Title :DARK MATTER AND DARK ENERGY OF THE UNIVERSE
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Abstract:

In this article, we propose a new model of dark matter. According to this new model, dark matter is a substance, that is a new physical element not constituted of classical particles, called *dark substance* and filling the Universe. Assuming some very simple physical properties to this dark substance, we will theoretically justify the flat rotation curve of galaxies and the baryonic Tully-Fisher's law. Then we will give a physical interpretation of the CMB Rest Frame (CRF). With the new model of dark matter, we will be naturally led to propose a new geometric model of the Universe, finite and not proposed by the Standard Cosmological model (SCM). We then will propose a first mathematical model of expansion of the Universe, based on General Relativity as the SCM, and in which the CMB rest frame plays an important role. This 1st model leads to the same mathematical predictions as the SCM. But we will propose also a 2nd mathematical model of expansion of the Universe, which is mathematically much simpler than General Relativity, but with theoretical predictions in agreement with the experimental data given by astronomical observations. Moreover this 2nd mathematical model does not need the existence of a dark energy, and consequently brings a solution to the enigma of dark matter. After this we will study according to the new proposed theory the different models of distribution of dark matter in galaxies. Then we will study the velocities of galaxies in clusters according to this distribution of dark matter, the evolution of the temperature of dark substance in the Universe and we will make appear the existence of a dark energy, due to our model of dark matter and to the expansion of the Universe.

Key words: Tully-Fisher's law, dark matter, dark halo, CMB, galaxy clusters, gravitational lensing, galaxy rotation curve, orbital velocity galaxies.

1.INTRODUCTION

In this article, we propose that a new physical element, called *dark substance*, constitutes the dark matter. According to our model, this dark substance fills all the Universe and has physical properties close to the physical properties of an ideal gas. We then show that it is possible, using those properties, to justify theoretically the flat rotation curve that is observed for some galaxies. If moreover we assume simple thermal properties to this dark substance, we see that we can justify theoretically the baryonic Tully-Fisher's law, despite the great specificity of this law. We recall that up to date, neither the flat rotation curve of galaxies nor the baryonic Tully-Fisher law have been justified theoretically in a satisfying way. It is true that a simple density of dark matter (in $1/r^2$) permitting to obtain this flat rotation curve has already been proposed, but this expression of density (in $1/r^2$) has not been theoretically justified. A theory called MOND theory [(1)] proposes also a theoretical justification of the flat rotation curve, but it is contrary to Newton's attraction law (which is difficultly acceptable) and moreover it is contradicted by some astronomical observations.

We also know that the CMB (Cosmic Microwave Background) Rest Frame (CRF), has not physical interpretation, concerning its nature and its main physical properties, in the Standard Cosmological Model (SCM). In this article, we are going to give a Physical Interpretation of the CRF, which permits new definitions of Cosmological variables (in particular the Cosmological time and the different kinds of distances used in Cosmology), that are in agreement with their definitions in the SCM. Considering the importance of this frame in Cosmology, we will also call it the local Cosmological frame. This will lead us to propose a new geometric model of Universe, flat and finite, that is not predicted by the SCM. Nonetheless, our Physical Interpretation of the CRF is compatible with Special and General Relativity. This Physical Interpretation of the CRF proposes 2 mathematical models of expansion of the Universe. The 1st model is as the SCM based on the equations of General Relativity. We then show that in this 1st model the observable Universe is identical to the observable Universe predicted by the SCM (Provided that some conditions be verified). Indeed in this 1st mathematical model, the different kinds of distances used in Cosmology and Hubble's constant , and also the cosmological redshift z have the same mathematical expression as in the SCM.

The 2nd mathematical model of our Interpretation of the CRF is not based on the equations of the SCM but is much simpler. Despite of this, its theoretical astrophysical predictions (In particular Hubble's law and Cosmological distances) are in agreement with astronomical observations. Moreover this 2nd model solves the enigma of the dark energy.

We will then study according to our model of dark matter the different models of distribution of dark matter in galaxies. We will also give a theoretical explanation to experimental data linked to the dark mass of clusters, in particular the velocities of galaxies in clusters, and the gravitational lensing that is the deviation of luminous rays, predicted by General Relativity, by the mass of clusters. Then we will study the density of dark matter in the Universe, that is at the origin of some anisotropies of the CMB. Finally we will study the evolution of the temperature of the dark substance in the Universe.

We remind that for many astrophysicists and physicists, the enigmas in the SCM, in particular the enigmas concerning dark matter and dark energy, make necessary a new paradigm for the SCM (2). Our article proposes such a new paradigm.

In this article we will express the main physical properties of the dark substance and the CRF in some Postulates, divided in points a),b)..

In our model of dark substance and in our Physical Interpretation of the CRF, we will keep all the points of the SCM, except the points of the SCM that are not compatible with our Postulates or that become useless because of them.

2. DARK SUBSTANCE-CMB REST FRAME

2.1 Physical properties of the dark substance.

As we have seen in 1.INTRODUCTION, we admit the Postulate 1 expressing the physical properties of the dark substance:

Postulate 1:

a)A substance, called *dark substance*, fills all the Universe.
b)This substance does not interact with photons crossing it.
c)This substance has a mass and obeys to the Boyle's law (called also Mariotte's law), to the Charles'law (called also Gay-Lussac's law), and to the following law that is their synthesis:
An element of dark substance with a mass m, a volume V, a pressure P and a temperature T verifies, k_0 being a constant:
$PV=k_0mT$

The preceding law is valid for a given ideal gas G_0, replacing k_0 by a constant $k(G_0)$, and this is a consequence of the *universal gas equation*, which is also obtained using Boyle and Charles'laws. For this reason we will call it the *Boyle-Charles'law*.

We have 2 remarks consequences of this Postulate1:
-Firstly despite of its name, the dark substance is not really dark but transparent. Indeed, because of the preceding Postulate 1b) it does not interact with photons crossing it.
-Secondly because of the Postulate 1a), what is usually called "vacuum" is not empty in reality: It is full of dark substance.

2.2 Flat rotation curves of galaxies.

Using the fact that the dark substance behaves as an ideal gas (Postulate 1c), we are going to show that a spherical concentration of dark substance in thermodynamic and gravitational equilibrium can constitute the dark matter in a galaxy with a flat rotation curve.

According to Postulate 1c) an element of dark substance with a mass m, a volume V, a pressure P and a temperature T verifies the law, k_0 being a constant:

$$PV=k_0mT \qquad (1)$$

Which means, setting $k_1=k_0T$:

$$PV=k_1m \qquad (2)$$

Or equivalently, ρ being the mass density of the element:

$$P=k_1\rho \qquad (3a)$$

We then emit the natural hypothesis that a galaxy can be modeled as a concentration of dark substance with a spherical symmetry, at an homogeneous temperature T.

We then consider the spherical surface S(r) (resp. the spherical surface S(r+dr)) that is the spherical surface with a radius r (resp. r+dr) and whose the center is the center O of the galaxy. S(O,r) is the sphere full of dark substance with a radius r and the center O.

S(O,r) (full sphere)

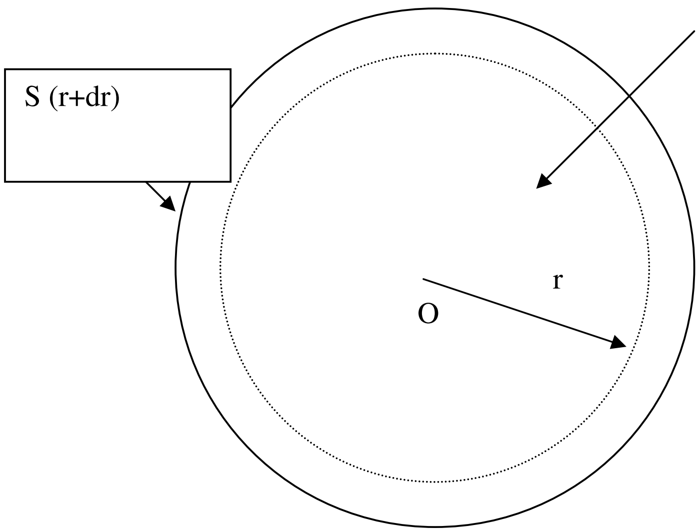

Figure 1:The spherical concentration of dark substance

The mass M(r) of the sphere S(O,r)is given by:

$$M(r) = \int_0^r \rho(x) 4\pi x^2 dx \qquad (3b)$$

Using Newton's law (**ΣF=0** for a material element in equilibrium, in the case of a spherical symmetry $\mathbf{F}_G$(r)=m**G**(r), $\mathbf{F}_G$(r) gravitational force acting on the element, **G**(r) gravitational field defined by Newton's universal law of gravitation) and Gauss theorem in order to obtain **G**(r),we obtain the following equation (4) of equilibrium of forces on an element dark substance with a surface dS, a width dr, situated between the 2 spheres S(O,r) and S(r+dr):

$$dSP(r+dr) + \frac{G}{r^2}(\rho(r)dSdr)(\int_0^r \rho(x)4\pi x^2 dx) - dSP(r) = 0 \qquad (4)$$

Eliminating dS, we obtain the equation:

$$\frac{dP}{dr} = -\frac{G}{r^2}(\rho(r))(\int_0^r \rho(x)4\pi x^2 dx) \qquad (5)$$

And using the equation (3) obtained using the Boyle-Charles'law assumed in the Postulate 1, we obtain the equation:

$$k_1 \frac{d\rho}{dr} = -\frac{G}{r^2}(\rho(r))(\int_0^r \rho(x)4\pi x^2 dx) \qquad (6)$$

We then verify that the density of the dark substance ρ(r) satisfying the preceding equation of equilibrium is:

$$\rho(r) = \frac{k_2}{4\pi r^2} \qquad (7)$$

(A density of dark matter expressed as in Equation (7) has already been proposed in order to explain the flat rotation curve of spiral galaxies, but it has not been justified theoretically. Here we give a theoretical justification of this expression (7), consequence of the model of dark substance as an ideal gas, Postulate 1)

The constant k_2 is given by, G being the Universal attraction gravitational constant:

$$k_2 = \frac{2k_1}{G} = \frac{2k_0 T}{G} \qquad (8)$$

Using the preceding equation (7), we obtain that the mass M(r) of the sphere S(O,r) is given by the equation:

$$M(r) = \int_0^r 4\pi x^2 \rho(x) dx = k_2 r \quad (9)$$

We then obtain, neglecting the mass of stars in the galaxy, that the velocity v(r) of a star of a galaxy situated at a distance r from the center O of the galaxy is given by $v(r)^2/r = GM(r)/r^2$ and consequently :

$$v(r)^2 = Gk_2 = 2k_1 = 2k_0 T \qquad (10)$$

So we obtain in the previous equation (10) that the velocity of a star in a galaxy is independent of its distance to the center O of the galaxy.

2.3 Baryonic Tully-Fisher's law.

2.3.1 Recall.

Tully and Fisher realized some observations on spiral galaxies with a flat rotation curve. They obtained that the luminosity L of such a spiral galaxy is proportional to the 4^{th} power of the velocity v of stars in this galaxy. So we have the Tully-Fisher's law for spiral galaxies, K_1 being a constant:

$$L = K_1 v^4 \qquad (11)$$

But in the case studied by Tully and Fisher, the baryonic mass M of a spiral galaxy is usually proportional to its luminosity L. So we have also the law for such a spiral galaxy, K_2 being a constant:

$$M = K_2 v^4 \qquad (12)$$

This 2^{nd} form of Tully-Fisher's law is known as the *baryonic Tully-Fisher's law*.

The more recent observations of Mc Gaugh [(3)] show that the baryonic Tully-Fisher's law (equation (12)) seems to be true for all galaxies with a flat rotation curve, including the galaxies with a luminosity not proportional to their baryonic mass.

We are going to show that using the Postulate 1 and a Postulate 2 expressing very simple thermal properties of the dark substance, (in particular its thermal interaction with baryonic particles), we can justify this baryonic law of Tully-Fisher despite of its great specificity.

2.3.2 Theory of quantified loss of calorific energy (by nuclei).

We saw in the previous equation (10) that according to our model of dark substance the square of the velocity of stars in a galaxy with a flat rotation curve is proportional to the temperature of the concentration of dark substance constituting this galaxy. So we need to determine T:
-A first possible idea is that the temperature T is the temperature of the CMB. But this is impossible because it would imply that all stars of all galaxies with a flat rotation curve be driven with the same velocity and we know that it is not the case.
-A second possible idea is that in the considered galaxy, each baryon interacts with the dark substance constituting the galaxy, transmitting to it a calorific energy. We can expect that this thermal energy is then very low, but because of the expected very low density of the dark substance and of the considered times (we remind that the diameter of galaxies is if the order of 100000 light-years), it can lead to appreciable temperatures of dark substance. A priori we could expect that this loss of calorific energy for each baryon (transmitted to the dark substance) depends on the temperature of this baryon and of the temperature T of the dark substance in which the baryon is immerged, but if it was the case, the total calorific loss for all baryons would be extremely difficult to calculate and moreover it should be very probable that we would then be unable to obtain the very simple baryonic Tully-Fisher's law.

We are then led to make the simplest hypothesis defining the thermal transfer between dark substance and baryons, expressed in the following Postulate 2a) (Postulate 2 gives the thermal properties of the dark substance):

Postulate 2a):
-Each nucleus of atom in a galaxy is submitted to a loss of calorific energy, transmitted to the dark substance in which it is immerged.
-This thermal transfer depends only on the number n of nucleons constituting the nucleus (So it is independent of the temperature of the nucleus). So if p is the thermal power dissipated by the nucleus, it exists a constant p_0 (thermal power dissipated by nucleon) such that:

$$p=np_0 \qquad (13)$$

According to the equation (13), the total thermal power transmitted by all the atoms of a galaxy towards the spherical concentration of dark matter constituting the galaxy is proportional to the total number of nucleons of the galaxy and consequently to the baryonic mass of this galaxy. So if m_0 is the mass of one nucleon, M being the baryonic mass of the galaxy, we obtain according to the equation (13) that the total thermal power P_r received by the spherical concentration of dark substance constituting the galaxy from all the atoms is given by the following equation, K_3 being the constant p_0/m_0:

$$P_r=(M/m_0)p_0=K_3M \qquad (14)$$

Concerning the preceding Postulate 2a):
-It is possible (but not compulsory) that it be true only for atoms whose temperature is superior to the temperature T of the concentration of dark substance.
-It permits to obtain the very simple Equation (14). We will see that this equation is essential in order to obtain the baryonic Tully-Fisher's law.

2.3.3 Obtainment of the baryonic Tully-Fisher's law.

In agreement with the previous model of galaxy (Section 2.2), we model a galaxy with a flat rotation curve as a spherical concentration of dark substance, at a temperature T and surrounded itself by a medium constituted of dark substance (called "intergalactic dark substance") at a temperature T_0 and with a density ρ_0.

In order to obtain the radius R of the concentration of dark substance constituting the galaxy, it is natural to make the hypothesis of the continuity of $\rho(r)$: R is the radius for which the density $\rho(r)$ of the concentration of dark substance is equal to ρ_0. So we have the equation:

$$\rho(R)=\rho_0 \qquad (15)$$

Consequently we have according to the equations (7) and (8):

$$\frac{k_2}{4\pi R^2} = \rho_0 \qquad (16)$$

$$\frac{2k_0 T}{G} \times \frac{1}{4\pi R^2} = \rho_0 \qquad (17)$$

So we obtain that the radius R of the concentration of dark substance constituting the galaxy is given approximately by the equation:

$$R = (\frac{2k_0 T}{4\pi G\rho_0})^{1/2} = K_4 T^{1/2} \qquad (18)$$

The constant K_4 being given by :

$$K_4 = (\frac{2k_0}{4\pi G\rho_0})^{1/2} \qquad (19)$$

We will call R the *dark radius* of the galaxy. We can then consider that the sphere with a radius R of dark substance constituting the galaxy at the temperature T is in thermal interaction with the medium constituted of intergalactic dark substance at the temperature T_0 surrounding it. The simplest and more natural thermal transfer is the classical convective transfer. We admit this in the Postulate 2b):

Postulate 2b):

The thermal interaction between the spherical concentration of dark substance constituting the galaxy (at the temperature T) and the surrounding intergalactic dark substance (at the temperature T_0) can be modeled as a classical convective thermal transfer.

