RELATION OF PRIME NUMBER THEORY AND ABSOLUTE METRIC OF SPACETIME

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Abstract: In the paper, relation of logarithmic integral (*li*) function to the metric of nondecelerative universe model is discussed. It opens the coupling of mathematic prime number theory and macroworld. Up-to-now there has been just a relation between zeros distribution of the zeta function, theory of random matrix and distribution of energy levels in quantum chaotic systems.

1: RELEVANCE OF MATHEMATICS AT DESCRIPTION OF THE WORLD

The physical world is recognised through observation. The observed things and phenomena are generalized and analysed by mathematical methods. Mathematics is remarkably successful at natural phenomena analyzing. Along with dominant significance and application in natural sciences, mathematics has penetrated also into humanities, psychology, game theory, economics etc.

It is natural to ask ourselves a question whether scientists create or discover the mathematical laws. It is right that within their natural language we create the basic rules of mathematics, its semantics, syntax as well as mathematical axioms. Subsequent to selection of main parameters, the following mathematical truths are discovered by specific modes. The actual world must be governed by certain laws and principles. They function on the basis of logic and causality. It explains the fact that mathematics is so successful at describing physical laws. There is no alternative mode for describing the physical laws as mathematical tools and procedures. If there are other civilizations in the Universe along with our one and are technologically advanced, they can recognize the identical laws as we do using their own mathematical methods. The answer to the question whether mathematics create or discover the things will lie somewhere „in between". It is also the truth that mathematical world seems to be more complicated and complex than the Universe itself. Using mathematics we are able to create also different types of universes. They must be, however, logical, causal and consistent.

In Tegmark's approach [1] everything being part of the Universe, man including, is defined by terms of fundamental mathematical structures. The matter is composed of particles possessing basic properties, such as spin, mass and parity and all these properties are purely of mathematical nature. Spacetime dimensions and motion in gravitational field are described also using mathematics. Mathematics is able to offer predictions preceding observations, the discovery of Pluto planet or the evidence of Higgs boson existence can serve as examples. The man brain is described as the most complex structure of the Universe since it allows us to understand the Universe and finally also us. According to Tegmark, the model of brain will be satisfactorily described through mathematics.

At present, string and loop theories have the highest ambitions in the application of mathematics at world description. Both theories are highly demanding from the viewpoint of mathematics and just a few professionals understand them. Loop quantum gravity (LQG) has provided a prediction that the high-frequency photons will need, due to the interaction with the spacetime structure, a longer time to pass a certain distance than low-frequency photons. This prediction was, however, recently disproved.

String theories have existed from the end of 1960s. In spite of high preferences, grants, sponsors, large scale advertising and a huge number of popular science books and films, these theories have not provided any prediction, which would not be known from the standard model or cosmology. The only benefit lies in the development of highly advanced mathematics. The string theories become resigned to an effort to describe exactly our Universe. According these theories, there is a set of universes and we are living accidentally in one of them.

Of course, the conditions must be suitable for life since we actually exist (anthropic principle). Such comprehension of multiverse is of no value for both cosmology and the standard model.

Even this benevolent approach was dramatically doubted in 2005. Based on string models, there are approximately 10^{500} types of universes with the positive value of the cosmological constant. However, in 2005 physicist Washington Taylor and his co-workers from MIT [2] documented that there is an infinite number of universes with the negative value of the cosmological constant. His proof has been verified and accepted. Shortly after that the South African scientist G. Ellis and some others give a notice of the fact that at a random distribution of the cosmological constant we must live (with an infinitive higher probability) in the Universe having the negative value of the cosmological constant. However, taking the issue of dark energy into account we know from 1998 that its value is positive. Based on the above ideas it is clear that the acceptability of string models – at least for application in cosmology – terminated. Nothing would be changed even at zero value of the cosmological constant.

