease significantly the risk the resulting posterior hypothesis has been caused by fusion error, sensor damage or intentional introduction of false data.

It is worth of notice, that the specific features of the presented mechanisms, demonstrated in the target threat attribute, may also be applied to other attributes. In the worst case, an application of the decomposition, described herein, may cause the creation of posterior hypotheses that refer to the targets that do not exist.
That, in turn, should be verified by mechanisms of information evaluation [2], [3] to eliminate such cases from the further analysis. On the other hand, remaining these cases will not degrade the quality of fusion, since the masses assigned to the improbable hypotheses are relatively small. However that will make the posterior hypotheses creation process less cost-effective.

As a direction for the forthcoming research works, related to the basic belief assignments the authors look forward to combination of ontological and evidential approaches [4], particularly an application of the former for creation of the posterior hypotheses.

References

[1] Chmielewski M., Kasprzyk R.: *Usage and characteristics of ontology models in Network Enabled Capability operations*, MCC Conference, Cracow, ISBN 83-920120-5-4, 2008.

[2] Hyder A.K. et al. (eds.): *Multisensor Fusion*, ISBN 1-4020-0722-1, 99-124, 2002.

[3] Krenc K., A. Kawalec: *An evaluation of the attribute information for the purpose of DSmT fusion in C&C systems*, Fusion2008, Cologne, ISBN 978-3-00-024883-2, 2008.

[4] Krenc K., A. Kawalec: *An application of DSmT in ontology-based fusion systems*, Fusion2009, Seattle, ISBN 978-0-9824438-0-4, 2009.

[5] NATO Standardization Agreement, *Identification Data Combining Process*, STANAG No. 4162, Ed. 2.

[6] NATO Standardization Agency, *Tactical Data Exchange – Link 16*, STANAG No. 5516, Ed. 3.

[7] NATO Standardization Agreement (STANAG 5511): *Tactical data exchange – Link 11/Link 11B*, Ed 6 2004

[8] Navy Center for Tactical Systems Interoperability, Operational specification for over-the horizon targeting gold revision D, 2000.

[9] Shafer G.: *A mathematical theory of evidence*, Princeton U.P., Princeton, NJ, 1976

[10] Smarandache F., Dezert J., *Advances and Applications of DSmT for Information Fusion*, Vol 1-Vol 3, American Research Press Rehoboth, 2004-2009.

[11] The Joint C3 Information Exchange Data Model, Edition 3.1b, 2007