We know that if φ is the thermal flow of thermal energy on the borders of the spherical concentration of dark substance with a radius R, P_l being the total power lost by the spherical concentration of dark substance constituting the galaxy is given by the equation:

$$P_l=4\pi R^2\varphi \qquad (20)$$

But we know that according to the definition a convective thermal transfer between a medium at a temperature T and a medium at a temperature T_0 and according to the previous Postulate 2b) the flow φ between the 2 media is given by the expression, h being a constant depending only on ρ_0:

$$\varphi=h(T-T_0) \qquad (21)$$

Consequently the total power lost by the concentration of dark substance is:

$$P_l=4\pi R^2 h(T-T_0) \qquad (22)$$

We can consider that at the equilibrium, the total thermal power P_r received by the spherical concentration of dark substance constituting the galaxy is equal to the thermal power P_l lost by this spherical concentration. Consequently according to the equations (14) and (22), (M being the baryonic mass of the galaxy), we have:

$$K_3 M=4\pi R^2 h(T-T_0) \qquad (23)$$

Using then the equation (18) :

$$K_3 M=4\pi K_4^2 hT(T-T_0) \qquad (24)$$

Making the approximation $T_0 \ll T$:

$$M = 4\pi \frac{K_4^2}{K_3} hT^2 \qquad (25)$$

Consequently we obtain the expression of T, defining the constant K_5 :

$$T = (\frac{K_3}{4\pi K_4^2 h})^{1/2} M^{1/2} = K_5 M^{1/2} \qquad (26)$$

And then according to the equation (10) :

$$v^2=2k_0T=2k_0K_5M^{1/2} \qquad (27)$$

So :

$$M = (\frac{1}{2k_0 K_5})^2 v^4 \qquad (28a)$$

So we finally obtain :

$$M=K_6 v^4 \qquad (28b)$$

The constant K_6 being defined by:

$$K_6 = (\frac{1}{2k_0 K_5})^2 = \frac{4\pi K_4^2 h}{4k_0^2 K_3}$$

$$K_6 = \frac{4\pi h}{4k_0^2 K_3} \times \frac{2k_0}{4\pi G\rho_0}$$

$$K_6 = \frac{m_0 h}{2k_0 G\rho_0 p_0} \qquad (28c)$$

So we obtain the baryonic Tully-Fisher's law (12), with $K_2=K_6$. It is natural to assume that h depends on ρ_0. The simplest expression of h is $h=C_1\rho_0$, C_1 being a constant. With this relation, K_6 is independent of ρ_0, and we can use the baryonic Tully-Fisher's law in order to define candles used to evaluate distances in the Universe.

2.4 Temperature of the intergalactic dark substance.

We introduced the temperature T_0 of the intergalactic dark substance. We could make the hypothesis that this temperature is the temperature of the CMB but we remind that in order to get the baryonic Tully-Fisher's law we supposed $T_0 \ll T$ (T temperature of the spherical concentration of dark substance in a galaxy). Consequently the previous hypothesis would lead to very high temperatures of spherical concentrations of dark substance constituting galaxies. We will see further that according to the theory of dark matter exposed here, the temperature T_0 of the intergalactic dark substance is not equal to the temperature of the CMB, except for a particular cosmological redshift z.

We could be in the following cases:
a)The temperature T_0 of the intergalactic dark substance at the present age of the Universe (equation (21)) is far less than the temperature of the CMB. (If the temperature of the dark halos of galaxies corresponding to the 1st model is inferior (approximately) to 300°K.)
b)Baryons can emit thermal power towards dark substance as assumed in the Postulate 2a) even if their temperature is inferior to the one of dark substance. (If the temperature of the dark halos of galaxies corresponding to the 1st model is superior to the temperature of gas whose the mass is used in the baryonic mass M intervening in the baryonic Tully-Fisher's law [3] $M=K_6v^4$ (equation (28b)) . We remind that dark substance being not ordinary baryonic matter, it can own very special thermal properties.)

We remind that according to the Postulate 1b), the dark substance does not interact with photons and in particular with the photons of the CMB. Consequently dark substance does not receive radiated energy.

2.5 Form of the Universe

If the Universe was completely isotropic, we could expect by symmetry that the thermal flow through a great surface be nil. Consequently the temperature of the dark substance inside a great sphere S of the Universe (For instance with a radius of 1 billion years) should increase and probably tend to a uniform temperature of dark substance inside the sphere S, because the thermal flow on S would be nil. We know that it is not possible in our model of dark substance because in this model spherical concentrations of dark substance constituting galaxies have not the same temperature (Because the velocity of stars is not always the same in all galaxies and we know that the temperature of the spherical concentration of dark substance is proportional to she squared velocities of stars inside this concentration (Equation (10)) and moreover because we admitted that the temperature T_0 of the intergalactic dark substance is by far inferior to the temperature of the spherical concentrations of dark substance constituting galaxies. So an infinite or finite isotropic Universe would contradict our model of thermal properties of the dark substance .

Nonetheless with our model of dark substance, it is much easier to define a finite Universe than in the SCM. Indeed we can consider that the Universe is a sphere (We could have chosen any other finite convex volume, but the spherical volume is by far the most attractive) constituted of dark substance surrounded by a medium called "nothingness" that is not constituted of dark substance. This was not possible in the SCM that admitted the Cosmological Principle according to which the Universe was isotropic observed from any point. Moreover the SCM did not assume the existence of the concept of a dark substance filling all the Universe and it is precisely this concept that permits us to define this new finite model of Universe with borders.

We can expect that in this new simple geometric model, the Universe appears to be isotropic not only if it is observed from O the centre of the sphere constituting the Universe, but also if it is observed from a point sufficiently far from its borders. We also remark that the existence of the medium that we called "nothingness" is also compatible with the MSC. Indeed we can consider that it was the medium before the Big-Bang.

In order to obtain the Cosmological redshift z with this new geometric model, we can apply the same equations as in the SCM. Indeed we keep the assumptions of the SCM according to which the densities of dark energy, of dark baryonic matter and of dark energy (if the latter exists) are homogeneous in all the Universe and we keep their values admitted in the SCM. The new model of Universe is no more isotropic (because of its borders), but nonetheless we can apply the same equations as in the SCM in any point situated at a distance sufficiently far from the borders of the Universe. And we will admit that this distance is quasi-nil or very small relative to the radius of the spherical Universe.

Concerning the CMB, we can admit as in the SCM that it appeared for an expansion factor 1+z of the order of 1500. The hypothesis according to which at the age of the Universe corresponding to this factor of expansion the temperature of dark substance and the temperature of the CMB were equal, is very attractive. Indeed with this hypothesis, assuming that the dark substance was homogeneous in temperature when the CMB appeared (for z of the order of 1500), because it is natural to assume that the dark substance in the Universe before the apparition of galaxies was homogeneous in temperature and density. In fact we assume that in the early Universe, the Homogenization Effect (concept defined in Section 2.4) prevailed in all the Universe. So the new theory of dark matter exposed here proposes a phenomenon different from the phenomenon called *inflation* in order to explain the quasi-isotropy of the CMB. But this theory remains sufficiently compatible with the SCM in order to explain the anisotropies of the CMB the same way as the SCM..

In the case in which Universe is a sphere (or any finite convex volume with a finite surface) constituted of dark substance, we avoid the previous problem concerning the temperature of the intergalactic dark substance. Indeed, we can assume, generalizing the Postulate 2b), that at the borders of the Universe, there is a convective thermal transfer. This new kind of thermal transfer is modeled as a convective transfer between a medium constituted of intergalactic dark substance at a temperature T_0 and a medium at a temperature equal to 0 (The nothingness). Then the thermal flow lost by the Universe is, h_n being a variable or a constant:

$$\varphi=h_n(T_0-0)=h_nT_0 \qquad (28d)$$

M being the baryonic mass of the Universe assumed to remain approximately constant, we obtain from equation (14) that the equation of thermal equilibrium is:

$$K_3M = 4\pi R_E(t)^2\ \varphi = 4\pi R_E(t)^2\ h_n T_0(t) \qquad (28e)$$

So we see that if the Universe increases from a factor 1+z, according to the equation (29a), if h_n is a constant (independent of the density of the intergalactic dark substance), the temperature $T_0(t)$ of the intergalactic dark substance diminishes from a factor $(1+z)^2$. If we had supposed that $h_n=C_2\rho_0$, ρ_0 being the mass density of the intergalactic dark substance and C_2 being a constant, we would have obtained that if the Universe increases from a factor 1+z, then T also increases by a factor 1+z which is impossible.

We also remark that the hypothesis of an infinite Universe, or a finite Universe without borders, that are geometric models proposed by the SCM [(6)(7)] , seems to be impossible to be conceived by the human mind, which is not the case with the finite spherical Universe, full of dark substance (or any finite convex volume with a finite surface), proposed by the theory exposed here.

2.6 Physical Interpretation of the CRF. Local and Universal Cosmological frames.

2.6.1 The 2 models of the Physical Interpretation of the CRF.

We remind that the CMB presents a Doppler effect that is canceled in a frame called for this reason the CMB Rest Frame (CRF). But this CRF has none physical interpretation in the SCM. We are going to give here a Physical Interpretation of the CRF, which permits to obtain a new model of Universe, that is spherical as in the preceding section 2.5. This new Physical Interpretation of the CRF is in agreement with the SCM in many points, in particular it admits Special and General Relativity. Also it permits to define *Cosmological variables* (Cosmological time, distances used in Cosmology, Hubble Constant) in a more precise way than in the SCM but nonetheless in a way that is in agreement with their definition in the SCM. Our Physical Interpretation of the CRF proposes 2 mathematical models of expansion of the Universe. (Because the Universe is in expansion in our Physical Interpretation of the CRF as it is in the SCM). The 1st mathematical model is based on General Relativity as the SCM. We will see that according to this 1st model the mathematical expressions of Cosmological variables are identical to their expression in the SCM. The 2nd mathematical model is much simpler, but nonetheless its theoretical predictions are in agreement with observation.

Concerning the physical properties of the CRF:

-Firstly it is natural that in each point of the Universe (and not only on the earth), we can define a CRF. We then can suppose that all CRF have parallel corresponding axis.

-Secondly we can think that the CRF permits to define very easily the Cosmological time, identified to the age of the Universe. The simplest definition of the Cosmological time would be that the time of the CRF (meaning the time given by the clocks at rest in the CRF) be precisely the Cosmological time. And we will see that this hypothesis is in agreement with observations. For instance we will see that its validity is illustrated by a very simple observation concerning the inertial frame linked to the sun. Indeed we recall that according to Special Relativity, if H_S is a clock linked to the sun and giving the time of the inertial frame

R_S linked to the sun, if R_{LC} is a local inertial frame giving the Cosmological time R_S being driven with a velocity V_S relative to R_{LC}, if T_S is a time measured by H_S corresponding to a Cosmological time T_C of R_{LC}, then: $T_S=T_C(1-V_S^2/c^2)^{1/2}$. Consequently if $V_S<<c$, we get $T_S \approx T_C$. Therefore it is completely impossible that locally all the inertial frames (with Lorentz transformations) give all the Cosmological time.
Consequently if $V_S<<c$ we get $T_S \approx T_C$.

-Thirdly we know that according to Special Relativity (We remind that we admit it as in the SCM) the velocity of a photon relative to the CRF in which it is situated is equal to c in norm. Moreover according to Special Relativity its velocity considered as a vector **c** keeps itself in this CRF. We will call *local velocity* this velocity **c**. An attractive hypothesis would be that the local velocity of the photon keeps itself the photon traveling in all the Universe. We will see that this hypothesis involves theoretical predictions that are in agreement with observation. In particular we will see that it permits to justify very simply the effect of the expansion of the Universe on the lengths of wave of photons and on the distances between 2 photons following one another. (This effect is also predicted by the SCM) .

So we express the preceding hypothesis in the following Postulate 3:

Postulate 3:
a)At each point of the Universe, we can define a CRF. We will assume that all CRF have parallel corresponding axis.
b)The Cosmological time (identified with the age of the Universe) is the time of all the CRF.
c)The *local velocity* of a photon, meaning measured in the CRF in which it is situated, keeps itself, the photon traveling in all the Universe.

We could think that the CRF are defined only after the apparition of the CMB, meaning at a very low Cosmological time but not at a Cosmological time equal to 0. In reality we will see in the Postulate 4 that in reality the RRC are defined since the beginning of the Universe. But CMB is presently the only way for detecting the CRF. This can be considered as a consequence of Special Relativity. We will see that the RRC is also the Referential in which the intergalactic dark substance is at rest. Considering the importance of this Referential we will also call it the *local Cosmological frame*.

Because of the Postulate 3b), and since we know that the inertial frame R_S linked to the sun is driven with a velocity $v_S<<c$ relative to the local CRF, the time of this frame R_S is very close to the time of the CRF, that is the Cosmological time, which is an agreement with observation. So the Postulate 3b) justifies that the time of R_S can be identified to the Cosmological time which was not at all evident. In fact, according with our models and astronomical observations, all galaxies of the Universe have a local velocity negligible (relative to c) relative to the local CRF and consequently the time given by the inertial frame linked to any star of any galaxy is very close to the Cosmological time.

We know need to define all the CRF. Each CRF has an origin and by analogy with the SCM, we can expect that if A(t) and B(t) are 2 origins of any 2 CRF (t Cosmological time), then the distance the distance $A(t_1)B(t_1)$ becomes $(1+z)A(t_2)B(t_2)$ if the factor of expansion of the Universe between t1 and t2 is equal to 1+z.

We saw in the previous section 2.5 that we could expect that the Universe had a finite convex volume with a finite surface, and we will assume in what follows that the Universe is a sphere (centre O), full of dark substance, surrounded but what we called “nothingness”. We remind nonetheless that what follows can be generalized if the Universe is a finite convex volume with a finite surface filled of dark substance and surrounded by what we called “the nothingness”. We saw that this medium can be identified with the medium preceding the Big-Bang:

If we consider that before the Big-Bang, a medium existed we call “nothingness” this medium and if we consider that before the Big-Bang nothing existed, we identify this “nothing” to the medium called “nothingness”.

In order to define completely the CRF, we introduce a new kind of frame, called *(Universal) Cosmological frame*, having its origin in O, centre of the sphere. This (Universal) Cosmological frame R_C will be used in order to define Cosmological variables. In particular the time of this Referential R_C is the Cosmological time of the CRF. Moreover we will assume that the axis of R_C are parallel to the corresponding axis of the CRF and that locally they give the same distances as the CRF. Nonetheless, the Cosmological frame R_C permits to measure distances between any 2 points of the Universe contrary to CRF that permit to measure only local distances. We will call *(primary) Cosmological distance* (in R_C) the distances measured in R_C. We will see that we can express all the classical Cosmological variables (For instance the comoving distances, the angular distance, the light-travel distance..) as a function of (primary) Cosmological distances measured in R_C ,of the time of R_C (Cosmological time) and of the expansion redshift z.

So we assume that the Universe is a sphere with a centre O, full of dark substance, and in expansion. Let $R_E(t)$ be the radius of this sphere , t being the Cosmological time. In analogy with the SCM, we assume that $R_E(t)=R_E(t_0)(1+z)$, 1+z being the factor of expansion of the Universe between t_0 and t. We will see further how we can get 1+z.

