A STUDY ON (T, S)-INTUITIONISTIC FUZZY SUBNEARRINGS OF A NEARRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of a (T, S)-intuitionistic fuzzy subnearring of a nearring.

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Key Words: T-fuzzy subnearring, anti S-fuzzy subnearring, (T, S)-intuitionistic fuzzy subnearring, product.

INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[4, 5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined the fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [13, 14]. In this paper, we introduce the some Theorems in (T, S)-intuitionistic fuzzy subnearring of a nearring.

1.PRELIMINARIES:

1.1 Definition: A (T, S)-norm is a binary operations T: [0, 1]×[0, 1] → [0, 1] and S: [0, 1]×[0, 1] → [0, 1] satisfying the following requirements;

(i) T(0, x)= 0, T(1, x) = x (boundary condition)
(ii) T(x, y) = T(y, x) (commutativity)
(iii) T(x, T(y, z))= T (T(x,y), z) (associativity)
(iv) if x ≤ y and w ≤ z, then T(x, w) ≤ T (y, z) (monotonicity).

(v) S(0, x) = x, S (1, x) = 1 (boundary condition)
(vi) S(x, y ) = S (y, x ) (commutativity)
(vii) S (x, S(y, z))= S ( S(x, y), z) (associativity)
(viii) if x ≤ y and w ≤ z, then S (x, w ) ≤ S (y, z) (monotonicity).

1.2 Definition: Let (R, +, . ) be a near ring. A fuzzy subset A of R is said to be a T-fuzzy sub nearring (fuzzy subnearring with respect to T-norm) of R if it satisfies the following conditions:

(i) µA(x−y)≥ T (µA(x), µA(y))
(ii) µA(xy)≥ T (µA(x), µA(y)) for all x and y in R.

1.3 Definition: Let (R, +, . ) be a nearring. An intuitionistic fuzzy subset A of R is said to be an (T, S)-intuitionistic fuzzy subnearring (intuitionistic fuzzy subnearring with respect to (T, S)-norm) of R if it satisfies the following conditions:

(i) µA(x−y)≥ T (µA(x), µA(y))
(ii) µA(xy)≥ T (µA(x), µA(y))
(iii) νA(x−y)≤ S (νA(x), νA(y))
(iv) νA(xy)≤ S (νA(x), νA(y)) for all x and y in R.

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1.4 Definition: Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by $A \times B$, is defined as $A \times B = \{(x, y), \mu_A(x, y), \nu_A(x, y)\}$ for all $x \in G$ and $y \in H$, where $\mu_A(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\nu_A(x, y) = \max\{\nu_A(x), \nu_B(y)\}$.

1.5 Definition: Let $A$ be an intuitionistic fuzzy subset in a set $S$, the strongest intuitionistic fuzzy relation on $S$, that is an intuitionistic fuzzy relation on $A$ is given by $\mu_A(x, y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x, y) = \max\{\nu_A(x), \nu_A(y)\}$ for all $x$ and $y$ in $S$.

1.6 Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two nearrings. Let $f : R \rightarrow R^1$ be any function and $A$ be an $(T, S)$-intuitionistic fuzzy subnearring in $R$, $V$ be an $(T, S)$-intuitionistic fuzzy subnearring in $f(R) = R^1$, defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$ for all $x$ in $R$ and $y$ in $R^1$. Then $A$ is called a preimage of $V$ under $f$ and is denoted by $f^{-1}(V)$.

1.7 Definition: Let $A$ be an $(T, S)$-intuitionistic fuzzy subnearring of a nearring $(R, +, \cdot)$ and $a$ in $R$. Then the pseudo $(T, S)$-intuitionistic fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$ and $((a\nu_A)^p)(x) = p(a)\nu_A(x)$ for every $x$ in $R$ and for some $p$ in $P$.

2- PROPERTIES

2.1 Theorem: Intersection of any two $(T, S)$-intuitionistic fuzzy subnearrings of a nearring $R$ is a $(T, S)$-intuitionistic fuzzy subnearring of a nearring $R$.