One of the most fundamental issues of mathematics is Riemann Hypothesis (RH). Along with mathematics, this issue concerns also quantum mechanics, quantum cryptography etc. Prime numbers are basic constituents of mathematics. It has been found that the distribution of zeros of the Riemann zeta function is considerably similar to the chaotic energy spectrum of a quantum system without symmetry to time inversion [3]. The corresponding distribution of adjacent zeros is nearly identical to a curve derived from the theory of random matrix. In this case there is a correlation between prime numbers and periodic orbits in quantum chaotic systems. Quantum chaotic systems have a certain hidden order and are responsible for the creation of complex structures, not excluding the creation of life and consciousness. There is thus obvious and deep relation between mathematics and quantum mechanics. This contribution is aimed at introducing this relation to the area of macroworld and cosmology.

2: DYNAMICS OF NONDECELERATIVE UNIVERSE

Our model of the Universe (Expansive Nondecelerative Universe, ENU) [4-6] is based on a simple assumption of constant velocity of the Universe expansion equal to the speed of light. It holds

$$a = c\,t_U \tag{1}$$

where *a* is the scale factor (1.32 x 10^{26} m), *c* is the speed of light and t_U is the cosmological time (1.38 x 10^{10} years). Moreover, it is declared that the cosmological member Λ is

$$\Lambda = 0 \tag{2}$$

and the Universe curvature *k* is

$$k = 0 \tag{3}$$

In classic models of flat Universe the gravity acts globally causing the Universe expansion deceleration. In our ENU model, gravitational forces act locally. The ENU dynamics can be expressed by Friedmann equations which subsequent to introducing dimensionless conformal time η adopt the following form

$$\frac{d}{d\eta}\left(\frac{1}{a}\frac{da}{d\eta}\right) = -\frac{4\pi G}{3c^4}a^2(\varepsilon + 3p) \tag{4}$$

$$\left(\frac{1}{a}\frac{da}{d\eta}\right)^2 = \frac{8\pi G}{3c^4}a^2\varepsilon - k \tag{5}$$

where ε is the energy density, *p* is the pressure and *a* is the scale factor expressed as follows

$$a = \frac{da}{d\eta} \tag{6}$$

Equations (1) to (6) lead to

$$\varepsilon = \frac{3c^4}{8\pi G a^2} \tag{7}$$

$$p = -\frac{\varepsilon}{3} \tag{8}$$

The energy density can also be written as follows

$$\varepsilon = \frac{3m_U c^2}{4\pi a^3} \tag{9}$$

where m_u is the Universe mass (m_u = 8.9 x 10^{52} kg)

It follows from (7) and (9)

$$a = \frac{2G\,m_\mathrm{U}}{c^2} \qquad (10)$$

Equation (10) expresses the necessity of matter creation. The amount of the matter formed at 1 sec, δ will be

$$\delta = \frac{dm_\mathrm{U}}{dt} = \frac{m_\mathrm{U}}{t_\mathrm{U}} = \frac{c^3}{2G} \qquad (11)$$

The amount of matter created during 1 sec corresponds to 10^5 Sun mass. It is the same amount of matter as that appearing from beyond the horizon in the inflationary model, *i.e*. just about 1 hydrogen atom in 1 km^3.

It is logical that in the Universe the positive energy of matter is compensated by the negative gravitational energy and the total energy of the Universe remains thus zero, *i.e*. the conservation laws are not violated.

If the Universe is an absolute system, all its physical characteristics must be of zero value. In an opposite case the characteristics should be observable from outside of the Universe and it would not be thus an absolute system.

3: THE UNIVERSE WITHOUT BIG BANG AS A CONSEQUENCE OF CHANGES IN CONSTANTS

One of the fundamental issues in cosmology is the mechanism of the Universe creation, its conditions and the existence of initial singularity. The issue represents not only mathematical and physical but also philosophical question. Scientists hopes, that the true theory will somehow eliminate this singularity. One of the theories expected to resolve the problem is the theory of quantum gravity. Such theory up to now has not been elaborated. Another way of eliminating the initial singularity lies in accepting changes in some physical constants. These changes cannot, however, be of dramatic nature and must respect the validity of fundamental physical laws, *i.e*. dimensionless fundamental physical interactions must be preserved. From the ENU viewpoint, change in some constants is directly required as a consequence of conservation laws and meaningful normalization of the Universe wave function and causality existence.