We are now going to define very important and particular points of the frame R_C, called *comoving points of the swelling sphere*.

We assume that P(t) is any point belonging to the border of the swelling sphere, t being the Cosmological time, with **OP**(t) (O is the centre of the swelling sphere) remaining in the same direction **u**, fixed vector R_C.

A comoving point A(t) of the swelling sphere is defined by :

-A(t) remains on the segment [O,P(t)]
-OA(t)=aOP(t), a being a constant belonging to [0,1]. (28f)

So in particular O and P(t) are comoving points of the swelling sphere. Moreover if A(t) and B(t) are 2 comoving points of the swelling sphere, belonging both to a radius [O,P(t)], and if t_1 and t_2 are 2 ages of the Universe, if $1+z=OP(t_2)/OP(t_1)$), (Here 1+z is the factor of expansion between t_1 and t_2) then we have the 2 relations:

$A(t_2)B(t_2)=(1+z)A(t_1)B(t_1)$ (28g)

And :

$[A(t_2),B(t_2)]//[A(t_1),B(t_1)]$ (28h)

(We classically note, P,Q being 2 points of R_C, PQ is the distance between P and Q measured in R_C, [P,Q] is the segment with extremities P and Q, (P,Q) is the straight line containing P and Q)

Using Thales theorem we obtain the 2 previous relation (28g) (28h) A(t) and B(t) being any comoving points of the swelling sphere (not compulsory belonging both to the same radius [O,P(t)]). We just use the relation: $OA(t_2)/OA(t_1)=OB(t_2)/OB(t_1)=f$.

So we see that the comoving points of the swelling sphere verify the expected relations between the origins of the CRF (Meaning that the distance between them increases by the factor of expansion of the Universe.)

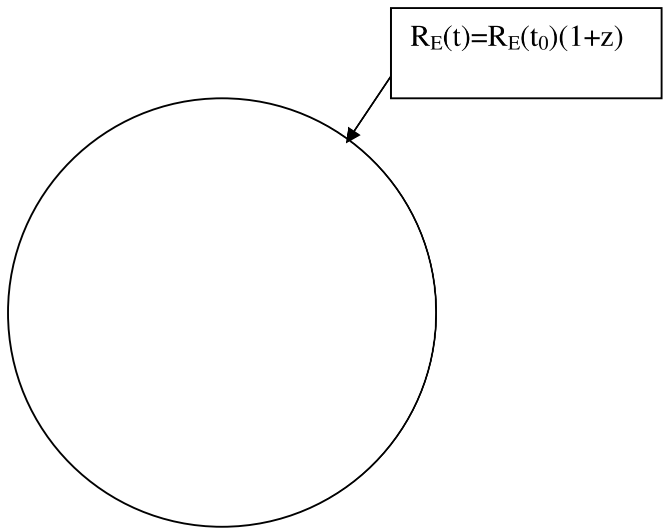

Figure 2:The model of the swelling sphere of the Universe.

Consequently the comoving points of the swelling sphere previously defined permit to complete the definition of the CRF, in the Postulate 4:

Postulate 4:
The origins of the CRF are the comoving points that we defined previously.

Now we need to express the factor of expansion 1+z as a function of the Cosmological time. We propose 2 models.

According to our 1st model, 1+z is obtained as it is obtained in the SCM: We apply locally the equations of General Relativity, assuming that the densities of dark substance, baryonic matter and dark energy own identical values to their values in the SCM and are homogeneous in all the Universe. A priori, we cannot apply the equations of General Relativity as in the SCM in a zone close to the borders of the Universe because we have no more isotropy of density in this zone. But we will assume that the dimensions of this zone are very small relative to the radius of the swelling sphere. Moreover, we will see that according to our model, in many cases this zone cannot be observed. And consequently in this 1st model, if the previous zone is sufficiently small, the factor of expansion 1+z used in the expression of $R_E(t)$ and to define the comoving points of the swelling sphere remains identical to its expression in the SCM. We will see that this equality involves that our 1st model of our Physical Interpretation of the CRF predicts distances used in Cosmology and a Hubble Constant that are mathematically equal to those predicted by the SCM.

Nonetheless, a priori, it is possible that the factor of expansion 1+z be not obtained by the equations of General Relativity. It is possible that as for the (local) velocity of light, the *Cosmological velocity* of the borders of the Universe relative to R_C (defined by $V_E(t)=d(R_E(t))/dt$, t Cosmological time) be as simplest as possible, meaning that it is equal to a constant C. There is no reason for which C should be equal or inferior to the velocity of light c because C is not the local velocity (defined in Postulate 3) of a photon or of a particle. So in our 2nd model, we assume that the Cosmological velocity of the borders of the Universe is equal to a constant C. We will see that we can give an inferior limit to this constant C. And we will also see that despite of this great simplicity, the predictions of this 2nd mathematical model are in agreement with all astronomical observations. Then if P(t) is a point of the border of the sphere OP(t)=Ct. And we have a very simple expression of 1+z: Between t_0 and t, $1+z=t/t_0$.

We saw that the SCM needed the existence of a mysterious dark energy, and it is also the case for our 1st model. But we see that in the 2nd model this enigma is solved because it does not need the existence of a dark energy. And this is a very attractive point of this 2nd model. This 2nd model is also clearly the simplest mathematical model of expansion of the Universe that can exist.

2.6.2 *The theoretical consequences of our Physical Interpretation of the CRF.*

As a consequence of our Physical Interpretation of the CRF, we can prove that as it was also the case in the MSC, if 2 photons ph1 and ph2 move in the same direction on a straight line towards the point O origin of R_C (We will see further that this remains true replacing O by any comoving point O' of the swelling sphere), then between 2 Cosmological times t_1 and t_2, the Cosmological distance measured in R_C between the 2 photons and the length of wave of each photon increase by the factor of expansion 1+z between t_1 and t_2 .

Indeed let us consider 2 photons defined as previously. So they have an identical local velocity **c** (with a direction being the direction of the straight line). We take the following notations: At the Cosmological time t ph1 is in the point ph1(t) of R_C, and ph2 is in the point ph2(t). Let us suppose that for a given Cosmological time t, ph1(t) coincides with a comoving point $A_1(t)$ and ph2(t) with a comoving point $A_2(t)$. Let 1+dz the factor of expansion of the swelling sphere between t and t+dt. Then we have according to the property (28g) of comoving points:

$A_1(t+dt)A_2(t+dt)=(1+dz)A_1(t)A_2(t)=(1+dz)ph1(t)ph2(t)$.

Moreover, the local velocity of photons being equal to c:

$A_1(t+dt)ph1(t+dt)=A_2(t+dt)(ph2(t+dt)=cdt$

And consequently :

$ph1(t+dt)ph2(t+dt)=A_1(t+dt)A_2(t+dt)=(1+dz)(ph1(t)ph2(t)$

We obtain the same way that because of the expansion of the Universe, the length of wave of a photon is also increased by the factor of expansion of the Universe 1+z. We identify a photon with a system between a segment [a(t),b(t)], the length a(t)b(t) being the length of wave of the photon, a(t) and b(t) being driven with the same local velocity **c**, **c** being the velocity of the photon, and the line (a(t),b(t)) being parallel to the velocity of the photon **c**.

We can show in an analogous way that if we suppose only that ph1 and ph2 own the same local velocity (as a vector), and are not compulsory moving on a straight line towards O, then the primary Cosmological distance between ph1(t) and ph2(t) is increased by the factor of expansion 1+z and moreover $(ph1(t_1),ph2(t_1))//(ph1(t_2),ph2(t_2))$

We remark that in any commoving point of the swelling sphere O'(t) we can define a Cosmological frame R_C' whose the axis are parallel to the corresponding axis of R_C and defining the same Cosmological variables as R_C (primary Cosmological distance at a given Cosmological time t and Cosmological time). We will call R_C' *secondary Universal Cosmological frame*.

Then if A(t) is any commoving point of the swelling sphere defined previously, t_1 and t_2 being 2 Cosmological times, according to the properties of commoving points (28g)(28h), if 1+z is the factor of expansion of the Universe between t_1 and t_2:

$O'(t_2)A(t_2)=(1+z)O'(t_1)A(t_1)$ et $(O'(t_2),A(t_2))//(O'(t_1),A(t_1))$

And consequently $(O'(t_1),A(t_1))$ et $(O'(t_2),A(t_2))$ are in the same direction **u**.

Consequently the properties (28f), replacing R_C by R_C' and O by O', remain valid, P(t) being still a point of the border of the sphere. (But here O'(t)P(t) is no more equal to $R_E(t)$). Consequently the expressions of distances used in Cosmology and Hubble's constant are obtained in R_C' exactly the same way as in R_C.

We will see that according to our Physical Interpretation of the CRF we cannot observe all the Universe from $O(t_0)$ (or $O'(t_0)$, (t_0 present age of the Universe), which was also the case in the SCM. Moreover the properties of R_C'(t) involve that if $O'(t_0)$ is sufficiently far from the borders of the Universe, then according to our Physical Interpretation of the CRF the Universe observable from $O'(t_0)$ is identical to the Universe observable from $O(t_0)$. In particular in that case the Universe is isotropic observed from $O'(t_0)$, as it was observed from O.

It is possible to elaborate a complete physical theory of the CRF [(4)], but the validity of the models exposed in this article is completely independent of this theory.

The spherical form of the Universe could be confirmed if some celestial bodies (quasars?) would not own a homogeneous density in the Universe, but a density presenting a spherical symmetry relative to a point O. According to our models, O would be then the centre of the spherical Universe.

2.7 Hubble's law-Distances used in Cosmology.

We keep the preceding model and notations. Let us suppose that a photon is emitted from a star S at a point $Q(t_E)$ of R_C (Q(t) is a commoving point of the swelling sphere) at a Cosmological time t_E towards $O(t_E)$ origin of R_C. We suppose that the photon reaches $O(t_0)$ at the present Cosmological time t_0. We assume that between t_E and t_0 the factor of expansion of the Universe is $1+z_0$.

Between t and t+dt, we know that the photon covers the local distance cdt. Consequently between t_E and t_0 the sum of the local distances covered by the photon will be :

$D_T=c(t_0-t_E)$ (29a)

We will call this distance, which is completely identical to the *light- travel distance* in the SCM, by the same name. We can also call it *time-back distance* because it permits to obtain the Cosmological time between the emission and the reception of the photon.

We will see further how in the 1st mathematical model of expansnsion distances used in Cosmology and Hubble's Constant have the same mathematical expressions as their expressions in the SCM, and are also obtained the same way.

But in the 2nd model we obtain very easily the Hubble's Constant using the light-travel distance defined previously.

Indeed according to this 2nd model:

$$1+z_0=(Ct_0)/(Ct_E)=t_0/(t_0-D_T/c) \qquad (29b)$$

When $D_T/ct_0<<1$ we obtain $z_0 \approx D_T/ct_0$ and consequently the Hubble's constant is equal to $1/t_0$. The preceding equation (29b) is very simple and can easily be verified. For instance taking t_0=15 billion years, for z_0=0.5,we obtain D_T=5 billion light years and for z_0=9 we obtain D_T=13.5 billion years. These predicted values are in agreement with the usual admitted experimental values for the light-travel distance D_T.

We took a present Cosmological time (age of the Universe) equal to 15 billion years corresponding to a Hubble's constant $H=1/t_0$ approximately equal to 65 km/sMpc^{-1} despite that it is generally admitted that the Hubble's constant H is approximately equal to 72km/sMpc^{-1} corresponding to a time $t_0=1/H$ approximately equal to 13,5 billion years.

Nonetheless many astrophysicists disagree with a Hubble's constant approximately equal to 72 km/s Mpc^{-1} and find a Hubble's constant approximately equal to 65km/sMpc^{-1}, for instance Tammann and Reindl [(5)] in a very recent article (October 2012). There is also a second possibility: light-travel distance could be superior to present estimations by a factor of 5% to 7%.

So it is very remarkable that according to the 2nd model, the value of Hubble's constant is very easily obtained and is equal to $1/t_0$, t_0 present age of the Universe, in agreement with the observation. In the SCM (and in the 1st model), the obtainment of Hubble's constant was much more complicated and moreover it was not exactly equal to $1/t_0$.

We assume that a photon is emitted by a star S (galaxy, star, cluster..) at a commoving point $Q(t_E)$, t_E age of the Universe, and reaches the origin $O(t_0)$ of the Universal Cosmological frame at the present age of the Universe t_0. We will obtain all the distances used in Cosmology assuming that the local velocity of any star S is small relative to c, meaning that S is approximately at rest in its local Cosmological frame. The predictions of the theoretical distances used in Cosmology are then in agreement with astronomical observations, which confirms the validity of this assumption. We remark that we could expect that the star S remain close to the commoving point Q(t) because its local velocity is small relative to c.

We then can define in our model of spherical Universe in expansion other kinds of distances used in Cosmology in a completely analogous way to their definition in the SCM:

We have seen that we can express the light-travel distance as:

$$D_T = \int_{tE}^{t0} cdt \qquad (29c)$$

The local distance covered by the photon between t and t+dt is, according to the Postulate 3 equal to cdt. This local distance, considered as a distance between 2 commoving points of the swelling sphere, is increased by the factor of expansion of the Universe $1+z=t_0/t$ between t and t_0 (See equation (28g)).

In complete analogy with the SCM, we will call *comoving distance* between O and S the primary Cosmological distance between $Q(t_0)$ and $O(t_0)$ (Meaning their distance measured in the Cosmological frame R_C), which is the sum of all the local distances cdt covered by the photon, increased by the factor 1+z. Let D_C be this distance:

$$D_C = \int_{tE}^{t0} c(1+z)dt \qquad (29d)$$

From this expression we define the *luminosity-distance* D_L between O and S (at the Cosmological time t_0) and the *angular-distance* D_A between O and S in complete analogy with their definition in the SCM:

$$D_L=(1+z_0)D_C$$

$$D_A=D_C/(1+z_0) \qquad (29e)$$

The distance D_A appears to be the primary Cosmological distance (distance in R_C) between $Q(t_E)$ and $O(t_E)$. In complete analogy with the SCM it permits to obtain some angles with a summit O in R_C.

The distance D_L , in complete analogy with its definition in the SCM, appears to be obtained measuring the luminous flow of a supernova taking into account the effect of the expansion of the Universe on the lengths of wave of the photons and on the distances between 2 photons (moving on the same axis). We saw in the section 2.6.2 that this effect, predicted by the SCM, was also true in the Physical Interpretation of the CRF.

The mathematical expressions of the different kinds of distances used in Cosmology (29c)(29d)(29e) are in agreement with their mathematical expression in the SCM, in which they are usually expressed as a function of the variable z.

In the 1st mathematical model of expansion, since 1+z has the same mathematical expression as in the SCM (as a function of the Cosmological time t) the final expression of those distances used in Cosmology as a function of z is identical to their final expression in the SCM. Consequently we also obtain an identical Hubble's constant.

In the 2nd model, the expressions of distances used in Cosmology are much simpler. Using $1+z=t_0/t$ we obtain:

$$D_C = \int_{tE}^{t0} c(1+z)dt = \int_{tE}^{t0} c(t_0/t)dt$$

So we obtain finally the expression of the comoving distance, using $1+z_0=t_0/t_E$:

$$D_C=ct_0\mathrm{Log}(t_0/t_E)=ct_0\mathrm{Log}(1+z_0) \qquad (29f)$$

Here also this simple expression is in agreement with the usual admitted experimental values for the comoving distance. We remark that in our 2nd model, according with the previous equations we have as in the SCM for $z_0<<1$, $D_T\approx D_C\approx D_A\approx D_L\approx ct_0z_0$.

