

# A Neutrosophic Approach of MRI Denoising

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**Abstract**— In this paper, a new filtering method to remove Rician noise from magnetic resonance image is presented. This filter is based on Neutrosophic set (NS) approach of median filtering. A Neutrosophic set, a part of neutrosophy theory, studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Here the MRI image is transformed into NS domain, which is described using three membership sets: T, I, F. The entropy of the neutrosophic set is defined and employed to evaluate the indeterminacy. The  $\gamma$ -median filtering operation is used on T and F to decrease the set indeterminacy and remove noise. We have conducted experiments on real MR image with Rician noise added. The performance of this filter is compared with the median filtering and the classical non local mean (NLM) de-noising approaches. This filter outperforms the median filter for different levels of noise. For low signal to noise ratio (SNR), this filter provides high peak signal to noise ratio (PSNR) than the NLM approach.

**Keywords**- Denoising, entropy, magnetic resonance imaging, neutrosophic set, PSNR, Rician distribution.

## I. INTRODUCTION

Magnetic Resonance Imaging (MRI) is a powerful diagnostic technique used in radiology to visualize detailed internal structures of the human body. MRI is based on the principles of nuclear magnetic resonance (NMR), to map the spatial location and associated properties of specific nuclei or protons in a subject using the interaction between an electromagnetic field and nuclear spin [1]. MRI is primarily used to demonstrate pathological or other physiological alterations of living tissues and is a commonly used form of medical imaging. Despite significant improvements in recent years, MR images often suffer from low signal to noise ratio (SNR) especially in cardiac and brain imaging. This is problematic for further tasks such as segmentation of important features; classification of images for computer aided diagnostics, three dimensional image reconstruction and image registration. Therefore, noise reduction techniques are of great interest in MR imaging.

The MR image is commonly reconstructed by computing the inverse Discrete Fourier Transform of the raw data. The signal component of the measurement is present in both real and imaginary channels. Each of the orthogonal channels is affected by additive white Gaussian noise. The noise in the reconstructed complex valued data is thus complex white Gaussian noise. Most commonly, the magnitude of the reconstructed MRI image is used for visual inspection and for automatic computer analysis. Since the magnitude of the MRI signal is the square root of the sum of the squares of two independent Gaussian variables, it follows Rician distribution [2]. The Rician noise is signal dependent and is therefore difficult to separate from the signal.

Numerous approaches of denoising MR images have been proposed including approaches based on anisotropic diffusion [3-7], the wavelet transform [8-13], bilateral and trilateral filtering [14-16], the non-local means algorithm [17- 22] in the literatures. In this paper, the neutrosophic set approach (NS) filter for image denoising proposed in [23] is adapted to remove the Rician noise in MRI. First the noisy MRI is transformed into the neutrosophic set and the  $\gamma$ -median filtering is employed to reduce the indeterminacy degree of the image, which is evaluated by the entropy of the indeterminate subset, after filtering, the noise will be removed. The experiments on real MR images having different Rician noise levels demonstrate that this approach can perform denoising better.

## II. NEUTROSOPHIC MR IMAGE DENOISING

Neutrosophy, a branch of philosophy introduced in [24] as a generalization of dialectics, studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophy theory considers proposition, theory, event, concept or entity,  $\langle A \rangle$  is in relation to its opposite  $\langle \text{Anti-}A \rangle$  and the  $\langle \text{Neut-}A \rangle$  which is neither  $\langle A \rangle$  nor  $\langle \text{Anti-}A \rangle$ . The neutrosophy is the basis of the neutrosophic logic, neutrosophic probability, and neutrosophic set and neutrosophic statistics [24]. In neutrosophic set, the indeterminacy is quantified explicitly and the truth-membership, indeterminacy-membership and falsity-

membership are independent. The neutrosophic set is a general formal frame work which generalizes the concept of the classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set and tautological set [24]. The neutrosophic set had been applied into image processing such as denoising [23], segmentation [25, 26]. The definition of a neutrosophic set and its properties are described briefly.