Let us look at dimensionless constants of fundamental physical interactions. The constant of strong interaction α_s is

$$\alpha_\mathrm{s} = 1 \qquad (12)$$

Dimensionless constant of electromagnetic interaction α_em is

$$\alpha_\mathrm{em} = \frac{e^2}{2\varepsilon_0 hc} = \frac{1}{137} \qquad (13)$$

Dimensionless constant of weak interaction α_w is

$$\alpha_{\mathrm{w}} = \frac{8\pi^3 g_{\mathrm{F}} m_{\mathrm{p}}^2 c}{h^3} \cong 10^{-6} \tag{14}$$

Dimensionless constant of gravitational interaction α_{g} is

$$\alpha_{\mathrm{g}} = \frac{2\pi G m_{\mathrm{p}}^2}{hc} \cong 10^{-38} \tag{15}$$

In relations (14) and (15), m_{p} is the proton mass.

There are several ways of how to change constants to preserve the value of dimensionless constants. The simplest and most probable is the way of increasing gravitational constant G and Fermi constant g_{F} proportionally to cosmological time and the same time, decreasing the mass of all elementary particles and Boltzmann constant (inverse to the square root of cosmological time). It would explain the Universe without beginning and end. The ratio of physical interactions constants would be conserved. Planck length and time would gradually increase and Planck mass decrease. These changes in Planck quantities would shift the moment of the Universe creation according to observer location on the time axis. This time axis itself would be relative. The question of constant changing is discussed for a long time [7]. An original approach of fundamental constants variation induced by dark matter or some yet-to-be-discovered cosmic field offered recently Stadnik and Flambaum [8, 9].

In the next part we put forward and explain how this unobservable change in fundamental constants functions and how it influences the illusive matter creation in the Universe.

The Universe expansion and the corresponding change in constants can be illustrated after introducing logarithmic time τ.

Such approach was many years ago discussed also by Milne [10]. It holds:

$$\tau = \ln t \tag{16}$$

where τ is real logarithmic time and t is the intrinsic cosmological time of the Universe.

At present, real time equals zero, it was negative in the past and will be positive in the future. Cosmological time at present is normalized to 1 and is proportional to the Universe scale factor a. For any observer his/her present real time is zero at any moment (see Fig. 1).

Three moments documenting advantage of logarithmic time introducing deserve attention.

1: If $t \to 0$, $\tau \to -\infty$

It means that the Universe if infinite, it has no beginning and no end. If cosmological time approaches zero, the units of Planck time and Planck length are proportionally reduced due to changes in some constants. This is why no Big Bang could exist. The corresponding changes in constants have no impact on the change in fundamental physical interactions and natural physical laws.