We obtain easily that according to the 2nd model, the Cosmological velocity of the borders of the sphere being constant and equal to C (in R_C), then the Cosmological velocity of any comoving point of the swelling sphere is constant and inferior or equal to C (measured in R_C, using that with our notations $OA(t)=aR_E(t)$ (equation (28f)). Let V_Q be the Cosmological velocity of Q(t). Then consequently the distance in R_C between $O(t_0)$ and $Q(t_0)$, that we called D_C is also equal to $V_Q t_0$. Consequently because of the previous equation (29f) we have:

$$V_Q=cLog(1+z_0)$$

We can interpret in our model of spherical Universe in expansion the observation of the explosion of a supernova [(6)] the same way as in the SCM, taking into account the effect of the expansion of the Universe on the lengths of wave of photons and on distances between photons moving on the same axis. We remind that we obtained this effect, that is also true in the SCM, in the section 2.6.2.

2.8 Cosmological limits of the observable Universe.

In our model of spherical Universe in expansion we cannot, as it was also the case in the SCM, observe the Universe (observing the galaxies) before a given time t_{OU}. This implies that observing the Universe from a comoving point $O'(t_0)$ (t_0 present Cosmological time) sufficiently far from the borders of the Universe, the observable Universe is isotropic and also that in many cases, the borders of the Universe cannot be observed from $O'(t_0)$. Here we are going to see how we can obtain this time t_{OU} according to our model of finite Universe in expansion, and more precisely according to the 2^{nd} mathematical model of expansion of the Universe, that is much simpler than the mathematical model of the SCM.

It is clear that according to our model of spherical Universe in expansion, as in the SCM, the Universe cannot be observed before the end of the dark age, at a Cosmological time t_D, because we admit as in the SCM that before t_D light cannot propagate inside the Universe. Moreover, galaxies cannot be observed before the Cosmological time t_G, that is the time of the apparitions of the first galaxies. It exist another limit according to our model of spherical Universe in expansion. This is very clear in our 2^{nd} model:

According to the equation (29g), V_Q being compulsory inferior to C, we have:

$$C \geq cLog(1+z_0) \qquad (29h)$$

Consequently, with the notations of the previous section:

$$t_0/t_E=1+z_0=\leq exp(C/c) \qquad (29i)$$

Which implies that the Universe cannot be observed in $O(t_0)$ before the time t_I defined by:

$$t_I=t_0 exp(-C/c) \qquad (29j)$$

So in our Physical Interpretation of the CRF, t_{OU} is the greatest time between t_I, t_G and t_D. Moreover if $t_{OU}>t_I$, we cannot observe the borders of the Universe from $O(t_0)$.

We remark that the equation (29h) permits to give an inferior limit to the constant C of the 2^{nd} model: The fact that we have observed some redshift z equal to 10 implies that C>2,3c. If we take C=10c, we obtain t_I of the order of 1million years.

The previous equations permit to obtain, according to the 2^{nd} model, the minimal distance in R_C' (Cosmological frame with an origin O'(t) defined in section 2.6.2) between $O'(t_0)$ and the borders of the Universe (at the Cosmological time t_0) for which the Universe appears to be isotropic observed from $O'(t_0)$ (Which means that the borders of the Universe cannot be observed from $O'(t_0)$).

2.9 The Cosmic Microwave Background.

In complete agreement with the SCM, we admit the apparition of a CMB at a Cosmological time very close to the Big-Bang (We admit as in the SCM that the Big Bang occurs at a Cosmological time equal to 0). Proceeding exactly as in the SCM, taking into account the effect of the expansion of the Universe on the lengths of wave of photons and on photons moving on the same axis (effect obtained in section 2.6.2) , we obtain in the Physical Interpretation of the CRF that if the CMB appears at a Cosmological time t_{iCMB} corresponding to a temperature T_{iCMB}, then at an absolute time t superior to t_{iCMB}, if the factor of expansion between t_{iCMB} and t is 1+z, then the CMB at a Cosmological time t corresponds to a temperature $T_{CMB}(t)=T_{iCMB}/(1+z)$. (This is obtained exactly the same way as in SCM, because we have in both Cosmological models that with the same notations the density of photons is divided by $(1+z)^3$ and the lengths of wave of photons are increased by a factor (1+z)). And consequently our Physical Interpretation of the CRF is in agreement with the observation of the CMB corresponding to a great redshift z_0 [7][8] .

We remind that we saw in section 2.5 that with the hypothesis of an initial equality of the temperature of the CMB and the temperature of the dark substance, taking a thermal model similar to the thermal model used in order to obtain the baryonic Tully-Fisher’s law, then at the present age of the Universe the temperature of the intergalactic dark substance (evolving in $1/(1+z)^2$) is approximately 1500 times less than the temperature of the CMB (evolving in 1/(1+z)).

But now we have given a very complete physical interpretation of the CRF that did not exist in the SCM. In our Physical Interpretation of the CMB we interpret the interpretation of the anisotropies of the CMB as the SCM.

It is important to know what happens to a photon reaching the borders of the spherical Universe. It could be absorbed but it is not the only possible hypothesis. The simplest hypothesis according which the photon is not absorbed, that we will admit in our Physical Interpretation of the CRF, expresses that the photon is reflected, taking exactly the opposite of its local velocity (as a vector). With this last hypothesis we could expect to see reflected images of some galaxies. But there are several explanations to the fact that it is not the case:

We keep the notations of the previous section 2.8, defining the limits of the Cosmological time before which it cannot be observed:

We obtain easily that if $t_G>t_I$ or $t_I<t_D$ then we cannot observe the reflection of images of galaxies on the borders of the Universe. Indeed in the 1st case the reflected images of galaxies reach O after t_0 and in the 2nd case the reflected photons are absorbed.

2.10 Dipole contribution of the CMB.

We know that according to the SCM we have the following fluctuations of temperature of the CMB [7]:

$$(\frac{\Delta T}{T}) = \frac{1}{4\pi}\sum_l l(2l+1)C_l \qquad (30)$$

In the previous expression l=1 is the dipole contribution, corresponding to the motion of the earth relative to the CRF. In our Physical Interpretation of the CRF, we keep the previous expression, but then we can interpret the dipole contribution of this equation, which was not the case in the SCM.

3.COMPLEMENTS

In the Part 2 of this article, we presented a new model of dark matter, called dark substance, and a Physical Interpretation of the CRF. In this Part 3, we study the consequences of these models, as for instance the motion of a spherical concentration of dark substance (constituting some galaxies with a flat rotation curve according to the preceding article), the thermal effects on the spherical concentration of dark substance due to this motion, and the effects of this motion on the mass and the velocity of this spherical concentration. We will see that it exists 2 kinds of radius in a galaxy, the 1st one being the baryonic radius (visible) and the 2nd one, called *dark radius*, being the radius of the spherical concentration of dark substance. We will give the mathematical expression of this dark radius as a function of the Cosmological time, and we will study a particular case, the case of the Milky Way at a Cosmological time equal to 5 billion years. We will also study the concentration of dark substance around stars and planets, and we will make appear the existence of new kinds of galaxies. Then we will propose a distribution of dark matter in clusters, and different dynamical models of clusters permitting to obtain the mass of clusters from the observation of velocities of galaxies in clusters. To end we will study according to our Cosmological theory the evolution of the temperature of the intergalactic dark substance as a function of the age of the Universe.

3.1 Motion of a galaxy inside the intergalactic dark substance.

We could think that a spherical concentration of dark substance constituting a galaxy, moving through the intergalactic dark substance, is submitted to some modifications of its mass and velocity because of this motion.

In fact, we have the 2 following properties for the concentration of dark substance:

a) The moving spherical concentration keeps its mass.
b) The moving spherical concentration keeps its velocity: It is not slowed down nor accelerated.

Indeed, let us consider a spherical concentration of dark substance constituting the dark matter of a galaxy (centre O) driven with a local velocity **V** relative to the intergalactic dark substance (In fact we can assume that locally, the dark substance is at rest relative to the local CRF, and consequently **V** is also the local velocity (relative to the local CRF) of the spherical concentration of dark substance). Let us consider the disk whose the center is O, the radius is the radius of the spherical concentration, and that is perpendicular to the velocity **V**. Let S be the surface of the disk. Then in an interval of Cosmological time dt, we have the 2 phenomena:

c) A volume SVdt of dark substance is absorbed by the spherical concentration.(In front of the sphere).
d) A volume SVdt is emitted by the spherical concentration (to the back of the sphere).

Moreover we remark that according to our model the emitted and the absorbed dark substance have the same density, that is the one of the intergalactic density. Consequently the emitted mass and the absorbed mass are equal, which implies that the spherical concentration keeps its mass (Property a)). Moreover we can assume that the emitted dark substance(in its final state) and the absorbed dark substance have the same local velocity (velocity of the surrounding intergalactic dark substance, which we can assume being equal to 0), and consequently the velocity of the spherical concentration is not modified (Property b)).

We have a second possible justification:

Let us suppose that the moving spherical concentration of dark substance lose a little more dark substance than it absorb. Let us suppose for instance that the total loss be δm. Then the equation of equilibrium (6) remaining the same, we can assume that the spherical concentration of dark substance will absorb also the missing mass δm, coming back to the equilibrium. Consequently the mass of the concentration of dark substance remains the same. Moreover we can assume as previously that lost dark substance (in its final state) and absorbed dark substance have the same velocity (velocity of the surrounding intergalactic dark substance). Consequently, this is a second and more general justification that the spherical concentration of dark substance is not accelerated nor slowed down.

It is also possible that lost dark substance and absorbed dark substance have not exactly the same local velocity. Then the velocity of the traveling concentration of dark substance is slightly modified, but it is possible that this effect be completely negligible and that the velocity of this galaxy in its galaxy cluster as a function of the Cosmological time remains constant. We remark also that it is very difficult to observe the evolution of the local velocity of a galaxy as a function of the Cosmological time.

3.2 Baryonic and dark radius of a galaxy.

We know that the galaxy Andromeda is approximately at 2.5 billions year-light of our galaxy the milky way. We consider for instance the case of the milky way in order to study the 2 kinds of radius of a galaxy. We suppose that we are in the 2^{nd} mathematical model of the Physical Interpretation of the CRF (Section 2.6.1) because of its great simplicity.

We saw in the Section 2.2 that if r is the distance to the center O of a spherical concentration of dark substance constituting a galaxy, then the expression of the density of dark substance ρ(r) is given by, k_3 being a constant (See section 2.2, equation (7) $k_3=k_2/4\pi$):

$$\rho(r) = \frac{k_3}{r^2} \qquad (31)$$

So we obtain, M(r) being the mass of the sphere having its center in O and a radius r (See equation (9)):

$$M(r)=4\pi k_3 r \qquad (32)$$

Consequently, v being the velocity of a star at a distance r of O (see equation (10)):

$$v^2 = \frac{GM}{r} = 4\pi k_3 G \qquad (33)$$

Consequently:

$$k_3 = \frac{v^2}{4\pi G} \qquad (34)$$

We know also that if ρ_0 is the local density of the intergalactic dark substance surrounding the spherical concentration of dark substance constituting the galaxy, then the radius R of this concentration of dark substance is given by the expression (See equation (15)):

$$\rho(R) = \frac{k_3}{R^2} = \rho_0 \qquad (35)$$

Consequently:

$$R = \sqrt{\frac{k_3}{\rho_0}} = v\sqrt{\frac{1}{4\pi G\rho_0}} \qquad (36)$$

In previous sections, we called R the *dark radius* of the considered galaxy.

So in a galaxy for which it exists a spherical concentration of dark substance with a density in $1/r^2$, we have 2 different kinds of radius:

The 1st kind of radius, called *dark radius*, is the radius of the spherical concentration of dark substance. The 2nd kind of radius is the radius of the smallest sphere containing all the stars. We will call *baryonic radius* this second kind of radius. We remark that at a given time, the dark radius must be greater than the baryonic radius.

Let $\rho_0(5)$ be the density of the intergalactic dark substance when the age of the universe (Cosmological time) was 5 billion years, and $\rho_0(15)$ this density at an age of 15 billion years (meaning presently).
We will see further that $\rho_0(z)$ is approximately the mean density of the Universe corresponding to a Cosmological redshift z. Consequently, if we admit that the total mass of dark substance keeps itself, we obtain that $\rho_0(z)=\rho_0(0)(1+z)^3$. Consequently if f=1+z is the factor of expansion of the universe between 5 and 15 billion years we obtain:

$$\rho_0(15)=\rho_0(5)/f^3 \qquad (37)$$

Moreover according to the 2nd mathematical model of expansion that we exposed previously, f=15/5=3 (See Section 2.6.1).

We note $r_B(15)$ the present baryonic radius of the milky way. We know that $r_B(15)$ is approximately equal to 50000 years light . If R(15) is the present dark radius of the milky way, let us suppose that R(15) is approximately 10 times greater than $r_B(15)$ (meaning approximately 500000 light-years):

$$R(15)\approx 10r_B(15) \qquad (38)$$

Of course we ignore the real value of R(15),we can only know its minimal value (It must be superior to the baryonic radius). We are going to see that our hypothesis (38) leads to coherent results. Let $r_B(5)$ be the baryonic radius of the milky way when the age of the Universe was 5 billion years. Considering that the baryonic radius increases with time, we have the relation:

$$r_B(15)\geq r_B(5) \qquad (39)$$

We have seen and justified theoretically in the Section 2.3 of this article that according to the baryonic Tully-Fisher's law the velocity of stars in a galaxy with a flat rotation curve depended only on the baryonic mass of this galaxy. Consequently if we suppose that between 5 and 15 billion years, the baryonic mass of the galaxy remains approximately the same, the velocity v used in the equation (36) remains unchanged between 5 and 15 billion years. Using this equation (36) and the equation (37), taking f=3 and $\sqrt{(27)}\approx 5$, we obtain, R(5) being the dark radius of the milky way at an age of the Universe equal to 5 billion years:

$$R(5)\approx R(15)/5\approx 2r_B(15) \qquad (40)$$

Using the equations (39) and (40) we obtain that at an age of the Universe of 5 billion years, the dark radius was greater than the baryonic radius:

$r_B(5) \leq r_B(15) \approx R(5)/2 \leq R(5)$ (41)

We remark that the previous relation (41) would have also be valid for a galaxy with the same dark radius R'(15)=500000 light-years but with a baryonic radius $r'_B(15)$ twice greater than the radius of the milky way meaning 100000 light-years. (We just take $r'_B(15) \approx$ 100000 years light and replace the equation (38) by the equation: $R'(15) \approx 5r'_B(15)$). Our model remains obviously valid if the final baryonic radius is reached after 5 billion years.

3.3.Thermal transfer of a moving galaxy.

We remark that the phenomenon of absorption and of emission of dark substance by a galaxy that we described in the Section 3.1 modifies the thermal equilibrium that we used in the Section 2.3 of this article in order to obtain the Tully-Fisher's law. Indeed the absorbed dark substance (cold, because it is intergalactic dark substance) is not at the same temperature than the lost dark substance (hot, because it is the temperature of the spherical concentration of dark substance).

Nonetheless we can consider that the previous phenomenon leads to a power $\varepsilon(t)$ dissipated by the spherical concentration of dark substance. $\varepsilon(t)$ mainly depends on the radius of the moving spherical concentration, of its velocity relative to the local intergalactic dark substance, of the density of the intergalactic dark substance, and of the temperature of the concentration of dark substance.

If we assume that $\varepsilon(t)$ is negligible compared with the power emitted by the baryons of a galaxy towards the spherical concentration of dark substance (whose we supposed the existence in order to obtain the baryonic Tully-Fisher's law, see Postulate 2a in section 2.3), then our thermal model used in order to get the Tully-Fisher's law remains valid. We can a priori neglect $\varepsilon(t)$ because in one year, the distance covered by the moving spherical concentration (the local velocity of the spherical concentration of dark substance is assumed to be of the order of 300km/s ($10^{-3}c$)), is very low relative to the dark radius of the considered galaxies (At least of the order of 100000 light-years).

