The Algorithm Simulation of the Orbits of Objects Differential Equation Second-Order Curves

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Abstract. In this paper we consider the algorithm for calculation of motion of bodies, using the coherent graph. The graph is built through solving a differential equation of second order curves, using theorem of the center of mass and transition from a consecutive number of relative coordinates to absolute system of coordinates.

keywords: material point, ellipse, angular acceleration, eccentricity, differential equation, connected graph.

1. Introduction.

Since ancient times, people have been interested in the motion of planets. For example the new seaworthy and overland routes were the motion and relative positions of the moon, sun, constellation affect the earth. With development of astronomy there were been physical study about difficult interaction of the massive objects which are located at huge distances from each other and moving on orbits - planets of solar system. Solutions of these studies led to fast development of mathematical apparatus in this area thanks to works of such most famous scientists as Newton, Kepler, etc. In the presents time a task of the relative movement of the 3rd and more bodies have no analytical decision. However the well developed differential calculus [1] allows us to present the numerical solution of this task including the computer modeling. “To solve the mass point motion problem we need differential equations for the motion. The way we derive these equations doesn’t matter”: [2, §11]. One way to describe the motion of bodies is to present it through the coherent graph [3]. In this work we present the graph building through solving a differential equation of second order curves (14), using theorem of the center of mass and transition from a consecutive number of relative coordinates to absolute system of coordinates [2, 4].

2. Differential equation for 2nd-order curves.

In this article we consider a differential equation for the 2nd-order curve of the motion of a parametric pendulum in zero gravity [5, 6].

We will derive a differential equation for the mass point motion in the case of zero gravity and movement on the ellipse under external force. Let us place the point into zero gravity space. Some external force makes this point to move along a 2nd-order curve. Let this curve be an ellipse and the point moves around a left center.
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Figure 1. The mass point movement around the left point.

N - pendulum.
Q – the force acting on the pendulum.
F1 - left center.
F2 - right center.
a(t) - angle between X axis and the line connecting left center and the point.

Let us place the left center into the origin of coordinates.

\[ r = \frac{p}{1 - e \cdot \cos \alpha} \]  

\[ p = \frac{B^2}{A} \]  

\((A, B – semi-major and semi-minor axis)\)
r - radius.
e – eccentricity

\[ m \dot{x} = -Q \cos |a|t| \]  
\[ m \dot{y} = -Q \sin |a|t| \]  

From equation (3) we can get

\[ Q = \frac{-m \dot{x}}{\cos |a|t|} \]  

Let us substitute equation (5) into equation (4)
Let us calculate the first and second time derivative.

\[
\dot{x} = \frac{d}{dt} \left( \frac{p}{1 - e \cos |a(t)|} \right) \cos |a(t)| = \dot{\alpha}
\]

\[
- p \cdot \cos |a(t)| \cdot e \cdot \sin \left( \frac{|a(t)|}{d} \right) \cdot \frac{d}{dt} |a(t)| - p \cdot \sin \frac{|a(t)|}{d} \cdot |a(t)| \left( \frac{1 - e \cos |a(t)|}{|a(t)|^2} \right)
\]

\[
\dot{y} = \frac{d}{dt} \left( \frac{p}{1 - e \cos |a(t)|} \right) \sin |a(t)| = \dot{\beta}
\]

\[
- p \cdot \dot{\beta} \cdot e \cdot \sin |a(t)| \cdot \frac{2 \cdot |a(t)|}{d} \cdot \frac{d}{dt} |a(t)| + p \cdot \cos |a(t)| \cdot \frac{d}{dt} |a(t)| \left( \frac{1 - e \cos |a(t)|}{|a(t)|^2} \right)
\]

\[
\dot{x} = \frac{2 \cdot p \cdot e^2 \cdot \cos |a(t)| \cdot \sin |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2}{\left( 1 - e \cos |a(t)| \right)^3} + \frac{2 \cdot p \cdot e \cdot \sin |a(t)| \cdot \cos |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2}{\left( 1 - e \cos |a(t)| \right)^2} - \frac{p \cdot e \cdot \cos |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2}{\left( 1 - e \cos |a(t)| \right)} - \frac{p \cdot \sin |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2}{\left( 1 - e \cos |a(t)| \right)}
\]

\[
\dot{y} = \frac{2 \cdot p \cdot e^2 \cdot \sin |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2}{\left( 1 - e \cos |a(t)| \right)^3} - \frac{3 \cdot p \cdot e \cdot \sin |a(t)| \cdot \cos |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2}{\left( 1 - e \cos |a(t)| \right)^2} - \frac{p \cdot e \cdot \sin |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2 \cdot |a(t)|^2}{\left( 1 - e \cos |a(t)| \right)^2} - \frac{p \cdot \sin |a(t)| \cdot \left( \frac{d}{dt} |a(t)| \right)^2 \cdot |a(t)|}{\left( 1 - e \cos |a(t)| \right)^2}
\]

Let us substitute equation (11) and equation (12) into equation (6)
3. Motion of material points.

We consider the motion of material points. By the definition of the material point, it follows that such object has mass and does not have dimensions.

