# Radiation from rotating dielectric disc

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The power spectral formula of the radiation of an electron moving in a rotating dielectric disc is derived. We suppose the index of refraction is constant during the rotation. This is in accord with the Fermi dielectric rotating disc for the determination of the light polarization gyration. While the well-known Čerenkov effect, transition effect, the Čerenkov-synchrotron effect due to the motion of particles in magnetic field are experimentally confirmed, the new phenomenon - the radiation due to a charge motion in rotating dielectric medium and the Čerenkov-synchrotron radiation due to the superluminal motion of particle in the rotating dielectric medium is still in the state of the preparation of experiment.

To consider the radiation by charges moving in the rotating dielectric medium (disc), it is suitable to describe such system from the viewpoint of the Lagrange dynamics and then to investigate the special cases including the Čerenkov-synchrotron radiation. So, we define the non-inertial mechanical systems by the differential equations following from the Lagrange formulation of the mechanical systems in the non-inertial systems. We follow the Landau et al. monograph (Landau et al. 1965) and author article (Pardy, 2007).

The Lagrange equation describing moving particle vith velocity $\mathbf{v}$ in the system $K'$ rotating with frequency $\mathbf{\Omega}$ and moving with velocity $\mathbf{V}(\mathbf{t})$ is as follows (Landau et al., 1987; Pardy, 2007):

$$m\frac{d\mathbf{v}}{dt} = -\frac{\partial U}{\partial \mathbf{r}} - m\mathbf{W} + m(\mathbf{r}\times\dot{\mathbf{\Omega}}) + 2m(\mathbf{v}\times\mathbf{\Omega}) + m\mathbf{\Omega}\times(\mathbf{r}\times\mathbf{\Omega}). \tag{1}$$

We observe three so called inertial forces. The force $m\mathbf{r}\times\dot{\mathbf{\Omega}}$ is connected with the nonuniform rotation of the system $K'$ and the forces $2m\mathbf{v}\times\mathbf{\Omega}$ and $m\mathbf{\Omega}\times\mathbf{r}\times\mathbf{\Omega}$ correspond to the uniform rotation. The force $2m\mathbf{v}\times\mathbf{\Omega}$ is so called the Coriolis force and it depends on the velocity of a particle. The force $m\mathbf{\Omega}\times(\mathbf{r}\times\mathbf{\Omega})$ is called the centrifugal force. It is perpendicular to the rotation axes and the magnitude of it is $m\varrho\Omega^2$, where $\varrho$ is the distance of the particle from the rotation axes.

We can apply equation (1) the special case with the harmonic oscillator in the rotating plane

We get from eq. (1) equations for harmonic oscillator in the rotating system:

$$\ddot{x} + \omega^2 x = 2\Omega_z\dot{y} \tag{2}$$

$$\ddot{y} + \omega^2 y = -2\Omega_z \dot{x}. \tag{3}$$

Let us introduce $\xi = x + iy$. Then instead of equations (2-3), we have:

$$\ddot{\xi} + 2i\Omega_z\dot{\xi} + \omega^2\xi = 0. \tag{4}$$

The solution of the last equation is for $\Omega_z \ll \omega$

$$\xi = e^{-i\Omega_z t}(A_1 e^{i\omega t} + A_2 e^{-i\omega t}), \tag{5}$$

which is in the x-y representation as

$$x + iy = e^{-i\Omega_z t}(x_0(t) + iy_0(t)), \tag{6}$$

where $x_0(t), y_0(t)$ describes the trajectory of oscillator without of the rotation of system. This case is identical with the so called Foucault pendulum and it evidently gives no substantial contribution to the synchrotron radiation by oscillator moving in the dielectric rotating disc. So, let us consider the second limiting case, namely the situation with $\Omega_z \gg \omega$.

Then, instead of eq. (4), we have:

$$\ddot{\xi} + 2i\Omega_z\dot{\xi} = 0. \tag{7}$$

By the separation of variables, we get the solution in the form:

$$\xi = Ae^{-i2\Omega_z t} = x + iy, \tag{8}$$

or,

$$x = A\cos(-2i\Omega_z t), \;\; y = A\sin(-2i\Omega_z t) \qquad (9),$$

The charged particle (electron) which moves according to the last parametric equations evidently produces the synchrotron radiation in the presence of the rotating medium with the index of refraction. The produced radiation is so called synergic synchrotron-Čerenkov radiation in case that the velocity of the particle in dielectric medium is greater than the velocity of light in this medium. So, we are prepared to determine the spectral formula for such situation. Let us remember that basic formula which is used is the vacuum to vacuum amplitude (Schwinger, 1970; ibid., 1976):

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W(S)}, \tag{10}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after the space-time region where sources are manipulated. The exponential form is introduced to account for the existence of physically independent experimental arrangements, which has a simple consequence that the associated probability amplitudes multiply and the corresponding $W$ expressions add (Schwinger, 1970; ibid., 1976).