3.4 Other models of distribution of dark matter in galaxies.

We have previously exposed a 1st model of distribution of dark matter in galaxies with a flat rotation curve. In this 1st model we could neglect the gravitational effect due to the baryonic mass of the galaxy.

At least 2 more models of distribution of dark matter in galaxies are possible:

In the 2nd model, in order to obtain the density $\rho(r,\mathbf{u})$ of the dark substance, we also apply Newton's laws, but we neglect contrary to the 1st model the gravitational attraction due to the dark substance and we consider only the gravitational attraction due to the baryonic matter of the galaxy. This case is most of the time very complex because the density of baryonic matter usually does not own a spherical symmetry and moreover its is difficult to obtain it. To begin with, we consider the case in which we have a spherical symmetry for the density of baryonic matter. If the galaxy is immerged in a medium of dark substance with a density ρ_0 and a temperature T_0, (ρ_0 and T_0 are not compulsory the density and the temperature of the intergalactic dark substance. For instance in the case of a galaxy G1 satellite of another galaxy G2, G2 belonging to the 1st model and G1 being inside the dark halo of G2. In this case we know the value of ρ_0 (equation (8)) we can assume as for the 1st model that it exists a minimal radius R_S, called *dark radius* of the galaxy, such that for $r>R_S$ we have $\rho(r)=\rho_0$ and a temperature of the dark substance equal to T_0. If we know R_S, we obtain $\rho(r)$ for $r<R_S$ using Newton's laws and the condition $\rho(R_S)=\rho_0$.

A priori we ignore R_S but it is interesting to consider the case in which we have $R_S=R_B$, R_B baryonic radius of the galaxy. We will justify further the possibility to consider this particular case.

Let us now consider an example in which the distribution of baryonic matter is the simplest possible, with a constant baryonic density ρ_B inside a sphere of radius R_B. It is very possible that such galaxies do not exist, but this example permits to show how we can get $\rho(r)$ and to obtain its order of magnitude. We first consider the case $R_S=R_B$.

We proceed as in the 1st model:

We assume that we have a spherical concentration of dark substance with a homogeneous temperature T and a radius $R_S=R_B$. We consider an element of dark substance with a surface dS, perpendicular to the radius, a width dr and situated at a distance r from the center of the galaxy. Applying the Newton's law, using the Gauss theorem and the Boyle-Charles'law (Postulate 1), we obtain proceeding as in the 1st model the equation:

$$\frac{1}{\rho(r)}\frac{d(\rho(r))}{dr} = -\frac{4}{3}\frac{G\pi\rho_B}{k_0 T} \qquad (42)$$

With the condition $R_S=R_B$ and consequently $\rho(R_B)=\rho_0$, we obtain for $r<R_B$:

$$\rho(r) = \rho_0 \exp\left(-\frac{4}{3}\frac{G\pi\rho_B}{k_0 T}(r - R_B)\right) \qquad (43)$$

We remark that if the Newton's laws could be applied for $r>R_B$ then we would have obtained the differential equation for $r>R_B$, M_B being the baryonic mass of the galaxy :

$$\frac{1}{\rho(r)}\frac{d(\rho(r))}{dr} = -\frac{GM_B}{k_0 T_0}\frac{1}{r^2} \qquad (44)$$

And so we would have obtained for $r>R_B=R_S$, with the condition $\rho(R_S)=\rho_0$:

$$\rho(r) = \rho_0 \exp\left(\frac{GM_B}{k_0 T_0}\left(\frac{1}{r} - \frac{1}{R_B}\right)\right) \qquad (45)$$

We see in this example that if we apply the equations of Newtonian mechanics with $R_S=R_B$, then we obtain $\rho(r)\leq\rho_0$ for $r>R_S$, and it was also the case in the 1st model. This justifies the possibility to consider the case $R_S=R_B$, in this example and more generally in the 2nd model when we have a spherical symmetry for the distribution of baryonic matter.

In this 2nd model, assuming a spherical symmetry and $R_S=R_B$, let R_{MD} be a value in $[0,R_B]$ such that $\rho(R_{MD})$ be maximal. We will distinguish the 1st case "R_{MD} is superior to ρ_0." and the 2nd case "R_{MD} is inferior to ρ_0". We will also consider the 3rd case in which for r in $[0,R_B]$, we have always $\rho(r)\approx\rho_0$.

In the preceding example (constant baryonic density ρ_B), we have according to the expression of $\rho(r)$ R_{MD}=0, and :

$$\rho(R_{MD}) = \rho_0 \exp(\frac{4}{3}\frac{G\pi\rho_B R_B}{k_0 T}) \qquad (46)$$

We have $\rho(r) \approx \rho_0$ if :

$$\frac{4}{3}\frac{G\pi\rho_B R_B}{k_0 T} << 1 \qquad (47)$$

We will see that in the case $\rho(R_{MD}) \geq \rho_0$, it is interesting to consider the case $R_S = R_{MD}$. We justify the possibility to consider this case the same way we justified the possibility to consider the case $R_S = R_B$, because with $\rho(R_{MD}) = \rho_0$, the equations of Newtonian mechanics give in this case $\rho(r) \leq \rho_0$ for $r > R_{MD}$.

We consider now the case, inside the 2nd model, in which we have no more a spherical symmetry of the distribution of baryonic matter. We assume that we know the baryonic density for any unitary vector **u** $\rho_B(r,\mathbf{u})$. For any vector **u** and any radius r, we can obtain (using $\mathrm{div}(\mathbf{G}_B(r,\mathbf{u}) = \rho_B(r,\mathbf{u})/\varepsilon_0))$ the gravitational field due to baryonic matter $\mathbf{G}_B(r,\mathbf{u})$. Moreover we can define for any vector **u** the baryonic radius $R_B(\mathbf{u})$. We proceed then as in the case with a spherical symmetry defining for any vector **u** a dark radius $R_S(\mathbf{u})$. We first consider, in analogy with when we had a spherical symmetry, the case $R_S(\mathbf{u}) = R_B(\mathbf{u})$ for any vector **u**. Then using the equations of Newton mechanics we can obtain the density $\rho(r,\mathbf{u})$. But the calculation could be very difficult, and could need a computer, especially when we have not always $G_B(r,\mathbf{u})//\mathbf{u}$. We can then define for any vector **u** $R_{MD}(\mathbf{u})$ the same way as when we had a spherical symmetry.

A 3rd possible model of distribution of dark substance in a galaxy is the model in which we have R_S=0, meaning that we have for any radius r $\rho(r) = \rho_0$, and the temperature of the dark substance is always equal to T_0, ρ_0 and T_0 being the density and the temperature of the dark substance in which the galaxy is immerged. This model is due to an effect, called *Homogenization Effect* ,according to which the intergalactic dark substance tends to be homogeneous in temperature and density. In the 2 preceding models, this effect prevails for $r > R_S$, and the laws of Newtonian mechanics prevail for $r < R_S$. This effect of homogenization is also the origin of the homogeneity in density and temperature of the intergalactic dark substance. We can then easily obtain the gravitational field $\mathbf{G}_S(r)$ that is due to dark matter. This field, added to the gravitational field $\mathbf{G}_B(r)$ due to baryonic matter, permits to obtain the velocities of stars in the galaxy, using Newton's laws. We can expect that usually $\mathbf{G}_S(r)$ be small relative to $\mathbf{G}_B(r)$. It should be possible to obtain for some galaxies the experimental values of $\mathbf{G}_B(R_B)$ and $\mathbf{G}_S(R_B)$ and to compare $\mathbf{G}_S(R_B)$ with its theoretical prediction.

We obtain using Newton's law:

$$\mathbf{G}_S(R_B) = -G(4/3)\pi R_B \rho_0 \, \mathbf{u}_r \qquad (48)$$

It is possible that the 2nd model exposed previously do not exist, and that only galaxies belonging to the 1st and the 3rd model do exist. Indeed, we remind that in the 1st model, if we applied the Newton's law we found $\rho(r)<\rho_0$ for $r>R_S$. It is possible that we can generalize this property in the following property HF1, that would be an illustration of the Homogenization Effect defined previously:

Property HF1: If, in a model A of distribution of dark matter (around a galaxy of any star, planet..), we find for some r, **u** $\rho(r,\mathbf{u})<\rho_0$, then we must replace the model A by a model B in which we replace always $\rho(r,\mathbf{u})=\rho_0$ when we had in the model A $\rho(r,\mathbf{u})<\rho_0$.

So in the 2nd model of distribution of dark matter in galaxies we have, under the condition $R_S=R_B$ $\rho(R_{MD})<\rho_0$, then according to the preceding property, $R_S=0$. If under the condition $R_S=R_B$ $\rho(R_{MD}\geq\rho_0$, taking a new condition $R_S=R_{MD}$ and applying the preceding property, we obtain $R_S=0$.

It is also possible that we have a 2nd property HF2, that is also an illustration of the Homogenization Effect :

Property HF2: If, in a model A of distribution of dark matter (around a galaxy of any star, planet..), we find always $\rho(r)\approx\rho_0$, then $R_S=0$.

It appears that the 2nd model is by far the most complicated, but we at not sure that it really exists. Some astronomical observations should permit to know if it really exists. Nonetheless, the observation of galaxies shows that some of them could belong to the 2nd or to the 3rd model, for instance [(4)].

So further in this article we will assume that only the 1st and the 3rd models (around galaxies) exist because we saw that the 2nd model could not exist and moreover we will see that this assumption leads to theoretical predictions, in particular concerning the distribution of dark matter in clusters, that are confirmed by astronomical observations.

We remark that the distribution of dark matter around stars and planets should belong to the 2nd or to the 3rd model exposed previously. The 2nd model is nonetheless easier to be studied because stars and planets present a spherical symmetry. It is possible as for galaxies that the 2nd model does not exist around stars and planets.

3.5 Other observations of dark matter.

We are now going to interpret using the new theory of dark matter experimental data linked to the velocities of galaxies in clusters obtained by astronomical observations.

According to what precedes, the velocity of a galaxy in a cluster is determined by:

-The baryonic mass inside the cluster (stars, gas..)
-The mass of the dark halos of galaxies.
-The mass of the intergalactic dark substance.

We suppose that only the 1st model and the 3rd model of distribution of dark substance presented in the section 2.4 exist. Consequently all dark halos of galaxies belong to the 1st model.

We obtain a very interesting result concerning the mean density of galaxies corresponding to the 1st model (density of dark substance in $1/r^2$):

Indeed, according to the equation (18), for those galaxies the dark radius is:

$$R_S=(2k_0T/4\pi G\rho_0)^{1/2} \qquad (49)$$

According to the equation (8) :

$k_2=2k_0T/G$ (50)

Consequently :

$R_S=(k_2/4\pi\rho_0)^{1/2}$ (51)

So according to the equation (9) the total mass of the dark halo is:

$$M_S(R_S)=\frac{k_2^{3/2}}{(4\pi\rho_0)^{1/2}} \qquad (52)$$

Let us now calculate the mass of a sphere with the same radius R_S and a density equal to the density of the intergalactic dark substance ρ_0 :

$$M_I(R_S)=\rho_0\frac{4}{3}\pi(\frac{k_2}{4\pi\rho_0})^{3/2}=\frac{1}{3}\frac{k_2^{3/2}}{(4\pi\rho_0)^{1/2}} \qquad (53)$$

Consequently :

$M_I(R_S)=M_S(R_S)/3$ (54)

So the mean density of the halos of galaxies belonging to the 1st model is equal to $3\rho_0$, whatever be the radius and the temperature of the considered halo, and consequently whatever be the orbital velocity of stars in the considered galaxy.

According to the previous equation (54) we can expect that the dark mass of a cluster be much greater than the baryonic matter in the galaxies of this cluster. Indeed we have seen that according to the theory of dark matter exposed here, for a galaxy corresponding to the 1st model, R_B being the baryonic radius of the galaxy, then the mass $M_B(R_B)$ of baryonic matter contained in the sphere with a radius R_B (centre O, centre of the galaxy) was much smaller than the mass $M_S(R_B)$ of the dark substance contained in the same sphere. And consequently, because $R_B<R_S$, the total mass of the dark halo $M_S(R_S)$ is much greater than the total mass of baryonic matter contained by the galaxy . But according to the equation (54), the mean density of the halo is only 3 times of the minimum density of dark matter inside the cluster. (Because we supposed that only the 1st and the 3rd model did exist for galaxies). Consequently we can expect that the dark mass of clusters be much greater than the baryonic mass of the galaxies belonging to this cluster.

So for a cluster A with a mean density ρ_{mA}, we obtain if we neglect the baryonic density :

$\rho_0<\rho_{mA}<3\rho_0$ (55)

Consequently the mean densities of clusters permit to obtain an estimation of the density ρ_0 of the intergalactic dark substance. Moreover if A1 and A2 are 2 clusters with mean densities ρ_{mA1} and ρ_{mA2} with for instance $\rho_{mA1}<\rho_{mA2}$, then according to the previous relation :

$\rho_{mA2}<3\rho_{mA1}$ (56)

We will see that the preceding theoretical prediction is in agreement with astronomical observations.

It is interesting to introduce the mean volume of dark halo per galaxy Vol_{SG}. Then if clusters contain the same kind of galaxies in the same proportions (which is not always the case), we can express the mean density of dark substance ρ_{mA} as a function of N_A the number of galaxies inside the cluster A, and Vol_{SG}. Indeed we immediately obtain, Vol_A being the volume of the cluster and assuming that the cluster contains at least 400 galaxies (in order to be able to use the mean volume of dark halo per galaxy Vol_{SG}).

$$\rho_{mA} = \frac{1}{Vol_A}[3\rho_0 N_A Vol_{SG} + \rho_0 (Vol_A - N_A Vol_{SG})] \quad (57)$$

So we obtain, ρ_{mAG} being the mean density of the number of galaxies in the cluster, $\rho_{mAG}=N_A/Vol_A$:

$$\rho_{mA}=\rho_{mAG}(2\rho_0 Vol_{SG})+\rho_0 \quad (58)$$

So if we draw the curve $\rho_{mA}(\rho_{mAG})$ we obtain a straight line permitting to obtain precisely ρ_0 and Vol_{SG}. But a 1st particular case is the case in which we have for all clusters, ρ_{mAG} is approximately the same. Then the prediction is that ρ_{mA} is also approximately constant. A second particular case is the case in which we have always $\rho_{mAG}Vol_{SG}<<1$. Then we have always $\rho_{mA}\approx\rho_0$ whatever be ρ_{mAG}. The previous expression is valid for a given z. It is not true for 2 clusters that do not contain the same kind of galaxies. But, $Vol_A(H)$ being the volume of dark halo of galaxies belonging to the 1st model in the cluster A, we have always:

$$\rho_{mA} = \frac{1}{Vol_A}[3\rho_0 Vol_A(H) + \rho_0 (Vol_A - Vol_A(H))] \quad (59)$$

$$\rho_{mA} = 2\rho_0 \frac{Vol_A(H)}{Vol_A} + \rho_0 \quad (60)$$

We remind that we assumed that we could neglect the contribution of baryonic matter in order to obtain the mean density of the cluster ρ_{mA}. In what follows we will assume that we have generally for clusters $Vol_A(H)/Vol_A<<1$ and consequently $\rho_{mA}\approx\rho_0$. We remind that ρ_0 depends on t, age of the Universe. We will see further that the previous assumption is in agreement with experimental data given by astronomical observations.

Now we are going to propose different dynamical models of clusters permitting to obtain some new relations between the mass of clusters and the velocities of galaxies belonging to clusters, and also to obtain an estimation of the density of intergalactic dark substance $\rho_0(z)$.