We consider three cases:

a. The motion of one material point around another. The second point is in the fixed position.

The mass of the first point is much less than the mass of the second one. We know the max and min distance between the points. We know that the second point is in one of the focuses of the ellipse. In this case we can calculate the motion of the first point using the equation (14).

b. The movement of two points around a common center of mass.

The masses of points $p_1$, $p_2$ are comparable. Let $m_1$ denote the mass of the first point and $m_2$ denote the mass of the second one. Let $a_{12}$ denote the max distance and $b_{12}$ denote the min distance between two points. Put the center of mass in the coordinate origin.
Now we can calculate semi-axes of the ellipse \( a_1, b_1, a_2, b_2 \):

\[
\begin{align*}
a_{12} &= a_1 + a_2 \\
b_{12} &= b_1 + b_2 \\
0 &= a_1 \cdot m_1 + a_2 \cdot m_2 \\
0 &= b_1 \cdot m_1 + b_2 \cdot m_2
\end{align*}
\]

Using equations (15) – (18) we get

\[
\begin{align*}
a_1 &= \frac{a_{12} \cdot m_2}{m_2 - m_1} \\
b_1 &= \frac{b_{12} \cdot m_2}{m_2 - m_1} \\
a_2 &= \frac{a_{12} \cdot m_1}{m_1 - m_2} \\
b_2 &= \frac{b_{12} \cdot m_1}{m_1 - m_2}
\end{align*}
\]

Now we can calculate the eccentricity. Taking into account that the points move synchronously, we can calculate the motion of the point using the equation (1). If the point move asynchronously, the \( m \) they will collide.

c. The motion of three or more points around the center of mass under following conditions:

a) We know all the masses of every point, \( m_1, m_2, m_3 \).

b) The system of points has the common center of rotation. We place the common center of rotation in the coordinate origin.

c) We know the min and max distances for every pair points.

d) Points rotate on the orbits in pairs and synchronously.

4. Algorithm

We calculate orbits of the given points. The orbit of each point is the sum of its orbit graph. From basic data we calculate the coordinates of objects of rather general the center of rotation. We consider the common center of rotation motionless.

a) Build the coherent graph for one point from the common center through intermediate centers of rotation of each pair where this point is present.
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b) Specify the starting point of the graph.
c) Group other points into non-intersecting pairs.
d) Calculate the coordinates of the rotation centers of pairs of points and assign the center of mass equal to the sum of the masses of the corresponding pair of points.
e) Calculate the motion of points in each pair using method from (3.b). "The movement of two points around a common center of mass." The result is the new system of point consisting of the starting point and centers of mass of pairs in the case when the number of points is odd, or the stating point, centers of mass of pairs and one point in the other case.
f) Repeat steps d), e), f) for a new system of points until the system reduces to two points which are the starting point and center of masses of all original points.
g) Repeat steps d), e), f) one more time.
h) Execute addition of coordinates of relative reference systems for each point.

We can see a sample calculation of angles, angular velocities, angular accelerations, parametric radii in files named “elli*.txt”. All calculations were done by using the equation (14). ZweiObjekte1Winkel.exe was written to provide these calculations.

5 Examples

Let the rotation period of all points be given by T=1. The accuracy of the calculation is the number of intervals forming the track. In the examples below the accuracy is 100. The interval is T / 100 = 0.01.

Example 1. Figure 2. Two points. Video clip Objectes2.swf.html is in the folder ObjectesVideosHTML.

Example 2. Figure 3.
Suppose we have three objects with equal mass, i.e. $m_1:m_2:m_3=1:1:1$ Suppose that every orbit is an ellipse. Let the distance between points be given by $p_{12}=p_{23}=p_{31}=8$ and the eccentricity be given by $e_x=0.6$, Taking into account that the center of rotation coincides with center of mass, we can build a graph of orbits. There exist three possible graphs, i.e. $p_1(p_2,p_3)$, $p_2(p_1,p_3)$, $p_3(p_1,p_2)$. It follows from the symmetry that all these three graphs are equivalent. Take $p_3(p_1,p_2) = (p_3,O_{12})$. The notation means that a section of graph is replaced by the center of mass, i.e. $(p_1p_2) = O_{12}$. We begin calculations from inner expressions.

Now we can calculate major and minor axes, velocities of points moving with the given time interval. In the folder Objectes3-1-1-1 we can see a set of “ellp*.*txt” files with results of the calculation, i.e. angles, angular velocities, angular accelerations, parametric radii. Also we can find source codes and executable file “Objecte3-1-1-1.exe”. Objecte3-1-1-1 provide calculation using the described above algorithm. Objectes3-1-1-1.swf.html displays a swf file with a video demonstrating the motion of points.

From symmetry, all these three point must have similar orbits. It is what we see on Figure 4.
Example 3. Figure 5.