Proof: Let $A$ and $B$ be any two $(T, S)$-intuitionistic fuzzy subnearrings of a nearring $R$ and $x$ and $y$ in $R$. Let $A = \{(x, \mu_A(x), \nu_A(x)) : x \in R\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) : x \in R\}$ and also let $C = A \cap B = \{(x, \mu_A(x), \nu_A(x)) : x \in R\}$ where $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$. Now $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $\nu_{A \cap B}(x) = \max\{\nu_A(x), \nu_B(x)\}$. Therefore $\mu_{A \cap B}(x) \geq \mu_A(x)$ and $\nu_{A \cap B}(x) \leq \nu_B(x)$.

2.2 Theorem: The intersection of a family of $(T, S)$-intuitionistic fuzzy subnearrings of a nearring $R$ is an $(T, S)$-intuitionistic fuzzy subnearring of a nearring $R$.

Proof: It is trivial.

2.3 Theorem: If $A$ and $B$ are any two $(T, S)$-intuitionistic fuzzy subnearrings of the nearrings $R_1$ and $R_2$ respectively, then $A \times B$ is an $(T, S)$-intuitionistic fuzzy subnearring of $R_1 \times R_2$.

Proof: Let $A$ and $B$ be two $(T, S)$-intuitionistic fuzzy subnearrings of the nearrings $R_1$ and $R_2$ respectively. Let $x_1, y_1$ and $x_2, y_2$ be in $R_1$ and $R_2$, respectively. Then $A = \{(x_1, y_1) : (x_2, y_2) \in R_1 \times R_2\}$ and $B = \{(x_2, y_2) : (x_1, y_1) \in R_1 \times R_2\}$. Now $\mu_{A \times B}(x_1, y_1) = \min\{\mu_A(x_1), \mu_B(y_1)\}$ and $\nu_{A \times B}(x_1, y_1) = \max\{\nu_A(x_1), \nu_B(y_1)\}$. Therefore $\mu_{A \times B}(x_1, y_1) \geq \mu_A(x_1)$ and $\nu_{A \times B}(x_1, y_1) \leq \nu_B(y_1)$.

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2.4 Theorem: If A is a (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, ), then \( \mu_A(x) \leq \mu_A(0) \) and \( v_A(x) \geq v_A(0) \) for x in R, the identity element 0 in R.

**Proof:** For x in R and 0 is the identity element of R. Now \( \mu_A(0) = \mu_A(x-x) \geq T(\mu_A(x), \mu_A(0)) \geq \mu_A(x) \) for all x in R. So \( \mu_A(x) \leq \mu_A(0) \). And \( v_A(0) = v_A(x-x) \leq S(\mu_A(x), v_A(0)) \leq v_A(0) \) for all x in R. So \( v_A(x) \geq v_A(0) \).

2.5 Theorem: Let A and B be (T, S)-intuitionistic fuzzy subnearring of the nearrings R_1 and R_2 respectively. Suppose that 0 and 0 are the identity element of R_1 and R_2 respectively. If A\times B is an (T, S)-intuitionistic fuzzy subnearring of R_1\times R_2, then at least one of the following two statements must hold. (i) \( \mu_{A \times B}(0) \geq \mu_A(x) \) and \( v_{A \times B}(0) \leq v_A(x) \) for all x in R_1, (ii) \( \mu_{A \times B}(0) \geq \mu_B(y) \) and \( v_{A \times B}(0) \leq v_B(y) \) for all y in R_2.

**Proof:** Let A\times B be an (T, S)-intuitionistic fuzzy subnearring of R_1\times R_2. By contrapositive, suppose that none of the statements (i) and (ii) holds. Then we can find a in R_1 and b in R_2 such that \( \mu_{A \times B}(a) > \mu_A(0) \), \( v_{A \times B}(b) < v_B(0) \) and \( \mu_{A \times B}(b) > \mu_B(0) \), \( v_{A \times B}(b) < v_B(0) \). We have \( \mu_{A \times B}(a, b) = \min\{\mu_A(a), \mu_B(b)\} = \min\{\mu_A(0), \mu_B(0)\} = \mu_{A \times B}(0, 0) \). And \( v_{A \times B}(a, b) = \max\{v_A(a), v_B(b)\} = \max\{v_A(0), v_B(0)\} = v_{A \times B}(0, 0) \). Thus A\times B is not an (T, S)-intuitionistic fuzzy subnearring of R_1\times R_2. Hence either \( \mu_{A \times B}(0) \geq \mu_A(x) \) and \( v_{A \times B}(0) \leq v_A(x) \) for all x in R_1 or \( \mu_{A \times B}(0) \geq \mu_B(y) \) and \( v_{A \times B}(0) \leq v_B(y) \) for all y in R_2.