According to a 1st dynamical model of clusters, galaxies turn around the centre of a cluster the same way planets turn around the sun or stars turn around the centre of the milky way. So we will call the *planetary dynamical model* of clusters this 1st model.

R_A being the radius of a cluster A, V_{MA} being the orbital velocity of a galaxy at a distance R_A from the centre O_A of A (We will obtain that V_{MA} is also the maximal orbital velocity of galaxies according to this 1st model), M_A being the mass of the cluster A, we obtain assuming a spherical symmetry of the distribution of the dark substance and neglecting the baryonic matter, using as in the previous sections the Newton's Universal law of attraction, the Gauss theorem and the classical Newton's dynamic law $\mathbf{F}_G=m\boldsymbol{\gamma}$:

$$\frac{GM_A}{R_A^2}=\frac{V_{MA}^2}{R_A} \qquad (61)$$

$$\frac{GM_A}{R_A}=V_{MA}^2 \qquad (62)$$

ρ_{mA} being the mean density of the cluster A, $M_A=(4/3)\pi R_A{}^3\rho_{mA}$ and therefore :

$$(4/3)\pi\rho_{mA}GR_A{}^2=V_{MA}{}^2 \qquad (63)$$

$$V_{MA}=R_A[(4/3)\pi\rho_{mA}G]^{1/2} \qquad (64)$$

If we assume that inside the cluster A the density is approximately constant and equal to ρ_{mA}, we obtain that V_{MA} is indeed the maximal orbital velocity of galaxies inside the cluster A. Consequently V_{MA} can be easily obtained experimentally measuring the maximal and the minimal recession velocity of galaxies belonging to the cluster A.

Nonetheless, some astronomical observations that are very important in order to study the validity of our different dynamical models of clusters have been realized concerning the Coma cluster that we will name A4 [(10)]. Using some astronomical observations of the Coma cluster, some astrophysicists realized a graph giving for some galaxies G belonging to the Coma cluster the recession velocity v(G) observed from a point O_T linked to the earth as a function of the angle a(G) between the lines (O_T,O_4) and (O_T,O_G), with O_4 the centre of the Coma cluster and O_G the centre of the galaxy G.

According to this graph, the gap between the maximal recession velocity and the minimal recession velocity is maximal for an angle a(G)=0 (5000 km/s). Then it decreases.

And this contradicts the 1st planetary dynamical model of clusters because according to this model for a galaxy with a(G)=0 the velocity of G (as a vector) is perpendicular to the line (O_T,O_G) and consequently the recession velocity v(G) should be close to 0 for a(G)=0. And also according to this model the gap between the maximal recession velocity and the minimal recession velocity should increase with a(G). So the previous astronomical observations concerning the Coma cluster contradict the 1st planetary dynamical model of clusters.

A 2nd possible dynamical model of clusters is the model generally used in the Standard Cosmological Model (SCM) based on the Virial's theorem. So we will name this model the *Virial's dynamical model* of clusters.

According to this model, if σ_A is the velocity dispersion inside a cluster A, M_A being the mass of the cluster and R_A its radius:

$$\frac{GM_A}{R_A}\approx\alpha_A\sigma_A^2 \qquad (65)$$

In the previous expression, α_A is of the order of the unity and depends on the cluster A. Some authors [(6)] replace in the previous expression R_A by a fixed radius called the Abel radius.

Nonetheless in order to establish the Virial's theorem, we consider N objects with masses $m_1,\ldots,m_N$, and we suppose that they are in equilibrium. But according to our model of dark matter, we are not in this hypothesis. Indeed, we can consider that we have N galaxies, but those galaxies interact not only between themselves, but also with the dark substance in which they are immerged.

Nonetheless, despite that the hypothesis of the Virial's theorem are not verified, the Virial's dynamical model permits theoretical predictions that are in good agreement with experimental data obtained using astronomical observations. We will justify further the origin of this good agreement. Let us for instance consider 3 clusters with $z<<1$ ($z<0,01$), A1 the Antlia cluster, A2 the Virgo cluster and A3 the Fornax cluster. M_i and σ_i being respectively the mass and the dispersion velocity of galaxies of the cluster Ai, we have the experimental data (obtained using Virial's theorem): $M_1 \approx 3{,}3\ 10^{14}$ s.m, $360\text{km/s}<\sigma_1<560\text{km/s}$, $M_2 \approx 1{,}2\ 10^{15}$ s.m, $\sigma_2 \approx 700$km/s, $M_3 \approx 2\ 10^{14}$ s.m, $\sigma_3 \approx 374 \pm 24$km/s We are going to see that according to the 2nd Virial's dynamical model of clusters, these experimental data are compatible with our model of distribution of dark matter in clusters, and in particular with the fact that according to this model, not only the mean densities of clusters must verify the equation (56), but also the ratio of 2 mean densities of 2 different clusters (with the same Cosmological redshift z) must be close to 1. (We assume that in the equation (60), $Vol_{Ai}(H)<<Vol_{Ai}$ for i=1,2,3).

Taking α_A constant and equal to α in the equation (65), we obtain the same way we obtained the equation (64):

$$\sigma_A=R_A[(4/3)\pi\rho_{mA}G\alpha]^{1/2} \qquad (66)$$

Applying the previous equation to the clusters A1 and A2 we obtain :

$$\frac{R_2}{R_1}=\frac{\sigma_2}{\sigma_1}\left(\frac{\rho_{m1}}{\rho_{m2}}\right)^{1/2} \qquad (67)$$

Moreover according to the definition of the mean density ρ_{mi} of the cluster Ai :

$$M_1=(4/3)\pi R_1^2\rho_{m1}$$

$$M_2=(4/3)\pi R_2^2\rho_{m2} \qquad (68)$$

Consequently according to the previous equalities:

$$\frac{R_2}{R_1}=\left(\frac{M_2}{M_1}\right)^{1/3}\left(\frac{\rho_{m1}}{\rho_{m2}}\right)^{1/3} \qquad (69)$$

Using the 2 previous equations (43) and (45) we obtain :

$$\frac{\rho_{m1}}{\rho_{m2}}=\left(\frac{M_2}{M_1}\right)^{2}\left(\frac{\sigma_1}{\sigma_2}\right)^{6} \qquad (70)$$

In the hypothesis $\rho_{m1}/\rho_{m2})^{1/6}\approx 1$, we obtain :

$$\frac{\sigma_1}{\sigma_2} \approx (\frac{M_1}{M_2})^{1/3} \qquad (71)$$

And we obtain the same relation replacing in (71) σ_1 and M_1 by σ_3 and M_3. We can verify that the previous theoretical prediction is in agreement with the previous experimental data of the σ_i and M_i for the clusters Ai. Indeed according to the relation (47) we should have $\sigma_1 \approx (M_1/M_2)^{1/3}\sigma_2 \approx 480$km/s in agreement with the experimental value of σ_1, and $\sigma_3 \approx (M_3/M_2)^{1/3}\sigma_3 \approx 360$km/s in agreement with the experimental value of σ_3.

Let us now consider the theoretical prediction of the Virial's dynamical model of clusters concerning the Coma cluster A4 ($z \approx 0{,}03$) and the Virgo cluster A2 ($z<0{,}01$). The experimental data are $\sigma_2 \approx 700$km/s, $\sigma_4 \approx 1000$ km/s and $R_4 \approx 2R_2 \approx 10$ millions l.y

Using the equation (67) with the hypothesis $(\rho_{m2}/\rho_{m4})^{1/2} \approx 1$, we obtain (We assume that in the equation (60), $Vol_{Ai}(H) << Vol_{Ai}$ for i=2,4):

$$\frac{R_2}{R_4} \approx \frac{\sigma_2}{\sigma_4} \qquad (72)$$

With the previous experimental data the left term of the previous relation is equal to 0,5 and the right term is equal to 0,7. We also obtain according to the equation (67):

$$\frac{\rho_{m4}}{\rho_{m2}} = (\frac{\sigma_4}{\sigma_2})^2 (\frac{R_2}{R_4})^2 \approx 0{,}5 \qquad (73)$$

Even if this result is close to the unity and in agreement with the equation (56) it does not give the result closer to 1 that we expected. We remind that if N_i is the number of galaxies of the cluster Ai, the experimental data obtained by astronomical observations are $N_2 \approx 1000$ et $N_4 \approx 10000$. ρ_{Gi} being the density of galaxies in the cluster Ai we therefore obtain using the previous experimental data of R_2 and R_4 $\rho_{G4} \approx 1{,}2\rho_{G2.}$

According to the equation (57) we obtain that if $\rho_0(z)$ is the density of the intergalactic dark substance for a Universe corresponding to a Cosmological redshift z, Vol_{SGi} being the mean volume of dark halo per galaxy in the cluster Ai :

$$\frac{\rho_{m4}}{\rho_{m2}} = \frac{\rho_0(0{,}03)}{\rho_0(0)} \frac{1+2Vol_{SG4}\rho_{G4}}{1+2Vol_{SG2}\rho_{G2}} \qquad (74)$$

With the approximations $\rho_0(0{,}03) \approx \rho_0(0)$ and $Vol_{SG4} \approx Vol_{SG2}$, we obtain:

$$\frac{\rho_{m4}}{\rho_{m2}} = \frac{1+2{,}4Vol_{SG2}\rho_{G2}}{1+2Vol_{SG2}\rho_{G2}} \qquad (75a)$$

So if the previous relation was true, we should obtain ρ_{m4}/ρ_{m2} very close to 1, which is not the case in the relation (73). This gap with the theoretical prediction of ρ_{m4}/ρ_{m2} could be due to the non validity of some of the numerous approximations that we made, for instance $\rho_0(0{,}03) \approx \rho_0(0)$, $Vol_{SG4} \approx Vol_{SG2}$, $\alpha_4 \approx \alpha_2$ (in the equation (65)). This gap could also be due to errors on the experimental values of σ_2, R_4 or R_2 .

But a more attractive possible origin of the gap between the experimental value and the theoretical prediction can be found analyzing the astronomical observations of the Coma cluster that we reminded previously and that invalidated the 1st planetary dynamical model of clusters [8]. Keeping our notations of v(G) and a(G), we note $\sigma_4(a)$ the velocity dispersion calculated for galaxies G such that a(G)=a. According to the astronomical observations, $\sigma_4(a)$ is maximal for a=0 and then decreases. But the experimental value of σ_4 that we used has been obtained considering galaxies corresponding to any angle. It is clear that then the obtained dispersion velocity σ_4 depends on the distance between O_4 centre of the cluster A_4 and O_T (observation point) despite that it should be independent of this distance. So it seems much more logical to calculate the dispersion velocity σ_4 considering the recession velocities observed from the centre of the cluster O_4. This is equivalent to identify σ_4 with $\sigma_4(0)$ (corresponding to the maximal dispersion velocity), because we assume a spherical symmetry for the dispersion velocity. The gap between the maximal recession velocity and the minimal recession velocity for a=0 being approximately 5000 km/s, it is very probable that $\sigma_4(0)$ be much greater than the value that we used for σ_4 (1000km/s). And with only $\sigma_4(0)\approx 1400$ km/s, we find according to the equation (73) $\rho_{m4}/\rho_{m2}\approx 1$.

So we see that despite that the hypothesis of the Virial's theorem are not verified, the theoretical predictions of the Virial's dynamical model of clusters are in a relative good agreement with the experimental data obtained by astronomical observations of clusters. We will further give an explanation of this relative good agreement.

We are now going to propose a 3rd dynamical model of clusters that seems to be the only one compatible with the experimental data and also with our model of distribution of dark matter in clusters. According to this model, G_A being a galaxy of a cluster A at a point P, we neglect the gravitational potential generated in P by the moving galaxies, and we consider only the gravitational potential generated in P by the dark substance. So we will name this model the *dynamical model of the dark potential* of clusters.

Then assuming a spherical symmetry in the distribution of dark substance in the cluster A, U(r) being the gravitational potential at a distance r from the centre O_A of the cluster, G_A being a galaxy situated at a distance r from O_A, $m(G_A)$ and $V(G_A)$ being the mass and the velocity of G_A the total energy $E_T(G_A)$ is therefore:

$$E_T(G_A)=(1/2)m(G_A)V(G_A)^2+m(G_A)U(r) \qquad (75b)$$

The total energy of all galaxies keeps itself. Consequently, if the cluster A contains N_A galaxies $G_{A1},..,G_{ANA}$:

$$\sum_{i=1}^{NA}\left(\frac{1}{2}m(G_{Ai})V(G_{Ai})^2+m(G_{Ai})U(r_i)\right)=E_T(A) \qquad (75c)$$

With $E_T(A)$ total energy of all galaxies of the cluster A being constant. So we see that the galaxies of the cluster A interact according to the previous equation, and that the energy of a given galaxy can evolve.

We are now going to give an estimation of the gravitational potential U(r). In order to get this estimation, we make the approximation that the density of dark substance in the cluster A is constant and approximately equal to ρ_{mA}, the mean density of the cluster A. Applying as in the previous sections the Newton's Universal law of gravitational attraction and the Gauss theorem, M(r) being the mass of the sphere with the centre O_A and the radius r, the gravitational field $\mathbf{G}$(r) is then:

$$\mathbf{G}(\mathrm{r}) = -\mathrm{G}\frac{\mathrm{M(r)}}{\mathrm{r}^2}\mathbf{u} \qquad (76)$$

And consequently :

$$\mathbf{G}(r) = -G\frac{4}{3}\pi r\rho_{mA}\mathbf{u} \qquad (77)$$

By definition **G**=-**Grad**(U), so we obtain, C_{AU} being a positive constant at a given age of the Universe:

$$U(r)=G(4/6)\pi r^2\rho_{mA}-C_{AU} \qquad (78)$$

This equation can also be written, in the approximation that the density of dark matter in the cluster is approximately constant an equal to ρ_{mA}, M(r) being the mass of the sphere with the centre O_A and a radius r :

$$U(r)=GM(r)/2r-C_{AU} \qquad (79a)$$

We cannot obtain C_{AU} using the Newtonian mechanics because of the expansion of the Universe. Nonetheless let us consider the very interesting following particular model in order to obtain C_{AU}:

In analogy with Newtonian mechanics, we admit in this particular model $U(R_A)=-GM_A/R_A$. Consequently we have, $M_A=M(R_A)$ being the mass of the cluster:

$$\frac{GM_A}{2R_A} - C_{AU} = -\frac{GM_A}{R_A} \qquad (79b)$$

So we finally obtain, with M_A and R_A depending a priori on t, age of the Universe:

$$C_{AU} = \frac{3}{2}\frac{GM_A(t)}{R_A(t)} \qquad (79c)$$

Moreover in the considered particular model, in order to obtain $R_A(t)$ we make the hypothesis that C_{AU} is constant, meaning that $M_A(t)/R_A(t)$ is constant, which involves great simplifications. Then we obtain that $R_A(t)$ evolutes in $1/(\rho_{mA}(t))^{1/2}$, and consequently in $1/(\rho_0(t))^{1/2}$, because of the equation (60) with $Vol_A(H)/Vol_A<<1$. We remark that this evolution in $1/(\rho_0(t))^{1/2}$ is the same as the evolution of dark radius of galaxies with a flat rotation curve (Equation (18)). Consequently in this considered particular model $Vol_A(H)/Vol_A$ is constant, and also $\rho_{mA}(t)/\rho_0(t)$.

We will consider further galaxies G_A with a radial velocity meaning that their velocity in vector is parallel to the line (O_A,O_{GA}) , O_A centre of the cluster and O_{GA} centre of G_A. We will see that such a galaxy G_{LA} situated at $r=R_A$ owns a nil velocity $V(G_{LA})$ and therefore its energy $E_T(G_{LA})$ is, according to the equation (74):

$$E_T(G_{LA}) = -m(G_{LA})\frac{GM_A}{R_A} \qquad (79d)$$

Therefore in the considered particular model, for a galaxy G_{LA} the ratio $E_T(G_{LA})/m(G_{LA})$ is constant because in this particular model we have $M_A(t)/R_A(t)$ is constant.