#### A. Neutrosophic Set

**Definition 1 (Neutrosophic Set).** Let  $U$  be a Universe of discourse and a neutrosophic set  $A$  is included in  $U$ . An element  $x$  in set  $A$  is noted as  $x(T, I, F)$ .  $T, I, F$  are real standard and non standard sets of  $]^{-0}, 1^{+}[$  with  $\sup T = t_{\sup}, \inf T = t_{\inf}, \sup I = i_{\sup}, \inf I = i_{\inf}, \sup F = f_{\sup}, \inf F = f_{\inf}$  and  $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}, n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$ .  $T, I$  and  $F$  are called the neutrosophic components. The element  $x(T, I, F)$  belongs to  $A$  in the following way. It is  $t\%$  true in the set,  $i\%$  indeterminate in the set, and  $f\%$  false in the set, where  $t$  varies in  $T$ ,  $i$  varies in  $I$  and  $f$  varies in  $F$ .

#### B. Transform the image into neutrosophic set

**Definition 2 (Neutrosophic image).** Let  $U$  be a Universe of discourse and  $W$  is a set of  $U$ , which is composed by bright pixels. A neutrosophic image  $P_{NS}$  is characterized by three membership sets  $T, I, F$ . a pixel  $P$  in the image is described as  $P(T, I, F)$  and belongs to  $W$  in the following way: It is  $t$  true in the set,  $i$  indeterminate in the set, and  $f$  false in the set, where  $t$  varies in  $T$ ,  $i$  varies in  $I$  and  $f$  varies in  $F$ . Then the pixel  $P(i, j)$  in the image domain is transformed into the neutrosophic set domain  $P_{NS}(i, j) = \{T(i, j), I(i, j), F(i, j)\}$ .  $T(i, j), I(i, j)$  and  $F(i, j)$  are the probabilities belong to white pixels set, indeterminate set and non white pixels set respectively, which are defined as:

$$T(i, j) = \frac{\bar{g}(i, j) - \bar{g}_{\min}}{\bar{g}_{\max} - \bar{g}_{\min}} \quad (1)$$

$$\bar{g}(i, j) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} g(m, n) \quad (2)$$

$$I(i, j) = \frac{\delta(i, j) - \delta_{\min}}{\delta_{\max} - \delta_{\min}} \quad (3)$$

$$\delta(i, j) = abs(g(i, j) - \bar{g}(i, j)) \quad (4)$$

$$F(i, j) = 1 - T(i, j) \quad (5)$$

where  $\bar{g}(i, j)$  is the local mean value of the pixels of the window.  $\delta(i, j)$  is the absolute value of difference between intensity  $g(i, j)$  and its local mean value  $\bar{g}(i, j)$ .

#### C. Neutrosophic image entropy

For a gray image, the entropy is utilized to evaluate the distribution of the gray levels. If the entropy is the maximum, the intensities have equal probability. If the entropy is small, the intensity distribution is non-uniform.

**Definition 3 (Neutrosophic image entropy).** Neutrosophic entropy of an image is defined as the summation of the entropies of three subsets  $T, I$  and  $F$ :

$$En_{NS} = En_T + En_I + En_F \quad (6)$$

$$En_T = - \sum_{i=\min\{T\}}^{\max\{T\}} p_T(i) \ln p_T(i) \quad (7)$$

$$En_I = - \sum_{i=\min\{I\}}^{\max\{I\}} p_I(i) \ln p_I(i) \quad (8)$$

$$En_F = - \sum_{i=\min\{F\}}^{\max\{F\}} p_F(i) \ln p_F(i) \quad (9)$$

where  $En_T, En_I$  and  $En_F$  are the entropies of sets  $T, I$  and  $F$  respectively.  $p_T(i)$ ,  $p_I(i)$  and  $p_F(i)$  are the probabilities of elements in  $T, I$  and  $F$  respectively, whose values equal to  $i$ .

#### D. $\gamma$ -median filtering operation

The values of  $I(i, j)$  is employed to measure the indeterminate degree of element  $P_{NS}(i, j)$ . To make the set  $I$  correlated with  $T$  and  $F$ , the changes in  $T$  and  $F$  influence the distribution of element in  $I$  and vary the entropy of  $I$ .

