To understand the elementary nuclear physics and quantum physics in the light of final unification

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Abstract: Boldly it is possible to show that, 1) There exists a strong interaction elementary charge of magnitude, \(e_s = 4.720586603 \times 10^{-19}\) C and squared ratio of electromagnetic and strong interaction charges is equal to the strong coupling constant. 2) Like quarks, the strong interaction elementary charge is experimentally undetectable and can be called as ‘invisible elementary nuclear charge’. 3) There exists a gravitational constant associated with strong interaction, \(G_s \approx 3.329561213 \times 10^{-28}\) m\(^3\)/kg\(\cdot\)sec\(^2\). 4) There exists a gravitational constant associated with electromagnetic interaction, \(G_e \approx 2.374335685 \times 10^{-37}\) m\(^3\)/kg\(\cdot\)sec\(^2\). Considering the proposed strong interaction elementary charge, magnetic moments of proton and neutron can be understood. Considering the proposed electromagnetic and strong gravitational constants currently believed quantum constants can be quantified and can be shown to be secondary physical constants. Based on these points, in this paper, an attempt is made to understand the mystery of origin of elementary particle rest masses, magnetic moments and quantum nature of electron in hydrogen atom. Proceeding further, the authors developed simple procedure for understanding stable atomic nuclides’ mass number and their corresponding nuclear binding energy.

Keywords: Schwarzschild’s interaction; Electromagnetic gravitational constant, Strong interaction gravitational constant; Strong elementary charge; Magnetic moments of proton and neutron; Quantum mechanics; Nuclear binding energy;

1. INTRODUCTION

Current paradigm of final unification is ‘to quantize gravity’ [1]. In the recently published papers and references therein [2,3,4,5], by introducing two different gravitational constants (one for the electromagnetic interaction and another for the strong interaction), the authors developed many characteristic unified relations along with a new strong interaction elementary charge. It may be noted that, considering the proposed strong interaction elementary charge, magnetic moments of proton and neutron can be understood. The authors would like to stress the fact that, by considering the proposed strong and electromagnetic gravitational constants, quantum constants can be fitted and can be shown to be secondary physical constants. By considering \((1/2n^2)\) as a probability factor of finding electron within \(n^{th}\) principal quantum shell, discrete nature of orbiting electron’s total energy can be understood. In this paper, the authors reviewed [4,6] the earlier proposed relations and made an attempt to understand the mystery of origin of elementary particle rest masses and quantum nature of electron in hydrogen atom. In addition, the authors proposed very simple relations for understanding proton-neutron beta stability, maximum nuclear binding energy per nucleon, nuclear binding energy coefficients and two term nuclear binding energy of stable atomic nuclides. Content of the paper is very critical and very sensitive and the authors sincerely request the readers to have minimum patience.

2. THE CLASSICAL LIMITS OF FORCE AND POWER

In order to unify cosmology, quantum mechanics and the four observed fundamental cosmological interactions, a ‘unified force’ is required. In this connection \((c^3/G)\) can be considered as the classical force or astrophysical force limit. For a detailed description on this characteristic limiting force, readers are strongly encouraged to see the historical paper by G.W.Gibbons [7]. Similarly, \((c^3/G)\) can be considered as the classical power limit. If it is true that \(c\) and \(G\) are fundamental physical constants,
then \((c^4/G)\) and \((c^4/G)\) can also be considered as fundamental compound physical constants. These classical limits are more powerful than the Uncertainty limit. These two characteristic limits are for future experimental verification with nuclear weapons, particle accelerators, nuclear reactors and rocket propulsion units etc. Moreover, these two characteristic limits can be understood with future astrophysical and cosmological interpretations, observations and inferences. In contrast to the current notion of black hole physics, the Schwarzschild radius of a black hole \([8,9]\) can be understood with the characteristic astrophysical limiting force of magnitude \((c^4/G)\). Note that by considering \((c^4/G)\), the famous Planck mass can be obtained very easily.

### 2.1 SIMPLE APPLICATIONS OF \((c^4/G)\)

a) Magnitude of force of attraction or repulsion between any two charged particles never exceeds \((c^4/G)\).
b) Magnitude of gravitational force of attraction between any two massive bodies never exceeds \((c^4/G)\).
c) Magnitude of mechanical force on a revolving/rotating body never exceeds \((c^4/G)\).
d) Magnitude of electromagnetic force on a revolving body never exceeds \((c^4/G)\).