Therefore the considered particular model is very interesting because we have as in Newtonian mechanics $U(R_A)=-GM_A/R_A$, and moreover C_{AU} is constant and the ratio $E_T(G_{LA})/m(G_{LA})$ of a galaxy with a radial velocity in $r=R_A$ is constant.

We can modify this particular model replacing R_A by a radius $R_{MA}>R_A$, keeping the hypothesis that C_{AU} is constant in order to obtain the evolution of $R_{MA}(t)$. We then obtain a 2^{nd} model very close to the 1^{st} particular model. Both previous models are justified if we admit that because of the physical properties of the expanding Universe it exists a radius R_{MA} such that for $r>R_{MA}$ we must not take into account the mass of dark substance situated between R_{MA} and r in order to calculate the gravitational field **G**(r) and the gravitational potential U(r). R_{MA} must depend only on the distribution of dark substance and the simplest hypothesis in order to obtain R_{MA} is that R_{MA} is the minimal radius for which we have strictly for $r>R_{MA}(t)$ a density equal to $\rho_0(t)$ ($\rho_0(t)$ density of the intergalactic dark substance at the age of the Universe t).

The preceding hypothesis, due to the physical properties of the Universe in expansion, can be expressed formally in the following law:

If we have an isolated celestial body S presenting a spherical symmetry immerged in the intergalactic dark substance (S can be a galaxy with or without dark halo, a star, a cluster..), we will define the *Newtonian radius* of S as being the minimal radius $R_N(S)$ such that for $r>R_N(S)$ we have the intergalactic dark substance with a density strictly equal to $\rho_0(t)$.

Then according to this law and in order to obtain the gravitational field **G**(r) and the potential U(r), we can proceed as in Newton's mechanics but without taking into account the mass of dark substance situated in $r>R_N(S)$.

We obtain with the previous law that for a cluster A, $R_N(A)$ is equal to R_A, baryonic radius of the cluster and consequently R_{MA} is equal to R_A. So we obtain exactly the particular model exposed previously. The previous law permits to determine the gravitational field **G**(r) and gravitational potential U(r) for any galaxy immerged in the intergalactic dark substance and at any point P the Universe, which would be not possible or would have led to incoherent results if we had kept Newton's mechanics.

For instance we obtain using the previous law that in a point P far from any cluster, galaxy, star, then the gravitational field and the gravitational potential are nil despite that in P the density is equal to $\rho_0(t)$ and is very close to the mean density of clusters. In order to obtain this property we identify P with an isolated celestial body S with $R_N(S)=0$, and we apply the previous law.

In what follows we will assume that we are in the hypothesis of the particular model exposed previously, with C_{AU} constant. Nonetheless we can generalize what follows even in the case in which C_{AU} depends on the age of the Universe t.

Therefore, using the equation (74) :

$$\frac{1}{2}m(G_A)v(G_A)^2 + Gm(G_A)\frac{M(r)}{2r} = C_A(G_A) \qquad (80a)$$

$C_A(G_A)$ being equal to $E_T(G_A)+m(G_A)C_{AU}$.

u being a given unitary vector, we consider all the galaxies G_A whose the velocity is parallel to **u** (in vector) and with moreover the line (O_A,O_{GA}) is parallel to **u**, O_{GA} being the centre of the galaxy G_A. If we assume that at any age t of the Universe, the maximal ratio $E_T(G_A)/m(G_A)$ is approximately the same for any distance r between G_A and O_A, then a galaxy situated at the limit of the cluster meaning in $r=R_A$ owns this maximal ratio, its velocity $V(G_{LA})$ is nil and consequently because of the previous equation:

$$C_A(G_{AL}) = Gm(G_{AL})\frac{M(R_A)}{2R_A} \qquad (80b)$$

So if moreover we assume that the energy $E_T(G_A)$ of most galaxies G_A keeps itself, then we obtain reciprocally that C_{AU} is constant (independent of t, age of the Universe) because of the equation (79c).

Let us now suppose that we can neglect the interactions between any galaxy G_A and the other galaxies, and that consequently any galaxy G_A keeps its energy and also the ratio $E_T(G_A)/m(G_A)$. Then we have moreover for any galaxy G_A, in the case with C_{AU} constant, $C_A(G_A)$ constant. We still consider the galaxies G_A whose the velocity is parallel to **u** (in vector) and with moreover the line (O_A,O_{GA}) is parallel to **u**. Such a galaxy G_A remains on the same straight line $(O_A,\mathbf{u})$, because the gravitational field $\mathbf{G}(r)$ is parallel to **u** (Equation (76)) and we neglect the interactions between G_A and the other galaxies. We obtain according to the equation (80b) that the maximal radius $r_M(G_A)$ reached by G_A before coming back towards O_A is given by the equation:

$$Gm(G_A)\frac{M(r_M(G_A))}{r_M(G_A)} = C_A(G_A) \qquad (80c)$$

Nonetheless even in the case with $C_A(G_A)$ constant, $r_M(G_A)$ is not always the same for a given galaxy G_A because $M(r_M(G_A))$ depends on ρ_{mA} that depends on $\rho_0(t)$.

In the same way the maximal velocity of G_A $v_M(G_A)$ is obtained for r=0 and is given by the equation :

$$\frac{1}{2}m(G_A)v_M(G_A)^2 = C_A(G_A) \qquad (81a)$$

Therefore, the maximal velocity of the considered galaxies is reached for galaxies with a maximal ratio $E_T(G_A)/m(G_A)$. Moreover we have always the equality, according to the equations (80c) and (81a), in the case with C_{AU} constant:

$$v_M(G_A)^2 = G\frac{M(r_M(G_A))}{r_M(G_A)} \qquad (81b)$$

Moreover assuming that at any age of the Universe, for any radius r the maximal value of $C_A(G_A)/m(G_A)$ is the same, meaning equal to $C_A(G_{AL})/m(G_{AL})$ (Equation (80b)), we obtain that at any age of the Universe, V_{MA} being the maximal velocity of galaxies in O_A (r=0), according to equations (80b) and (81a):

$$V_{MA}^2 = \frac{GM_A}{R_A} \qquad (81c)$$

In order to obtain the previous relation, we did not use that C_{AU} was constant. Consequently it remains valid even in the case in which C_{AU} depends on t, age of the Universe.

So it is remarkable that the previous equation be exactly the same form as the equations (61) and (65) despite that we obtained it using a completely different way. Nonetheless it owns only an approximate validity because in order to obtain this equation we neglected all the interactions between galaxies and moreover we assumed that for any radius r the maximal value of $C_A(G_A)/m(G_A)$ was the same. And in reality such interactions exist expressed by the equation (75). But the approximate validity of this equation explains why the 2 dynamical models of clusters (planetary model and Virial's model) exposed previously give some theoretical predictions that are in relative good agreement with experimental data despite that those 2 dynamical models are wrong. In order to take into account those interactions between galaxies and also of the fact that for any radius r the maximal value of $C_A(G_A)/m(G_A)$ is not compulsory exactly the same, we introduce a constant β_A depending on the cluster, of the order of the unity, such that, V_{MA} being the maximal recession velocity observed from O_A.

$$V_{MA}^2 = \beta_A \frac{GM_A}{R_A} \qquad (81d)$$

We have seen that the experimental data for the Virgo cluster A2 and the Coma cluster A4 were $R_4 \approx R_2 \approx 10$ millions l.y and we have the maximal recession velocity V_{M2} and V_{M4} observed from O_2 and O_4 (obtained taking the half of the gap between the maximal and the minimal recession velocities observed from O_T) are $V_{M2} \approx 1000$ km/s et $V_{M4} \approx 2500$ km/s.

If the equation (81c) was true, we would obtain, the same way we obtained the equation (67) :

$$\frac{\rho_{m4}}{\rho_{m2}} = (\frac{V_{M4}}{V_{M2}})^2 (\frac{R_2}{R_4})^2 \qquad (81e)$$

We find with this equation (81e) and the experimental data given previously the experimental value $\rho_{m4}/\rho_{m2} \approx 1,5$. And this experimental value, despite that it is acceptable, (of the order of the unity) is not satisfying. This means, in the hypothesis of the validity of the 3rd dynamical model of the dark potential, that we cannot neglect the interactions between galaxies and that we must use the equation (81d) with $\beta_2 \neq \beta_4$.

We did not take into account the fact that $\rho_0(0,03) \neq \rho_0(0)$ (We remind that for the Virgo cluster $z_2 < 0,01$ and for the Coma cluster, $z_4 \approx 0,03$). And we will see further, that according to our model of dark matter, $\rho_0(0,03) \approx 1,1\rho_0(0)$. So taking into account this correction, we should obtain $\rho_{m4}/\rho_{m2} \approx 1,1$. Moreover some astronomical observations give $R_4 \approx 12,5$ millions l.y. With this experimental data and keeping the other experimental data ($R_2 \approx 5$ millions l.y), we obtain $\rho_{m4}/\rho_{m2} \approx 1$.

It is remarkable that we always find, in agreement with the theoretical predictions of this 3rd dynamical model of clusters and with our model of distribution of dark matter in clusters, that ρ_{mi}/ρ_{mj} is always of the order of the unity for all the clusters Ai and Aj despite that we consider clusters with very different sizes.

The density of the intergalactic dark substance depends on the age of the Universe. We will use the symbol $\rho_0(0)$ in order to represent the density of dark matter at the present age of the Universe (z=0) and $\rho_0(z)$ in order to represent the density of the intergalactic dark substance at the age of the Universe corresponding to a cosmological redshift z. The estimation of the intergalactic density $\rho_0(0)$ obtained using the previous dynamical models of clusters permit other theoretical predictions.

Indeed, according to the equation (18), for a galaxy corresponding to the 1st model immerged in the intergalactic dark substance, the radius R_S of this galaxy is given by, at the present age of the Universe:

$$R_S = \left(\frac{2k_0 T}{4\pi G\rho_0(0)}\right)^{1/2} \qquad (82a)$$

Therefore, v being the orbital velocity of stars in this galaxy, according to the equation (10):

$$R_S = \frac{v}{(4\pi G\rho_0(0))^{1/2}} \qquad (82b)$$

But it is possible to determine a minimal experimental value of R_S: R_S is superior to the baryonic radius of the galaxy, but also to the distance between the centre of the galaxy and the galaxies satellites driven with the same orbital velocity of the stars belonging to this galaxy. For instance in the Milky Way it is the case of the Small and of the Large Magellanic Clouds. Let R_{mS} be the minimal experimental value of the dark radius of the galaxy. Then if the equation (82b) is true, we have, using the obtained estimation of $\rho_0(0)$:

$$\frac{v}{(4\pi G\rho_0(0))^{1/2}} \geq R_{mS} \qquad (82c)$$

And it is easy to compare the preceding relation with astronomical observations. Let us for instance consider the case of the Milky Way. In order to get $\rho_0(0)$, we apply the 3rd dynamical model of the dark potential to the Virgo cluster (z<0,01). According to the equation (62) we obtain, ρ_{mA} being the mean density of the cluster A:

$$\rho_{mA} = \frac{1}{(4/3)\pi G}\frac{V_{MA}^2}{R_A^2} \qquad (83a)$$

Identifying ρ_{mA} (mean density of the Virgo cluster or of a cluster with z<<1) with $\rho_0(0)$ (We assume that in the equation (60), $Vol_{A2}(H) << Vol_{A2}$ and consequently $\rho_{mA2} \approx \rho_0(0)$) we obtain:

$$R_S = \frac{v}{\sqrt{3}}\frac{R_A}{V_{MA}} \qquad (83b)$$

Taking as cluster A the Virgo cluster A2, with the experimental data R_2=5 millions l.y, $V_{M2} \approx 1000$ km/s and $v \approx 210$ km/s, we find the dark radius of the Milky Way $R_{SM.W} \approx 600000$ l.y. This result is not only coherent, but it gives also a dark radius of the Milky Way superior to the distance between the centre of the Milky Way and the Magellanic clouds (approximately 250000 l.y). It is also in agreement with the value of $R_{SM.W}$ used in Section 3.2 (500000 l.y)).

We know that we observe an effect called gravitational lensing, predicted by General Relativity, that consists in a deviation of luminous rays due to the mass of clusters. If we analyze this effect, we obtain that the mass of a cluster is mainly constituted of dark mass. Moreover, we obtain that the mass of a cluster calculated using the gravitational lensing is precisely equal to the mass of the cluster obtained using the previously exposed dynamical models of clusters. Therefore, the previous dynamical models of clusters and model of distribution of dark matter in clusters permit to justify theoretically the gravitational lensing observed for clusters.

Moreover we know that the study of the CMB shows the existence of anisotropies due to the density of dark substance in the Universe.

If $\rho_{mU}(z)$ is the mean density of dark substance in an Universe corresponding to a Cosmological redshift z, we obtain as in the case of clusters:

$$\rho_0(0)<\rho_{mU}(0)<3\rho_0(0) \qquad (84a)$$

As for clusters, it is interesting to introduce a density of dark halos in the Universe ρ_{UH} such that if $Vol_U(z)$ is the volume of the Universe corresponding to a cosmological redshift z and $Vol_U(H)(z)$ the volume of dark halos in the Universe, then $Vol_U(H)(z)=\rho_{UH}\, Vol_U(z)$. Then we obtain as for clusters the equality:

$$\rho_{mU}(0)=\rho_0(0)(1+2\rho_{UH}(0)) \qquad (84b)$$

Using the dynamical models of clusters exposed previously we obtain an estimation of $\rho_0(0)$. We also remark that if we assume that the dark mass of the Universe keeps itself, 1+z being the factor of expansion of the Universe between the age of the Universe corresponding to the redshift z and the present age of the Universe:

$$\rho_{mU}(z)=\rho_{mU}(0)(1+z)^3 \qquad (85)$$

We can expect $\rho_{mU}(0)<<1$. Then according to equation (84b) $\rho_{mU}(0)\approx\rho_0(0)$ and using the previous equation we obtain $\rho_{mU}(z)\approx\rho_0(z)\approx\rho_0(0)(1+z)^3$. Using the dynamical models of clusters exposed previously we obtain an estimation of $\rho_0(0)$ and it should be possible to verify the previous approximation of $\rho_{mU}(z)$ and $\rho_0(z)$, observing some galaxies or clusters situated far from us (z>3).

Moreover using the previous dynamical models of clusters in order to obtain an estimation of $\rho_0(0)$, we could compare this value with the value obtained from the anisotropies of the CMB.

3.6 Link between the CMB and the temperature of the intergalactic dark substance.

In the Sections 2.5 and 2.6 , we have seen that according to our Physical Interpretation of the CRF, the Universe was a sphere filled of dark substance, surrounded by a medium called “nothingness”. We saw in the Section 2.5 that we could model a convective thermal transfer between this spherical Universe and the nothingness. The convective flow F was then in agreement with the expression $F=h_nT_0(t)$, $T_0(t)$ being the temperature of the intergalactic dark substance at a Cosmological time t. It is easy to verify that it is impossible that we have a constant C_2 such than $h_n=C_2\rho_0(t)$ contrary to the case in which we had also a convective transfer but between 2 mediums constituted of dark substance in section 2.3. (Indeed in this case we would obtain that $T_0(t)$ increases). We saw in Section 2.5 that it is nonetheless possible that h_n be constant, independent of the density of the intergalactic dark substance. Indeed in this case, because of the Postulate 2a) we have the equation of thermal equilibrium $K_3M=4\pi R_E(t)^2(h_nT_0(t))$, with K_3 constant (Equation (14)) , M baryonic mass of the Universe,

$R_E(t)$ radius of the Universe at a Cosmological time t. We obtain that $T_0(t)$ evolutes in $1/(1+z)^2$, (1+z) factor of expansion of the Universe. We admit as in the SCM that the apparition of the CMB in the Universe corresponds to a redshift z approximately equal to 1500. If we admit that for this value of z, the temperature of the intergalactic dark substance was equal to the temperature of the CMB, we obtain that presently (with an age of the Universe of 15 billion years), the temperature of the intergalactic dark substance is 1500 times lower than the temperature of the CMB, which is an acceptable value, justifying our approximation in Section 2.3 expressing that the temperature of the intergalactic dark substance can be neglected in comparison with the temperature of spherical concentrations of dark substance (corresponding to galaxies with flat rotation curve, see Section 2.).