2.6 Theorem: Let A be an intuitionistic fuzzy subset of a nearring R and V be the strongest intuitionistic fuzzy relation on R. We have \( A(x1y1), V(x1) \) for all x in R and 0 is the identity element of R. Now \( A(0) = A(x-x) \geq S(\mu_A(x), \mu_A(0)) \geq \mu_A(x) \) for all x in R. So \( \mu_A(x) \leq \mu_A(0) \). And \( V(x-x) \leq S(\mu_A(x), v_A(0)) \leq v_A(0) \) for all x in R. So \( v_A(x) \geq v_A(0) \).
max \{v_\alpha(x,y), v_\alpha(x,y_2)\} \leq \max \{S(v_\alpha(x_1), v_\alpha(y_1)), S(v_\alpha(x_2), v_\alpha(y_2))\} \leq S(\max\{v_\alpha(x_1), v_\alpha(x_2)\}, \max\{v_\alpha(y_1), v_\alpha(y_2)\}) = S(v_\alpha(x_1, x_2), v_\alpha(y_1, y_2)) = S(v_\alpha(x), v_\alpha(y)). Therefore, v_\alpha(x,y) \leq S(v_\alpha(x), v_\alpha(y)), for all x and y in R \times R. This proves that V is an (T, S)-intuitionistic fuzzy subnearring of R \times R. Conversely assume that V is an (T, S)-intuitionistic fuzzy subnearring of R \times R, then for any x = (x_1, x_2) and y = (y_1, y_2) are in R \times R, we have min\{\mu_\alpha(x_1, y_1), \mu_\alpha(x_2, y_2)\} = \mu_\alpha(x_1, y_1, x_2, y_2) = \mu_\alpha((x_1, x_2), (y_1, y_2)) = \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)).

2.8 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, \cdot), then H = \{x / x \in R : \mu_\alpha(x) = 1, v_\alpha(x) = 0\} is either empty or is a subnearring of R.

Proof: It is trivial.

2.9 Theorem: If A be an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, \cdot), then (i) if \mu_\alpha(x, y) = 0, then either \mu_\alpha(x) = 0 or \mu_\alpha(y) = 0 for all x and y in R. (ii) if \mu_\alpha(x) = 0, then either \mu_\alpha(x) = 0 or \mu_\alpha(y) = 0 for all x and y in R. (iii) if v_\alpha(x, y) = 1, then either v_\alpha(x) = 1 or v_\alpha(y) = 1 for all x and y in R. (iv) if v_\alpha(x) = 1, then either v_\alpha(x) = 1 or v_\alpha(y) = 1 for all x and y in R.

Proof: It is trivial.