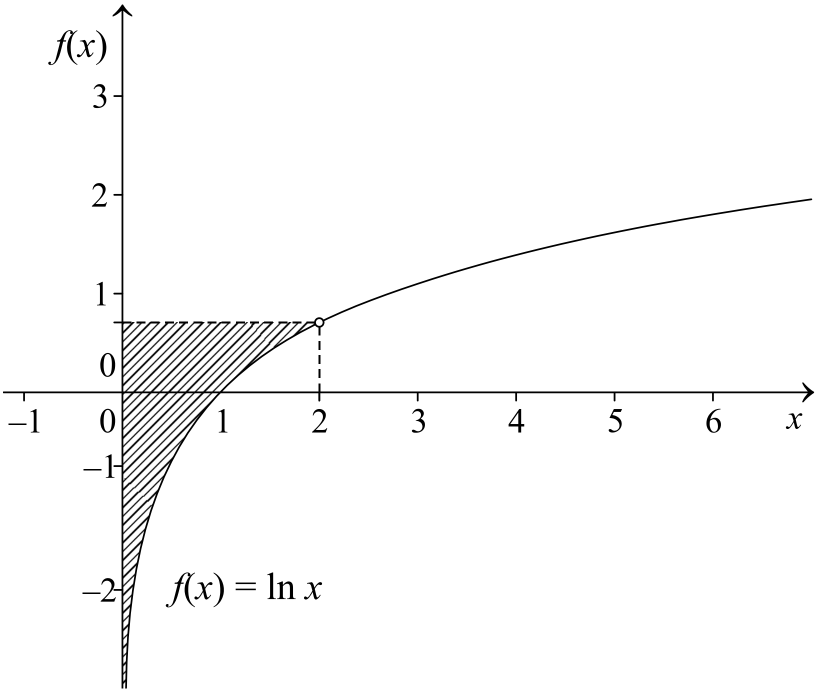

Fig. 1. Plot of logarithmic function

2: Any influence of real time could be observed only in the past. Due to the logarithmic function it seems that the Universe started to accelerate its expansion in the past. What is perceived as accelerated expansion of the Universe is in fact the gradual increase of space and time units. At the same time, any observer will have the real time equals zero at any cosmological time. Everybody will find that in a given time the Universe characteristics correspond the constant velocity of expansion equals the speed of light.

3: The area delineated by the curve (16) in the range from t = 0 to t = 1 has negative value and represents the past universe. The area for $t \geq 1$ has positive value and represents the future of the universe. To sum the negative and positive parts reaching zero value, it must hold:

$$\int_0^1 \ln t \, \mathrm{d}t + t \ln t - \int_1^t \ln t \, \mathrm{d}t = 0 \tag{17}$$

Solution of (17) leads to t = 2, which is exactly in agreement with Fig. 1. One must only realize that cosmological time is dimensionless and expressed in logarithmic scale. Causality is thus preserved just up to a limit predicted by another, independent mode. An increase of t from 1 to 2 represents 61 orders of magnitude and this is in accord with the ENU as for causality in the universe. For cosmological time it is 10^{71} years.

The question of why the area over the curve of the function is taken into account can be answered as follows. As regard to the observation of the universe past, the curve (16) may be understood as a worldline of the universe expanding in direction of the τ-

axis (real time) *i.e*. bottom-up. This is why we compare the areas delineated by the curve (16), τ-axis and the coordinate of real time over and under the *t*-axis.

It holds for the metric of such universe

$$ds^2 = dt^2 - t^2 \, d\tau^2 \tag{18}$$

and also

$$R_0^0 = R_1^1 = R_0^1 = 0 \tag{19}$$

It is flat Universe with the singularity in the point $t = 0$ (illusory beginning of the Universe). This is, in fact, de Sitter metric.

4: ABSOLUTE METRIC OF NONDECELERATIVE UNIVERSE

In Fig. 2 the plot of logarithmic integral function (*li*-function) is depicted.