Moreover the hypothesis of the initial temperature of the CMB and the temperature of the intergalactic dark substance implies, because we assumed that the latter was homogeneous in all the Universe (see the homogenization effect in the previous section) , that the initial temperature of the CMB was also homogeneous in all the Universe. And so this hypothesis justifies the isotropy of the CMB observed from the CRF, without needing to introduce the phenomenon of inflation, as it was the case in the SCM.

3.7 Evolution of the temperature of the dark substance.

We saw in section 3.6 that the hypothesis of an initial equality of the temperature of the CMB and the temperature of the dark substance (For $z \approx 1500$) and a thermal model similar to the thermal model used in order to get the baryonic Tully-Fisher's law, led to obtain that at the present age of the Universe the temperature of the intergalactic dark substance (evolving in $1/(1+z)^2$) was approximately 1500 times less than the temperature of the CMB (evolving in $1/(1+z)$). This is in agreement with our hypothesis used in order to obtain the baryonic Tully-Fisher's law according to which we could neglect the temperature of the intergalactic dark substance relative to the temperatures of dark halos of galaxies with a flat rotation curve.

Nonetheless in order to obtain the evolution in $1/(1+z)^2$ of the temperature of the intergalactic dark substance, we used in the section 3.6 the equation, M_B baryonic mass of the Universe and $R_U(t)$ radius of the Universe for the age of the Universe t:

$$K_3M_B=4\pi R_U(t)^2h_nT_0(t) \qquad (86)$$

With K_3 constant defined in the equation (14), and we did not take into account the evolution of the internal energy of the dark substance nor the energy lost because of the dilatation of the volume of the intergalactic dark substance. We will call 1st *model of the evolution of the temperature* of the intergalactic dark substance the preceding model. We remark that we assumed its validity only for z<1500.

Let us consider a 2nd model in which on the contrary we neglect (i) the thermal energy transferred from the baryons towards the dark substance (energy that is obviously nil before the apparition of baryons) and also (ii) the energy lost by the intergalactic dark substance through the convective transfer between intergalactic dark substance and the medium that we called nothingness and we consider only (iii) the variation of the internal energy of the intergalactic dark substance and also relative to (iv) the energy lost because of the variation of the volume of the intergalactic dark substance. We suppose that in this model, the dark substance is homogeneous in all the Universe, because we consider its validity only for z>1500, and for this cosmological redshift z the galaxies did not exist. Consequently the dark substance obeys to the Boyle-Charles law (Postulate 1) and moreover we assume that it also obeys to Joule's law for ideal gas: It exists a constant K_{ES} such that T(t) being the temperature of the dark substance, M_S being the total mass of the dark substance and U(T(t)) being the total internal energy of the dark substance for an age of the Universe t:

$U(T(t))=K_{ES}M_ST(t)$ (87).

Moreover the energy lost that is the work corresponding to a variation of the volume of the dark substance dV under the pressure P is equal to:

$W=-PdV$ (88)

We assume in this 2nd model of the evolution of the temperature of the dark substance that the transformation is adiabatic reversible. Consequently we can apply the Laplace's law: It exists a constant γ such that, V being the volume of the Universe for a temperature T at an age of the Universe t, and V_1 its volume for a temperature T_1 at an age t_1:

$TV^{\gamma-1}=T_1V_1^{\gamma-1}$ (89)

Consequently if 1+z is the factor of expansion of the Universe between t_1 and t, $V(t)=V(t_1)(1+z)^3$ and:

$T(t)=T(t_1)/(1+z)^{3(\gamma-1)}$ (90)

In a 3rd model of evolution of the temperature of the (intergalactic) dark substance we take into account every kind of energy received or lost by the dark substance. Nonetheless, we consider in this model that the dark substance is homogeneous in density and temperature in all the Universe, without taking into account the dark halos of galaxies with a flat rotation curve, and we have seen that this was justified because the total volume of those dark halos was very small relative to the total volume of the Universe. We will take the following notations:
dW(t,t+dt) is the energy received by the dark substance as a work (negative) due to the variation of volume of the dark substance between the ages of the Universe t and t+dt.

$dE_{TF}(t,t+dt)$ is the energy received by the dark substance (negative) due to the thermal transfer between the dark substance and the medium that we called "nothingness" between t and t+dt. $R_U(t)$ being the radius of the Universe at the age of the Universe t, we have seen (equation (86)):

$dE_{TF}(t,t+dt)=(-h_nT(t))(4\pi R_U(t)^2)dt$ (91)

$dE_{TB}(t,t+dt)$ is the energy transferred by the baryons to the dark substance (positive), (Equation (14)) between t and t+dt, $M_B(t)$ being the mass of the baryons at the age t of the Universe we have:

$dE_{TB}(t,t+dt)=K_3M_B(t)dt$ (92)

Then the equation of equilibrium of the energy received and lost by the dark substance is:

$dU(t,t+dt)=dW(t,t+dt) + dE_{TF}(t,t+dt) + dE_{TB}(t,t+dt)$ (93)

We remind that according to the Boyle-Charles law, M_S being the total mass of the dark substance (assumed to be constant):

$$P(t)V(t)=k_0M_ST(t) \qquad (94)$$

And, $R_U(t)$ being the radius of the Universe, $V(t)=(4/3)\pi R_U(t)^3$ and $d(R_U(t))=dzR_U(t)$ (1+dz being the factor of expansion of the Universe between t and t+dt), $dV(t)=4\pi R_U(t)^2dR_U(t)=4\pi R_U(t)^3dz$ and consequently $dV(t)/V(t)=3dz$. So we have:

$$dW(t,t+dt)=-PdV(t)=-k_0M_ST(t)(dV(t)/V(t))$$

$$dW(t,t+dt)=-3k_0M_ST(t)dz \qquad (95)$$

So we obtain the following differential equation in T(t), because dz and $R_U(t)$ can be expressed as a function of t:

$$d(K_{ES}M_ST(t))=-3k_0T(t)dz-h_nT(t)(4\pi R_U(t)^2)dt+K_3M_B(t)dt$$

$$K_{ES}M_S(dT(t)/dt)=-3k_0M_ST(t)(dz/dt)-h_n(4\pi R_U(t)^2)T(t)+K_3M_B(t) \qquad (96)$$

We remark that with the previous notations, the parameter γ used in Laplace's equation (89) can be expressed by:

$$\gamma=1+k_0/K_{ES}$$

Consequently k_0 should be of the order of K_{ES}. Using the previous equation (96) we can express the conditions of validity of the 1st model, in which we neglected the variation of internal energy and the work received by the dark matter due to the variation of its volume. Those conditions are:

$$-K_{ES}M_S(dT(t)/dt)<< K_3M_B(t)$$

$$-K_{ES}M_S(dT(t)/dt)<< h_n(4\pi R_U(t)^2)T(t)$$

$$3k_0M_ST(t)(dz/dt)<< K_3M_B(t)$$

$$3k_0M_ST(t)(dz/dt)<< h_n(4\pi R_U(t)^2)T(t) \qquad (97)$$

The conditions for which the 2nd model of the evolution of the temperature of dark substance be valid are the inverse conditions (replacing "<<" by ">>")

3.8 Evolution of the temperature of dark substance- 2nd model of expansion.

We are going to consider the application of the preceding section 3.7 in the case of the 2nd model of expansion of the Universe, meaning with $R_U(t)=Ct$, (C constant), and consequently between t and t+dt, $1+dz=(t+dt)/t$, so $dz=dt/t$.

We remark that in the 1st model of evolution of the temperature T(t) evolves in $1/(1+z)^2$, consequently for this 2nd model of expansion in $1/t^2$. In the 2nd model of the evolution of the temperature, T(t) evolves in $1/(1+z)^{3(\gamma-1)}$ with $\gamma>1$, consequently in this 2nd model of expansion in $1/t^{3(\gamma-1)}$. So in both cases T(t) evolves in $1/t^p$, with $p>0$. For such a

function T(t), we obtain that for t tending towards the infinite both functions T(t) and (dT(t)/dt)/T(t) tend towards 0. So for t sufficiently great the equations (97) are valid and the 1st model of evolution of the temperature of dark substance is also valid.
On the contrary for t tending towards 0, the functions (dT(t)/dt)/T(t) and T(t) tend towards the infinite and consequently for t sufficiently small (for instance just after the Big-Bang), the inverse of the relations (97) are valid and consequently the 2nd model of evolution of the temperature of dark substance is also valid.

3.9 Dark energy in the Universe.

We defined in the Postulate 1 the Boyle-Charles'law for an element of dark substance with a pressure P, a volume V, a temperature T and a mass m, k_0 being a constant:

$$PV=k_0mT \qquad (98)$$

Using the previous law and the Newton's Universal law of gravitation, we obtained the equation (10), valid for all galaxies with a flat rotation curve. For instance for the Milky Way, T_{MW} being the temperature of the dark halo of the Milky Way and v_{MW} being the orbital velocity of stars in Milky Way, we have the equation:

$$v_{MW}^2 \approx 2k_0T_{MW} \qquad (99)$$

Consequently taking $v_{MW} \approx 2.\ 10^5$m/s we obtain $k_0T_{MW} \approx 2.\ 10^{10}$ U.S.I .
Let us compare the equation (98) with the analogous equation valid for hydrogen modeled as an ideal gas. We know that it exists a constant k_H such that for a hydrogen element with a mass m_H, a volume V, at a temperature T and a pressure P:

$$PV=k_Hm_HT \qquad (100)$$

We know that for a mole of hydrogen, for $T=T_K=273°K$, $V=20.\ 10^{-3}$, $P=10^5$ Pa, $m_H=10^{-3}$ kg, we have:

$$k_HT_K \approx PV/m_H=10^5 \times 20.\ 10^{-3} \times 10^3= 2.\ 10^6 \text{ U.S.I} \qquad (101)$$

If we assume that dark substance and hydrogen obeys to Joule's law, we therefore obtain that the internal energy of a kg of hydrogen at the temperature T_K is of the order of k_HT_K meaning 2. 10^6 Joules despite that the internal energy of a kg of dark substance belonging to the halo of the Milky Way is of the order of k_0T_{MW} meaning 2. 10^{10} Joules, and therefore the latter energy is by far superior to the former. Considering this important difference of energy, we must consider a 2nd possible model of energetic transfer from baryons towards the dark substance, permitting a transmitted power much greater than a power corresponding to a diminution quasi imperceptible of the temperature of the baryonic matter. In this 2nd model, the transferred energy is dark energy. In this 2nd model, baryonic particles contain a very important quantity of dark energy, but this dark energy must not be taken into account in the mass appearing in the classical equations $E=mc^2$ or $E_p=mU$. Consequently we cannot detect this dark energy using classical experiments. The power of dark energy transmitted from baryons towards dark substance has the same expression as in the 1st model of power (calorific power):

$P_r = K_{3S} M$ (102)

With M the mass of the considered baryonic particles and K_{3S} constant. p_{0S} being the power of dark energy lost by nucleus and m_0 being the mass of a nucleus we obtain $K_{3S}=p_{0S}/m_0$. Moreover if we consider that baryonic particles have been created just after the Big-Bang with a total dark energy per nucleus equal to E_0, we get with the previous 2nd model that the dark energy of a mass M at an age of the Universe t is:

$E_S(M,t)=(M/m_0)\,(E_0-p_{0S}t)$ (103)

It is very possible that E_0 be of the order of m_0c^2. We ignore what happens in this 2nd model when baryonic particles have lost all their dark energy.

4.CONCLUSION

So in this article we proposed the existence of a dark substance whose physical properties are in agreement with observations connected to dark matter. In particular those physical properties, despite of their simplicity, permitted to us to justify theoretically the flat rotation curve observed for many galaxies and the baryonic Tully-Fisher's law. In order to obtain those laws, we interpreted galaxies with a flat rotation curve as spherical concentrations of dark substance in thermal equilibrium.

We have also exposed a Physical Interpretation of the CMB Rest Frame (CRF) that we also called the local Cosmological frame. This Interpretation permitted to us to define in a simple and new way the Cosmological time, in agreement with all astronomical observations and with the definition of Cosmological time in the SCM. This Interpretation has also permitted to us to introduce a new kind of frame, called (Universal) Cosmological frame, that is fundamental for the description of the Universe. Then using these new concepts, we proposed a new model of Universe, flat and finite, not proposed by the SCM. Despite of this difference we have seen that according to a 1st mathematical model of expansion of the Universe ,based as the SCM on General Relativity, the observable Universe was identical to the one predicted by the SCM (in particular it is isotropic), provided that it be observed from a point sufficiently far from the borders of the Universe. We also have proposed a 2nd mathematical model of expansion, much simpler than the mathematical model of the SCM, and we have seen that the theoretical predictions of this 2nd were nonetheless in agreement with astrophysical observations. Moreover this 2nd mathematical model did not need a dark energy, contrary to the SCM, and consequently brings a solution to the enigma of dark energy.

In section 3 we studied the effects of the motion of a spherical concentration of dark substance on its velocity and its mass. We also studied the 2 kinds of radius for a galaxy, the dark radius and the baryonic radius. We also studied the different possible models of distribution of dark matter in galaxies. Then we exposed the theoretical predictions concerning the velocities of galaxies in clusters and we saw that those predictions were in agreement with experimental data of some clusters having a cosmological redshift inferior to 0,03. We saw that the new theory permitted to predict the value of the dark radius of all galaxies (In particular galaxies with a flat rotation curve), and to obtain the mean density of the Universe, and also the value of the density of the intergalactic dark substance. Finally we studied the evolution of the temperature of the dark substance, just after the Big-Bang up to the present age of the Universe. And we have then seen the existence of a dark energy that is identified with the internal energy of the dark substance.

Concerning the Physical Interpretation of the CRF, finding some observations permitting to compare our 1st model and the SCM will be a greater challenge because we have seen that they both predicted the same observable Universe. It should be nonetheless possible to find astronomical observations permitting to compare the phenomenon used in our Physical Interpretation of the RRC to justify the isotropy of the CMB in our 1st model (equality of the initial (Cosmological time t_{ICMB}) temperature of the CMB and the temperature of intergalactic dark substance (also Cosmological time t_{ICMB}), Section 3.5) and the corresponding phenomenon in the SCM (inflation) permitting to justify the observed isotropy of the CMB.

It should be easier to find astronomical observations permitting to compare the predictions of our 2nd model with the predictions of the SCM because they are mathematically different. For instance we have seen that in our 2nd model, the Hubble's constant is precisely equal to $1/t_0$, t_0 age of the Universe. In the same way distances used in Cosmology have not the same mathematical expression in our 2nd model as in the SCM (See Section 2.7).

But a very attractive element in favor of the model of the Universe proposed by our Physical Interpretation of the CRF is that this geometric model of Universe can be conceived by the human mind, which was not the case for models of Universe proposed by the SCM that were either infinite or finite but without borders. It is our model of dark substance that permitted to us to define easily such a Universe, flat and finite.

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