**Definition 4 ( $\gamma$ -median filtering operation).** A  $\gamma$ -median filtering operation for  $P_{NS}, \hat{P}_{NS}(\gamma)$ , is defined as:

$$\hat{P}_{NS}(\gamma) = P(\hat{T}(\gamma), \hat{I}(\gamma), \hat{F}(\gamma)) \quad (10)$$

$$\hat{T}(\gamma) = \begin{cases} T & I < \gamma \\ \hat{T}_{\gamma} & I \geq \gamma \end{cases} \quad (11)$$

$$\hat{T}_\gamma(i, j) = \text{median}_{(m,n) \in S_{i,j}} \{T(m, n)\} \quad (12)$$

$$\hat{F}(\gamma) = \begin{cases} F & I < \gamma \\ \hat{F}_\gamma & I \geq \gamma \end{cases} \quad (13)$$

$$\hat{F}_\gamma(i, j) = \text{median}_{(m,n) \in S_{i,j}} \{F(m, n)\} \quad (14)$$

$$\hat{I}_\gamma(i, j) = \frac{\delta_{\hat{T}}(i, j) - \delta_{\hat{T}_{\min}}}{\delta_{\hat{T}_{\max}} - \delta_{\hat{T}_{\min}}} \quad (15)$$

$$\delta_{\hat{T}}(i, j) = \text{abs}(\hat{T}(i, j) - \bar{\hat{T}}(i, j)) \quad (16)$$

$$\bar{\hat{T}}(i, j) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} \hat{T}(m, n) \quad (17)$$

where  $\delta_{\hat{T}}(i, j)$  is the absolute value of difference between intensity  $\hat{T}(i, j)$  and its local mean value  $\bar{\hat{T}}(i, j)$  at  $(i, j)$  after  $\gamma$ -median filtering operation.

The new neutrosophic approach to image denoising is described as below:

Step 1: Transform the image into NS domain;

Step 2: Use  $\gamma$ -median filtering operation on the true subset

$$T \text{ to obtain } \hat{T}_\gamma;$$

Step 3: Compute the entropy of the indeterminate subset

$$\hat{I}_\gamma, \text{En}_{\hat{I}_\gamma}(i);$$

Step 4: if  $\frac{\text{En}_{\hat{I}_\gamma}(i+1) - \text{En}_{\hat{I}_\gamma}(i)}{\text{En}_{\hat{I}_\gamma}(i)} < \delta$ , go to Step 5;

Else  $T = \hat{T}_\gamma$ , go to Step 2;

Step 5: Transform subset  $\hat{T}_\gamma$  from the neutrosophic domain into the gray level domain.

### III. RESULTS AND DISCUSSION

In the experiments, we used the T2 Axial with TR 9000, TE 87 of 5th Slice and T1 Sagittal with TR 552, TE 17 of 5th Slice of brain MR image of 2 years old male child, which is obtained from SIEMENS 1.5 T MRI scanner. The simulated Rician noise with different level is added to the both the images.

The performance of the denoising algorithm is measured by using the peak-signal-to-noise ratio (PSNR). The higher the PSNR is, the better the denoising algorithm is.

First, the NS filter is compared with the median filter in removing the Rician noise with different noise level from the both T2 Axial and T1 sagittal brain MR images. Then, the NS Filter is compared with the Non Local Mean (NLM) filter proposed by Buades *et al* [17].