Suppose the masses of the point be at ratio $m1:m2:m3 = 1:2:3$. Let the center of rotation coincide with the center of mass. Let all orbits be an ellipse, eccentricity is equal to 0.3, $e = 0.3$. Now we can build a graph of orbits. There exist three possible graphs, i.e. $(p1(p2,p3))$, $(p2(p1,p3))$, $(p3(p1,p2))$. Take the $(p1(p2,p3)) = (p1,c23)$ graph. Now we can calculate major and minor axis. Let rotation period of every object be given by $T=1$. We can do it because all objects move synchronously. Take the accuracy as $0.01*T$. Angle, angular velocity, angular acceleration and parametric radius data can be found in the files ellpi*.txt. The Objecte3-1-2-3 program calculates orbits using this algorithm. Objectes3-1-2-3.swf.html contains a video demonstrating the calculated movement is in the folder ObjectesVideosHTML. Objecte3-1-2-3.exe is an executable file running the calculation, Objecte3-1-2-3.cpp contains source codes, data files ellpi*.txt are in the folder Objectes3-1-2-3.
Example 3a. Figure 7.

Let us repeat the example 2 but with the \((p_3(p_1,p_2)) = (p_3,c_{12})\) graph. Angle, angular velocity, angular acceleration and parametric radius data can be found in the files ellpi*.txt. The Objecte3-1-2-3a program calculates orbits using this algorithm. Objectes3-1-2-3a.swf.html contains a video demonstrating the calculated movement is in the folder ObjectesVideosHTML. Objecte3-1-2-3a.exe is an executable file running the calculation, Objecte3-1-2-3a.cpp contains source codes, data files ellpi*.txt are in the folder Objectes3-1-2-3a.
Example 4. Figure 9.

Suppose the masses of the point be at ratio \(m_1:m_2:m_3:m_4 = 1:2:3:4\). Let the center of rotation coincide with the center of mass. Let all orbits be an ellipse, eccentricity be equal to 0.3, \(e = 0.3\). Now we build a graph of orbits. Let us take the \((p_4(p_1(p_2,p_3))) = (p_4(p_1,c_{23})) = (p_4,c_{123})\) graph. Now we can calculate major and minor axis. Let all objects have \(T=1\) as a rotation period. We can do it because all objects move synchronously. We calculate the speed of the movement of objects in points of orbits with a step on time 0.01*T. Angle, angular velocity, angular acceleration and parametric radius data.
can be found in the files ellpi*.txt. The Objecte4-1-2-3-4 program calculates orbits using this algorithm. Objectes4-1-2-3-4.swf.html contains a video demonstrating the calculated movement is in the folder ObjectesVideosHTML. Objecte4-1-2-3-4.exe is an executable file running the calculation, Objecte4-1-2-3-4.cpp contains source codes, data files ellpi*.txt are in the folder Objectes4-1-2-3-4.

Figure 10. Objects - p1, p2, p3, p4. The centers of rotation – c0, c12, c123. From example 4.

**Example 5.** We will simulate rotation of two identical bodies round the third, similar to satellites of Saturn to Janus and Epimete. We will put the rotation period round the central body, it is equal 1 godu, for 1 period of 4 crossings of orbits. We receive 3-fold mutual rotation, round the center of rotation of two bodies (Janus and Epimete).
Example 5a. The opposite case, we set 5 rotations of the first two bodies relatively each other (like the Moon with Earth). We receive 8 crossings of orbits for 1 period of rotation round the central body 3. 6 periods – 10 crossings.

N of the periods = 2N-2 of crossings of orbits.

Angle, angular velocity, angular acceleration and parametric radius data can be found in the files ellpi*.txt. The Pendel3_1_2GraphOpenGl program calculates orbits using this algorithm. Ja-E_4p.swt.html contains a video demonstrating. Pendel3_1_2GraphOpenGl.exe is an executable file running the calculation, data files ellpi*.txt are in the folder Pendel3_1_2GraphOpenGl.

6. Conclusion:
The presented algorithm allows to calculate the orbits of the objects what is shown on examples in chapter 5. The algorithm is based on the decision of the differential equations (14). One of the equation properties is constant sectoral speed. However additional investigations of the equation properties, calculations and proofs are required to apply our algorithm in the case of systems the real objects. For example, such as:

a) To calculate an orbits using the presented algorithm with parameters of the systems of the real objects and to compare these with the orbits known from an astronomical observations.

b) Prove the equivalence of graphs with different starting points. Examples 3 and 3a.

c) To calculate the orbit taking into account a precession. In examples calculation of orbits was made without a precession. By $d_p$ denote the pericenter precession. Now we can integrate the equation (14) from 0 to $\pi + d_p$. As a result of calculation also we receive a precession of an apocenter of $d_a$. At the following cycle we displace 0 at a size $d_a$.

Support information:

All videos are to the address [http://www.fayloobmennik.net/4823487](http://www.fayloobmennik.net/4823487)

The executed files are to the address [http://www.fayloobmennik.net/4909818](http://www.fayloobmennik.net/4909818)

Source codes are available at address [http://copyright.gov/eco/](http://copyright.gov/eco/) Title: ClosedTrackSystemObjects, Registration Number TXu 1-946-230.

All calculations were made using well known program products [7-10].

Projects written are to the address [http://www.fayloobmennik.net/5411508](http://www.fayloobmennik.net/5411508)

List of reference

[2] Sivukhin D. V. General course of physics, Electricity, Moscow, Russia, Nauka, 1996