### 2.2 SIMPLE APPLICATIONS OF \((c^4/G)\)

a) Mechanical power never exceeds \((c^4/G)\)
b) Electromagnetic power never exceeds \((c^4/G)\)
c) Thermal radiation power never exceeds \((c^4/G)\)
d) Gravitational radiation power never exceeds \((c^4/G)\)

### 3. UNDERSTANDING THE ROLE OF \((c^4/G)\) IN BLACK HOLE FORMATION AND PLANCK MASS GENERATION

#### 3.1 SCHWARZSCHILD RADIUS OF A BLACK HOLE

The most fundamental properties of a black hole are its mass, charge, and angular momentum. Without going too deep into the mathematics of black hole physics, in this section, an attempt is made to understand the Schwarzschild radius of a black hole. In all directions, if a force of magnitude \((c^4/G)\) acts on the mass-energy content of the assumed celestial body, it approaches a minimum radius of \(\left(GM/c^2\right)\) in the following way. The origin of the force \((c^4/G)\) may be due to self-weight or internal attraction or external compression or something else.

\[
R_{\text{min}} \equiv \frac{Mc^2}{(c^4/G)} \equiv \frac{GM}{c^4} \tag{1}
\]

If no force (i.e. force of zero magnitude) acts on the mass content \(M\) of the assumed massive body, its radius becomes infinity. With reference to the average magnitude of \(\left(0, \frac{c^4}{G}\right)\), the presently believed Schwarzschild radius can be obtained as

\[
(R)_{av} \equiv \frac{Mc^2}{(c^4/2G)} \equiv \frac{2GM}{c^4} \tag{2}
\]

This proposal is very simple and seems to be different from existing concepts and may be a unified form of Newton’s law of gravity, the special theory of relativity and the general theory of relativity.

#### 3.2 TO DERIVE THE PLANCK MASS

So far no theoretical model has proposed a derivation for the Planck mass. To derive the Planck mass the following two conditions can be considered.

Assume a gravitational force of attraction between two particles of mass \((M_p)\) separated by a minimum distance \((r_{\text{min}})\) to be,

\[
\left[\frac{GM_pM_p}{r_{\text{min}}^2}\right] = \left(\frac{c^4}{G}\right) \tag{3}
\]

With reference to wave mechanics, let

\[
2\pi r_{\text{min}} \equiv \lambda_p = \left[\frac{h}{cM_p}\right] \tag{4}
\]

Here, \(\lambda_p\) represents the wavelength associated with the Planck mass. With these two assumed conditions, the Planck mass can be obtained as follows.

\[
M_p = \frac{hc}{2\pi G} \approx \sqrt{\frac{hc}{G}} \tag{5}
\]

#### 3.3 UNDERSTANDING THE STRENGTH OF ANY INTERACTION

From the above relations it is reasonable to say that,

1) If it is true that \(c\) and \(G\) are fundamental physical constants, then \((c^4/G)\) can be considered as a fundamental compound constant related to a characteristic limiting force.

2) Black holes are the most compact form of matter.

3) Magnitude of the operating force at the black hole surface is of the order of \((c^4/G)\).
4) Gravitational interaction taking place at black holes can be referred to ‘Schwarzschild interaction’.
5) Strength of this ‘Schwarzschild interaction’ can be assumed to be unity.
6) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical or astrophysical force magnitude \( (c^4/G) \).
7) If one is willing to represent the magnitude of the operating force as a fraction of \( (c^4/G) \) i.e.

\[
X \text{times of } (c^4/G) \equiv X \rightarrow \text{Effective } G \Rightarrow \frac{G}{X}
\]

If \( X \) is very small, \( \frac{1}{X} \) becomes very large. In this way, \( X \) can be considered as the strength of interaction. Thus, the strength of any interaction is \( \frac{1}{X} \) times smaller than the ‘Schwarzschild interaction’ and effective \( G \) becomes \( \frac{G}{X} \).

### 4. THREE BASIC ASSUMPTIONS OF FINAL UNIFICATION

The following three assumptions can be considered in a final unification program [6,10,11].

**Assumption-1:** The gravitational constant associated with the electromagnetic interaction, \( G_s \equiv 2.374335685 \times 10^{-37} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \).