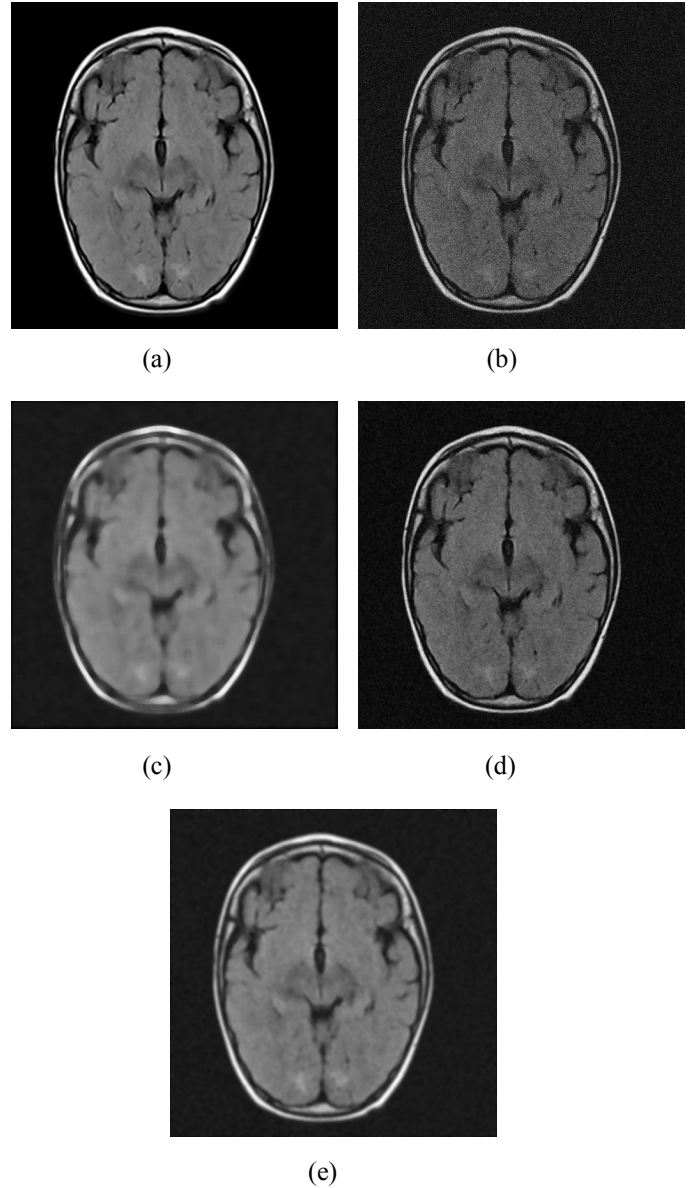


Fig. 1 a) Original Image (T2 Axial Brain MRI), b) Noisy image with PSNR 24.79, c) Denoised using Median Filter PSNR 25.96, d) Denoised using NLM PSNR 28.36, e) Denoised using NS Filter PSNR 26.86

For T2 Axial brain MRI, Fig. 1 (a) is the original image of the T2 Axial brain MRI, and Fig. 1 (b) is the image resulted from the Fig. 1 (a) added with Rician noise ( $s=20$ ; PSNR=24.79). Fig. 1 (c), Fig. 1 (d) and Fig. 1 (e) are the denoised results of Median filter (PSNR=25.96), NLM (PSNR=28.36) and NS Filter (PSNR=26.86) respectively. Fig. 2 shows the PSNR comparison of Median and NS filter. From these results, we conclude that the NS filter performs better than the Median filter for different noise level. Fig. 3 (a) and (b) shows the PSNR comparison of NLM and NS filter. From these performance results, we can conclude that at High SNR (low noise level), NLM performs well than NS filter and at low SNR (high noise level), NS filter performs better than that of NLM.

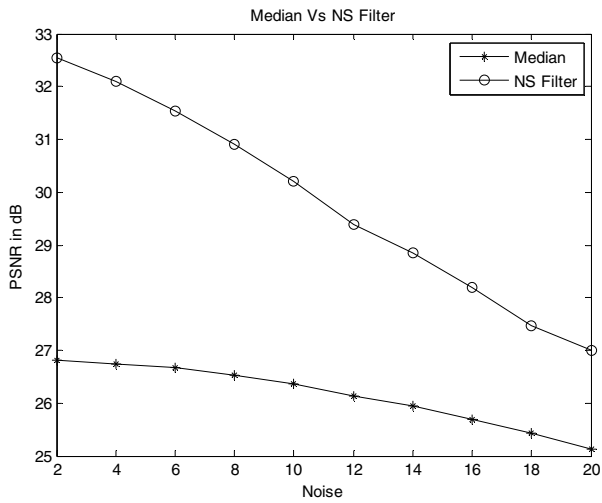
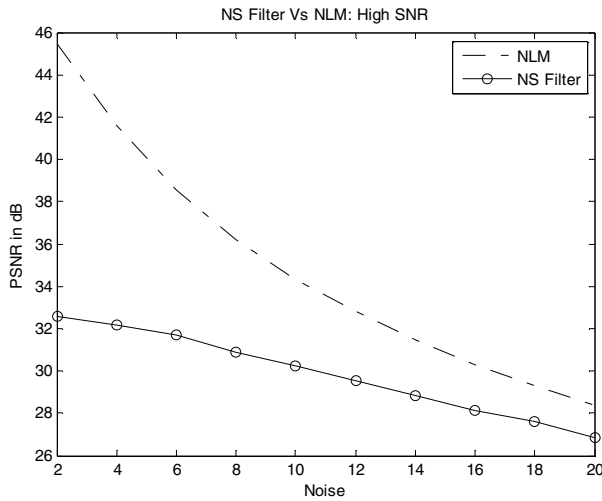
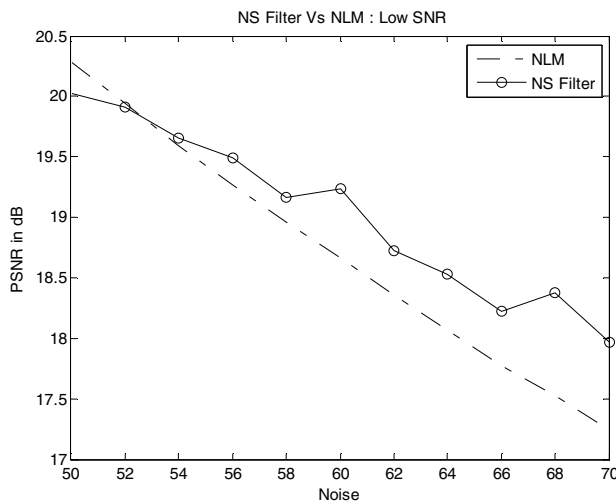