2.10 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, \cdot), then \Box A is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Consider A = \{x, \mu_\alpha(x), v_\alpha(x)\}, for all x in R, we take \Box A = B = \{x, \mu_\alpha(x), v_\alpha(x)\}, where \mu_\alpha(x) = \mu_\alpha(x), v_\alpha(x) = v_\alpha(x). Clearly \mu_\alpha(x, y) \geq \mu_\alpha(x, y) for all x and y in R and \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subnearring of R, we have \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R, which implies that 1 – \mu_\alpha(x, y) \leq 1 – T((1 – \mu_\alpha(x, y))) = 1 – \mu_\alpha(x, y). Therefore \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R. Moreover, \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R, which implies that 1 – \mu_\alpha(x, y) \leq 1 – T((1 – \mu_\alpha(x, y))) = 1 – \mu_\alpha(x, y). Therefore \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R. Hence B = \Box A is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.11 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, \cdot), then A is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. That is A = \{x, \mu_\alpha(x), v_\alpha(x)\} for all x in R. Let A = \{x, \mu_\alpha(x), v_\alpha(x)\} where \mu_\alpha(x) = 1 – v_\alpha(x), v_\alpha(x) = v_\alpha(x). Clearly v_\alpha(x, y) \leq S(v_\alpha(x), v_\alpha(y)) for all x and y in R and v_\alpha(x, y) \leq S(v_\alpha(x), v_\alpha(y)) for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subnearring of R, we have v_\alpha(x, y) \leq S(v_\alpha(x), v_\alpha(y)) for all x and y in R, which implies that 1 – v_\alpha(x, y) \leq 1 – S(1 – v_\alpha(x), v_\alpha(y)) = 1 – v_\alpha(x, y). Therefore \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R. Moreover, v_\alpha(x, y) \leq S(v_\alpha(x), v_\alpha(y)) for all x and y in R, which implies that 1 – v_\alpha(x, y) \leq 1 – S(1 – v_\alpha(x), v_\alpha(y)) = 1 – v_\alpha(x, y). Therefore \mu_\alpha(x, y) \geq T(\mu_\alpha(x, y)) for all x and y in R. Hence A = \Box A is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.12 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, \cdot), then the pseudo (T, S)-intuitionistic fuzzy coset (aA)^p is an (T, S)-intuitionistic fuzzy subnearring of a nearring R, for every a in R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. For every x and y in R, we have \((a_\alpha)^p(x, y) = p(a)\mu_\alpha(x, y) \geq p(a)T(\mu_\alpha(x, y)) = T(p(a_\alpha(x, y)), (a_\alpha)^p(y)). Therefore \((a_\alpha)^p(x, y) \geq T((a_\alpha)^p(x, y)), (a_\alpha)^p(y)). Now \((a_\alpha)^p(x, y) = p(a)(\mu_\alpha(x, y) = T(p(a_\alpha(x, y)), (a_\alpha)^p(y)) \geq T((a_\alpha)^p(x, y)), (a_\alpha)^p(y)). Therefore \((a_\alpha)^p(x, y) \geq T((a_\alpha)^p(x, y)), (a_\alpha)^p(y)). For every x and y in R, we have \((a_\alpha)^p(x, y) = p(a_\alpha(x, y) \leq p(a)S(v_\alpha(x, y)) = S(p(a)v_\alpha(x, y)), (a_\alpha)^p(y)). Therefore \((a_\alpha)^p(x, y) \leq S((a_\alpha)^p(x), (a_\alpha)^p(y)).

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Proof: Let \( x \) and \( y \) in \( R \) and \( A \) be an \((T, S)\)-intuitionistic fuzzy subnearring of a nearring \( H \). Then we have \( (\mu_A, \nu_A(x-y)) = (\mu_A(x) - \nu_A(y)) \geq T(\mu_A(x), \nu_A(y)) = T(\mu_A(y), \mu_A(x)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). And \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \geq T(\mu_A(x), \nu_A(y)) = T(\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). Then we have \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). Therefore \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). In the following Thm \( 6 \) is the composition operation of functions.

**2.13 Theorem:** Let \( A \) be an \((T, S)\)-intuitionistic fuzzy subnearring of a nearring \( H \) and \( f \) is an isomorphism from a nearring \( R \) onto \( H \). Then \( A^f \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \).

Proof: Let \( x \) and \( y \) in \( R \) and \( A \) be an \((T, S)\)-intuitionistic fuzzy subnearring of a nearring \( H \). Then we have \( (\mu_A, \nu_A(x-y)) = (\mu_A(x) - \nu_A(y)) \geq T(\mu_A(x), \nu_A(y)) = T(\mu_A(y), \mu_A(x)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). And \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \geq T(\mu_A(x), \nu_A(y)) = T(\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). Then we have \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). And \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). Therefore \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \).