**Assumption-2:** The gravitational constant associated with the strong interaction, \( G_s \equiv 3.32956123 \times 10^{-28} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \).

**Assumption-3:** There exists strong elementary charge, \( e_s \equiv \left( \frac{e_s}{\sqrt{\alpha_s}} \right) \) where \( e_s \) is the assumed strong interaction elementary charge, \( e_s \) is the currently believed electromagnetic elementary charge and \( \alpha_s \) is the currently believed strong coupling constant. Like quarks, the strong interaction elementary charge is experimentally undetectable and can also be called as ‘invisible elementary nuclear charge’.

With these three assumptions, the key features of nuclear and atomic structure can be understood. With reference to the Schwarzschild interaction, for electromagnetic interaction, \( X \equiv 2.8105 \times 10^{-46} \) and for strong interaction, \( X \equiv 2.004 \times 10^{-36} \). Here the authors would like to stress the fact that, as the magnitude of operating force is far less than the magnitude of \( (c^4/G) \), protons and electrons cannot be considered as black holes. Within the nuclear medium, in analogy with gravity and Schwarzschild interaction, nuclear phenomena can be understood with a large value of gravitational constant. Atomic phenomena can be understood with both nuclear and electromagnetic gravitational constants.

### 5. IMPORTANT RESULTS AND UNDERSTANDING THE MYSTERY OF QUANTUM NATURE

#### 5.1 RESULTS

A) **Strong coupling constant:** It can be understood as follows.

\[
\alpha_s \equiv \frac{e_s^2}{c G m_p} \equiv \frac{m_e}{m_p} \left( \frac{G m_p^2}{G m_e^2} \right) \tag{7}
\]

B) **Fine structure ratio:** It can be understood as follows.

\[
\alpha \equiv \frac{e e}{4 \pi e_0 G m_p^2} \tag{8}
\]

C) **Reduced Planck’s constant:** It can be understood as follows.

\[
\hbar \equiv \left( \frac{e_s}{e} \right) \left( \frac{G m_p^2}{c} \right) \equiv \frac{\alpha_s}{\alpha} \left( \frac{G m_p^2}{c} \right) \tag{9}
\]

\[
G_s \equiv \left( \frac{e_s}{e} \right) \left( \frac{\hbar}{m_p} \right) \left( \frac{1}{\sqrt{\alpha_s}} \right) \left( \frac{\hbar}{m_s} \right) \tag{10}
\]

D) **Down and Up quark mass ratio:** It can be understood as follows.

\[
\frac{m_d}{m_u} \equiv 2 \pi \left( \frac{e_s}{e} \right) \equiv 2 \pi \sqrt{\alpha_s} \tag{11}
\]

E) **Magnetic moment of proton:** It can be understood as follows.

\[
\mu_p \equiv \frac{e h}{2 m_p} \equiv \frac{G e c m_p}{2 e} \tag{12}
\]

F) **Magnetic moment of neutron:** It can be understood as follows.

\[
\mu_n \equiv \frac{e h}{2 m_n} \equiv \frac{e h}{2 m_n} \left( e_s - e_c \right) \tag{13}
\]

G) **Magnetic moment of electron:** It can be understood as follows.

\[
\mu_e \equiv \left( \frac{e_s}{m_p} \right) \left( \frac{G m_p^2}{2 e} \right) \tag{14}
\]

H) **Magnetic moment of muon:** It can be understood as follows.

\[
\mu_\mu \equiv \left( \frac{e_s}{m_p} \right) \left( \frac{m_\mu}{m_p} \right) \left( \frac{G m_p^2}{2 c} \right) \tag{15}
\]
I) **Nuclear charge radius:** It can be understood as follows.

\[ R_0 \approx \frac{2G_im_p}{c^2} \]  \hspace{1cm} (16)

J) **Root mean square radius of proton:** It can be understood as follows.

\[ R_p \approx \frac{\sqrt{2}G_im_p}{c^2} \]  \hspace{1cm} (17)

K) **Bohr radius of electron:** It can be understood as follows.

\[ a_0 \approx \left( \frac{4\pi\varepsilon_0G_im_e^2}{e^2} \right) \left( \frac{G_im_p}{e^2} \right) \]  \hspace{1cm} (18)

L) **Ratio of rest mass of proton and electron:** It can be understood as follows.

\[ \frac{G_im_pm_e}{hc} \approx \frac{hc}{G_em_p^2} \]  \hspace{1cm} (19)