Fig. 2 The performance comparison of Median and NS Filter on T2 Axial brain MR Image with Rician noise.



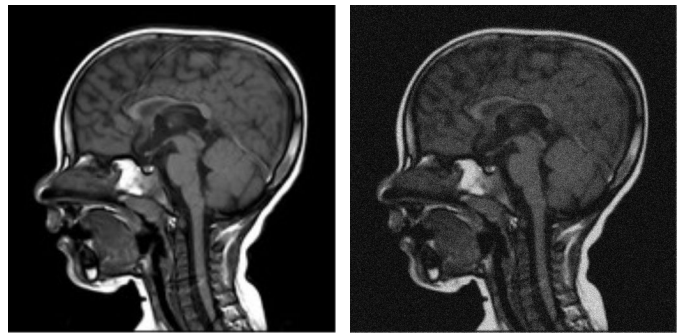
(a)



(b)

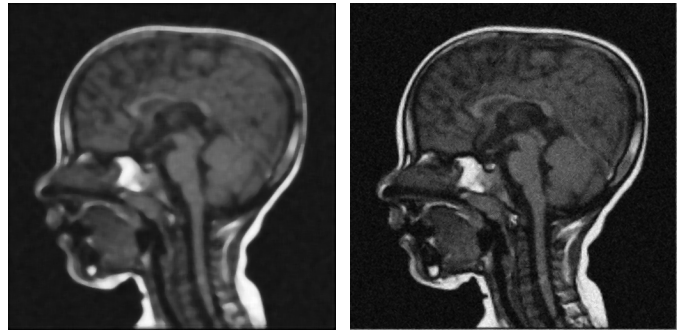
Fig. 3 The performance comparison of different filters on T2 Axial brain MR Image with Rician noise. a) High SNR, b) Low SNR.

For T1 Sagittal brain MRI, Fig. 4 (a) is the original image of the T1 Sagittal brain MRI, and Fig. 4 (b) is the image resulted from the Fig. 4 (a) added with Rician noise ( $s=20$ ; PSNR=26.80). Fig. 4 (c), Fig. 4 (d) and Fig. 4 (e) are the denoised results of Median filter (PSNR=27.19), NLM (PSNR=29.22) and NS Filter (PSNR=28.19) respectively. Fig. 5 shows the PSNR comparison of Median and NS filter. From these results, we conclude that the NS filter performs better than the Median filter for different noise level. Fig. 6 (a) and (b) shows the PSNR comparison of NLM and NS filter.



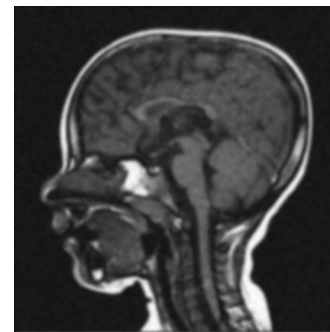
(a)

(b)



(c)

(d)



(e)

Fig. 4 a) Original Image (T1 Sagittal Brain MRI), b) Noisy image with PSNR 26.80, c) Denoised using Median Filter PSNR 27.10, d) Denoised using NLM PSNR 29.22 e) Denoised using NS Filter PSNR 28.23