**2.14 Theorem:** Let \( A \) be an \((T, S)\)-intuitionistic fuzzy subnearring of a nearring \( H \) and \( f \) is an anti-isomorphism from a nearring \( R \) onto \( H \). Then \( A^f \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \).

Proof: Let \( x \) and \( y \) in \( R \) and \( A \) be an \((T, S)\)-intuitionistic fuzzy subnearring of a nearring \( H \). Then we have \( (\mu_A, \nu_A(x-y)) = (\mu_A(x) - \nu_A(y)) \geq T(\mu_A(x), \nu_A(y)) = T(\mu_A(y), \mu_A(x)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). And \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \geq T(\mu_A(x), \nu_A(y)) = T(\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). Then we have \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). And \( (\mu_A, \nu_A(x-y)) = (\mu_A(x), \nu_A(y)) \) which implies that \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \). Therefore \( (\mu_A, \nu_A(x-y)) \geq T(\mu_A(x), \nu_A(y)) \).

**2.15 Theorem:** Let \( (R, +, .) \) and \( (R^1, +, .) \) be any two nearrings. The homomorphic image of an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \).

Proof: Let \( f : R \rightarrow R^1 \) be a homomorphism. Let \( V = f(A) \) where \( A \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \). We have to prove that \( V \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \). Now for \( f(x), f(y) \) in \( R^1 \), \( \mu_A(f(x)-f(y)) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \). Again \( \mu_A(f(x)-f(y)) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \). And \( \mu_A(f(x)-f(y)) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \). Therefore \( \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \).

**2.16 Theorem:** Let \( (R, +, .) \) and \( (R^1, +, .) \) be any two nearrings. The homomorphic preimage of an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \).

Proof: Let \( V = f(A) \), where \( V \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \). We have to prove that \( A \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \). Let \( x \) and \( y \) in \( R \). Then \( \mu_A(x-y) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \). And \( \mu_A(x-y) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \). And \( \mu_A(x-y) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \). Therefore \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \).

**2.17 Theorem:** Let \( (R, +, .) \) and \( (R^1, +, .) \) be any two nearrings. The anti-homomorphic image of an \((T, S)\)-intuitionistic fuzzy subnearring of \( R \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \).

Proof: Let \( f : R \rightarrow R^1 \) be an anti-homomorphism. Then \( f(x+y) = f(y) + f(x) \) and \( f(xy) = f(y)f(x) \) for all \( x \) and \( y \) in \( R \). Let \( V = f(A) \), where \( V \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \). We have to prove that \( V \) is an \((T, S)\)-intuitionistic fuzzy subnearring of \( R^1 \). Now for \( f(x), f(y) \) in \( R^1 \), \( \mu_A(f(x)-f(y)) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \). Again \( \mu_A(f(x)-f(y)) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \). And \( \mu_A(x-y) = \mu_A(f(x)-f(y)) \geq T(\mu_A(x), \mu_A(y)) \) which implies that \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \). Therefore \( \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) \).
2.18 Theorem: Let \((R, +, .)\) and \((R', +, .)\) be any two nearrings. The anti-homomorphic preimage of an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R'\).

Proof: Let \(V = f(A)\), where \(V\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\). We have to prove that \(A\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\). Let \(x, y\) in \(R\). Then \(\mu_A(x - y) = \mu_V(f(x - y)) = \mu_V(f(y) - f(x)) \geq T(\mu_V(f(y)), \mu_V(f(x))) = T(\mu_V(f(x)), \mu_V(f(y))) = T(\mu_V(f(x)), \mu_V(f(y))) = T(\mu_A(x), \mu_A(y))\) which implies that \(\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))\). Again \(\nu_A(x - y) = \nu_V(f(x - y)) = \nu_V(f(y) - f(x)) \leq S(\nu_V(f(y)), \nu_V(f(x))) = S(\nu_V(f(x)), \nu_V(f(y))) = S(\nu_V(f(x)), \nu_V(f(y))) = S(\nu_A(x), \nu_A(y))\) which implies that \(\nu_A(x - y) \leq S(\nu_A(x), \nu_A(y))\). Hence \(A\) is an \((T, S)\)-intuitionistic fuzzy subnearring of \(R\).

REFERENCE


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