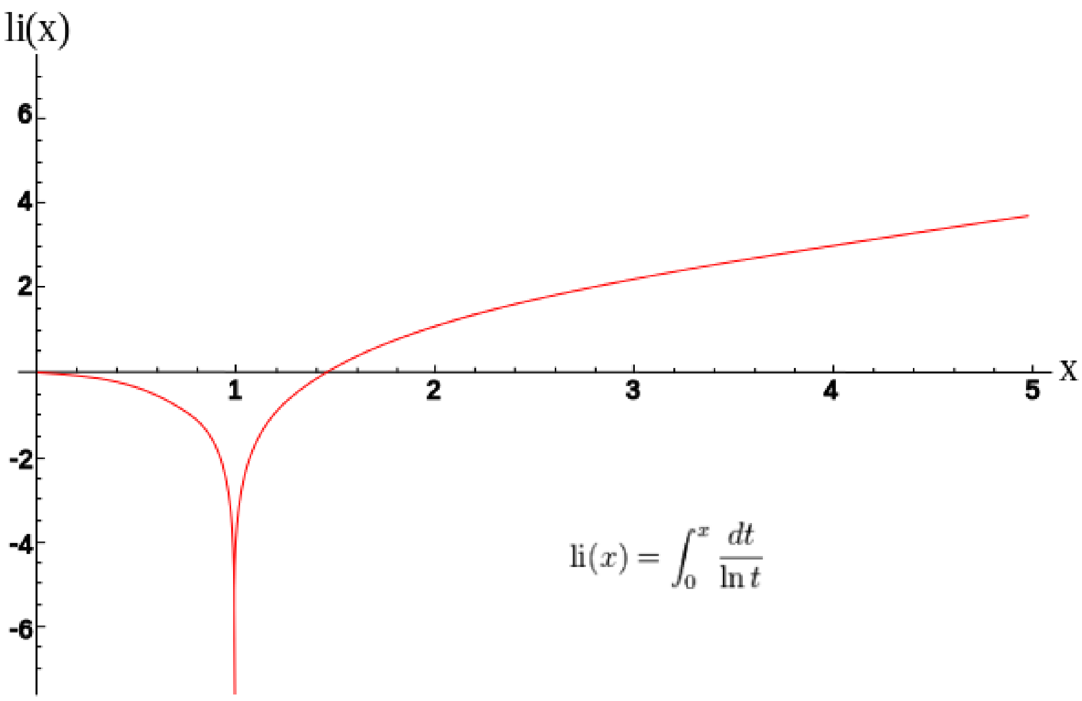

Fig.2 *Plot of li-function as a superposition of the causal past and future*

The *li*-function is defined as follows

$$li(t) = \int_0^t \frac{dt}{\ln t} \tag{20}$$

In case the lower limit of the integral equals 2, the function is denoted as *Li*-function. This function is of great importance for the estimation of a number of primes. For us, *li*-function in the interval $0 \le t \le 1.45$ is of importance. In this area the *li*-function is symmetrical to the axis $t = 1$. Function (20) has its singularity in the points $t = 0$ and $t = 1$. Now the *li*-function will be applied to the Universe. Let us take the *y*-axis as time which is analogous to real logarithmic time introduced in the previous part. This time spans from minus infinity up to the present represented on the time axis as the point

$t = 1$. Any observer during the infinity past was always in the point $t = 1$. For us as the present time observers, y-axis = 0. For observers in the past $t = 1$ and y-axis is negative (from the current point of view). The axis t represents our known spacetime expanding by the constant velocity identical to the speed of light c. When putting

$c = 1$, the Universe scale factor is equal to the cosmological time.

The entire causal history of the Universe is a series of „present moments“ (always $t = 1$). At the same time, the left part of the area confined by the straight line $t = 1$ and curve (20) illustrates the causal past and the right side the causal future of the observer. Of course the span in the past was smaller due to the Universe expansion. The point $t = 1.45$ is the starting point of chaos. This area is of no interest for us since the area following the causal horizon is logically not solvable. Our past (from the viewpoint of cosmological time) ranges in the interval from 0 to 1 on the axis t and our future is in the range (1 – 1.45). The causal intervals of the past observers are smaller. It is clear that the Universe has no beginning. This is a simple mode of expression of the entire Universe causal past and future. Now we can formulate the corresponding metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \ln^2 t \, \mathrm{d}\eta^2 \tag{21}$$

while

$$\mathrm{d}\eta = \frac{\mathrm{d}t}{\ln t} \tag{22}$$

Nonzero components of Ricci tensor are

$$R_0^0 = R_1^1 = \frac{1}{t^2 \ln t} \tag{23}$$

For scalar curvature and Einstein field equations it holds

$$R = \frac{2}{t^2 \ln t} \tag{24}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = 0 \tag{25}$$

It is obvious that the Universe is flat and all components of momentum-energy tensor are of zero value. It follows from (23) that for the past ($t < 1$) all components of Ricci tensor are negative, and for the future ($t > 1$) they are positive. This is why at observing the Universe, illusion expansive acceleration is observed.