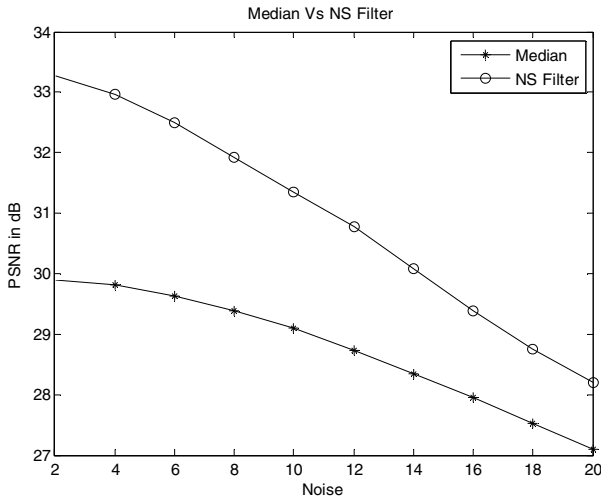
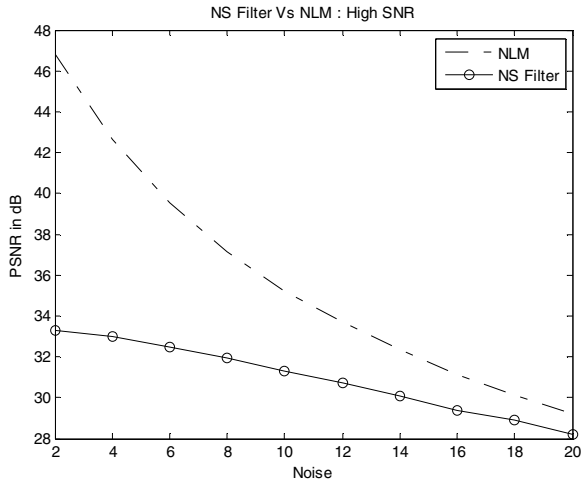
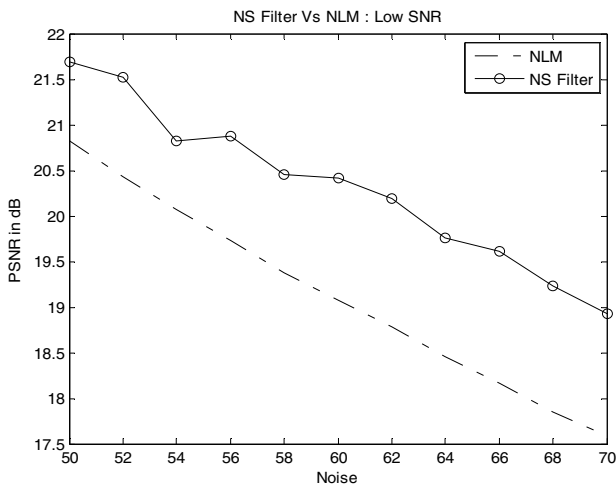


Fig. 5 The performance comparison of Median and NS Filter on MR Image T1 Sagittal with Rician noise.



(a)



(b)

Fig. 6 The performance comparison of different filters on T1 Sagittal MR Image with Rician noise. a) High SNR, b) Low SNR.

From these performance results, we can conclude that at High SNR (low noise level), NLM performs well than NS filter and at low SNR (high noise level), NS filter performs even better than that of NLM.

From Table I, we better understand that the NS filter perform well than Median filter for different noise level and also, for low SNR the NS filter performs better than NLM irrespective of the MR images.

TABLE I  
COMPARISON OF MEDIAN, NLM, NS FILTER BASED ON PSNR IN dB

Noise Level	PSNR (dB)					
	Median		NLM		NS filter	
	T2 Axial	T1 Sagittal	T2 Axial	T1 Sagittal	T2 Axial	T1 Sagittal
2	23.18	27.75	<b>35.65</b>	<b>46.56</b>	32.69	33.40
10	23.14	27.24	<b>34.36</b>	<b>35.23</b>	30.41	31.42
20	22.79	25.89	<b>28.36</b>	<b>29.19</b>	26.84	28.14
30	22.04	24.20	<b>24.81</b>	25.58	24.19	<b>25.59</b>
40	21.09	22.48	21.30	22.90	<b>21.99</b>	<b>23.28</b>
50	19.94	20.86	20.27	20.80	<b>21.05</b>	<b>22.00</b>
60	18.81	19.44	19.64	19.07	<b>20.02</b>	<b>20.36</b>
70	17.74	18.08	18.25	17.55	<b>19.90</b>	<b>19.11</b>

IV. CONCLUSIONS

This paper presents a neutrosophic denoising approach based on neutrosophic set for removing Rician noise from brain MRI. This filtering method tends to produce good denoised image not only in terms of visual perception but also in terms of the quality metric (PSNR). This filter performs better than Median filtering method for removing the Rician noise with different noise levels. Further, and also it outperforms the Non Local Mean approach when the noise level is high (low SNR).

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