The the following expression (Schwinger et al., 1976) for the power spectral formula was derived:

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int\int\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\left[\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|\right]}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]\times$$

$$\times \left\{ \varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t') \right\}. \tag{11}$$

We apply the last formula to the case of an electron moving in the rotating dielectric disc in the spirit of the Schwinger et al.( 1976) article.The radiation is the synergic photon production initiated by the motion of an electron in a rotating disc. This process with the superluminal particle motion is the synergic Čerenkov-symchrotron radiation. The process includes the effect of the medium, which is represented by the phenomenological index of refraction $n$ and magnetic permeability $\mu$, and it is well-known that these phenomenological constants depend on the external magnetic field. We consider here, that $n$ and $\mu$ do not depend on rotation velocity.

First, we write for the charge density $\varrho$ and for the current density, $\mathbf{J}$, the equations concerning only the circular motion of the electron and then we will show how to apply the derived formulas for the circular motion of an electron in a rotating disc. We write for the circular motion (Schwinger et al., 1976):

$$\varrho(\mathbf{x},t) = e\delta(\mathbf{x}-\mathbf{R}(t)), \qquad \mathbf{J}(\mathbf{x},t) = e\mathbf{v}(t)\delta(\mathbf{x}-\mathbf{R}(t)) \tag{12}$$

with

$$\mathbf{R}(t) = R(\mathbf{i}\cos(\omega_0 t) + \mathbf{j}\sin(\omega_0 t)), \tag{13}$$

where we will later identify $A = R$, and $-\Omega_z = \omega_0$ in order to get harmony with eq. (9).

In this specific case, we have:

$$\mathbf{v}(t) = d\mathbf{R}/dt, \quad \omega_0 = v/R, \quad \beta = v/c, \quad v = |\mathbf{v}|. \tag{14}$$

After insertion of eq. (12) into eq. (11), we get

$$P(\omega,t) = \sum_{l=1}^{\infty}\delta(\omega - l\omega_0)P_l(\omega,t) \tag{15}$$

with

$$P_l(\omega,t) = \frac{e^2}{4\pi^2 n^2}\frac{\omega\mu\omega_0}{v}\left(2n^2\beta^2 J'_{2l}(2ln\beta) - (1-n^2\beta^2)\int_0^{2ln\beta} dx J_{2l}(x)\right) \tag{16}$$

where during the derivation of eq. (16), we have used the relations:

$$t' - t = \tau, \quad dt' = d\tau \tag{17}$$

$$|\mathbf{R}(t+\tau) - \mathbf{R}(t)| = 2R\left|\sin\frac{1}{2}\omega_0\tau\right| \tag{18}$$

$$\mathbf{v}(t)\cdot\mathbf{v}(t+\tau) = v^2\cos\omega_0\tau \tag{19}$$

$$\omega_0\tau = \varphi + 2\pi l, \qquad \varphi \in (-\pi,\pi),\ l = 0,\ \pm 1,\ \pm 2,\ .... \tag{40}$$

Let us remark that formula (16) is for $n = 1$ and $\mu = 1$ identical with formula derived in monograph by Sokolov, et al. (1983).

So we can express the following cnclusion words with perspectives of our discovery. We have derived the power spectral formula of the synergic Čerenkov-synchrotron radiation for the superluminal motion of an electron moving in a rotating dielectric disc. The Grenoble accelerator produces the synchrotron radiation by motion of a charged particle along the circle with radius $R$. Here, the radius can be determined from the corresponding angular velocity $\omega_0 = v/R$ where $v$ is the initial velocity of an electron in the dielectric medium.

Fermi used the dielectric rotating disc to prove the gyration of the polarization plane of light. The formula derived by Fermi involves also the index of refraction, which is considered to be constant during the rotation (Landau et al., 1989). We suppose here that the index of refraction is not changed during the rotation. Such assumption enables to obtain simple formulas for the Čerenkov-synchrotron radiation. On the other hand, in case of the Faraday disc (Jackson, 1998), which is metal rotating disc, the physical parameters (density of free electrons, and so on) of the disc depend on its rotation velocity and on the local position of the elementary (infinitesimal) volume in the disc. It leads to new effects such as the potential difference between the axis of rotation and the edge of the disc, and so on.

The similar situation is in case of the rotating graphene disc. Such reality enables to observe new physical effects, such as the rotation Hall effect, or, the quantum fractional Hall effect, the formation of the Onsager quantum vortexes and so on, forming in a such a way the Nobelian experimental situation.

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