Given the metric (21) there is the singularity in the points $t = 0$ (beginning) and $t = 1$, (the present) which is also the horizon of our Universe. It is analogous to Schwarzschild metric of black holes, where there are also singularities at the horizon and the black hole centre. It indicates an equivalency of black hole and Universe. The singularity at the black hole horizon can be eliminated selecting a suitable coordinate system. The negative curvature could be observed in the Universe past, positive one in the vicinity of black holes, however, it relates to the same objects.

In spite of the positive curvature for the future, Ricci's component R_0^1 will be always of zero value and our Universe will be flat. Gravity is considered local; the space with our event horizon is, however, global for us. Only the observer beyond the horizon, considering our Universe as a local event – black hole, can attribute nonzero value to the component R_0^1 and register gravity. The *li*-curve for $t > 1$ expresses future observers of our Universe for who this Universe will always be flat when they are its part. Observers in the future outside the *li*-curve will register gravitation of our Universe – applying Schwarzschild metric. These observers will never be part of our Universe. The beginning and present singularity of metric (21) document that a question on the beginning of expansion is a meaningless question. Similarly, a question of metric exactly in the present moment is meaningless since exactly in a given moment there is no volume, energy and any other physical reality except the horizon surface and coupling information. Any, even the most short moment which we are able to register is just the past. Saying more exactly, singularity is identical to the present moment. It is an infinitely short time and distance and, at the same time, infinitely large curvature (see (23) and (24)) with the zero energy density (25). This is why Schwarzschild metric at the black hole horizon is identical to metric (21) in the point t = 1. The observer in the surroundings of our Universe differs from us just by phase change. When transferring the horizon, the matter is changed to antimatter which is in accord with Feynman idea expressing that an antiparticle is a particle returning in time. Any universe appears a black hole in the multiversum. Since this hierarchy proceeds interminably, there must be an infinite number of universes in all phases of development. Our Universe may appear either as a common point or a quantum of spacetime in an extremely large Multiversum.

The *li*-function contributes thus in a significant extent also to the spacetime structure. This function contains also the expansion of spacetime units for the past causal horizons, but not for the future causal horizons. This is why the zeroizing of the areas will not happen at t = 2, but at t = 1.45 and *li*-function displays an asymmetry between the past and future horizons. Based on the above hypothesis it follows that the expansion factor of spacetime is proportional approximatively to $t^{1/2}$ in accord with the decrease in the mass of elementary particles predicted by our theory where this factor is inversed. Summarizing the above ideas, metric (21) leads to a conclusion that our Universe is infinite both in time and space, it seems that it accelerated its expansion in the past, some constants and timespace intervals must change, and the Universe manifests itself flat. Moreover, a single metric describes all causal past and future in the form of a smooth variety.

5: CONCLUSIONS

There is a significant relation of *li*-function and the metric allowing predicting a number of primes and elementary quanta of timespace. Our attention has been focused on *li*-function trying to unveil deeper relationships between mathematics and cosmology. Thus, *li*-function can be found in mathematics, quantum mechanics and cosmology. Metric (21) represents a mirror image of de Sitter metric (18). *Li*-Function plot (Fig. 2 for $t \geq 2$) can be used directly for the estimation of a number of universes (primes) as a function of dimension t(x). It should be taken into account that this plot is valid for any scale and can thus be used to estimate a number of black holes – universes in our visible Universe, which is Multiverse for black holes.

Scale factor of our Universe is about 10^{26} m and black hole must have gravitational diameter at least 10^4 m. The dimensionless ratio of these numbers is approximately $z = 10^{22}$. A number of universes P (black holes) for z is

$$P \cong Li(z) \cong \frac{z}{\ln z} \cong 2 \times 10^{20} \tag{26}$$

The estimated number of stars in our visible Universe is 10^{23}. According to the latest detailed investigation [11] one from thousand stars will change to black hole. This is in a perfect accord with prediction (26) and, at the same time, a bonus for our theory.

NOTE: *It is necessary to take into account that identification of (26) with a number of black holes is not a trivial issue. What counts is not the volume but just a single dimension. It may function only in case when any black hole is a full-fledged universe even in the beginning phase of its development. It is also clear that spacetime must be infinite and only its scale is changed.*

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