On simplification,

\[ \frac{m_p}{m_e} \approx \left( \frac{4\pi\varepsilon_0G_im_e^2}{e^2} \right) \left( \frac{4\pi\varepsilon_0G_im_p^2}{e^2} \right)^{-\frac{1}{2}} \]  \hspace{1cm} (20)

\[ \Rightarrow \frac{m_p}{m_e} \approx \left( \frac{G_e}{G_p} \right) \]  \hspace{1cm} (21)

M) **Planck’s constant:** It can be understood as follows.

\[ h \approx \frac{\alpha_i m_p}{e^2} \left( \frac{G_em_e^2}{c} \right) \]  \hspace{1cm} (22)

From relations (21 and 22),

\[ G_e \approx 3.329561213 \times 10^{-28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \]

\[ G_e \approx 2.374353685 \times 10^{-7} \text{ m}^2\text{kg}^{-1}\text{sec}^{-2} \]

\[ \alpha_i \approx 0.1151937072, \]

\[ e_i \approx 4.720586603 \times 10^{-19} \text{ C} \]

\[ R_0 \approx 1.239290981 \times 10^{-15} \text{ m}, \]

\[ R_p \approx 0.87631106 \times 10^{-15} \text{ m} \]

\[ \mu_p \approx 1.488142 \times 10^{-26} \text{ J/Tesla} \]

\[ \mu_n \approx 9.817104 \times 10^{-27} \text{ J/Tesla} \]

\[ \mu_e \approx 9.27009 \times 10^{-24} \text{ J/Tesla} \]

\[ \mu_{\mu} \approx 4.485159 \times 10^{-26} \text{ J/Tesla} \]

**Note-1:** The very interesting point to be noted here is that, Proton’s magnetic moment is associated with its strong elementary charge. Neutron’s magnetic moment seems to be the difference of magnetic moment associated with strong interaction and magnetic moment associated with electromagnetic interaction.

**Note-2:** Strong gravitational constant, electromagnetic elementary charge and electromagnetic gravitational constant play a combined role in understanding the Bohr radius and discrete nature of orbiting electron. (Section-5.2)

**Note-3:** Strong elementary charge and strong gravitational constant play a combined role in fitting the up and down quark rest masses. (Section-7)

**Note-4:** Strong elementary charge, strong gravitational constant, electromagnetic elementary charge and electromagnetic gravitational constant play a combined role in fitting the muon and tau rest masses. (Section-8)

**Note-5:** Strong elementary charge and strong gravitational constant play a combined role in understanding the nuclear binding. (Section-9).

### 5.2 UNDERSTANDING THE MYSTERY OF QUANTUM NATURE OF ELECTRON IN HYDROGEN ATOM

Considering relations (7) to (22), the authors would like to say that,

A) Along with the strong elementary charge, within the atomic medium there exist two different gravitational constants and their existence is real, not virtual.

B) Considering \( G_e \) and \( G_p \) magnitudes of quantum constants like ‘basic unit of angular momentum’, ‘basic unit of electron’s distance’ etc. can be fitted and understood.

C) It may be noted that, according to Bohr’s theory of Hydrogen atom, number of electrons that can be accommodated in any principal quantum shell is \( 2n^2 \). Based on this idea, it is possible to assume that, probability of finding any one electron is \( \left( \frac{1}{2n^2} \right) \) It can be obtained in the following way.

D) Out of \( 2n^2 \) electrons, number of electrons that can be accommodated in \( s \) shell is 2. If one is willing to consider \( s \) shell as a basic entity in such a way that, \( p \) shell constitutes \( 3s \) shells, \( d \) shell constitutes \( 5s \) shells, \( f \) shell constitutes \( 7s \) shells etc, then, \( n^2 \) can be considered as a representation of total number of \( s \) shells that can be accommodated in any principal quantum shell.

E) **Notation point of view,** it can be assigned for \( p \) shell: \( ps1, ps2, ps3 \) and for \( d \) shell: \( ds1, ds2, ds3, ds4, ds5 \) etc. Transition of electron from \( 2^{nd} \) orbit \( p \) shell to 1st orbit \( s \) shell can be expressed as: \( 2ps1 \rightarrow 1s, 2ps2 \rightarrow 1s, 2ps3 \rightarrow 1s \). Thinking in this way different transition levels can be expected. With reference to \( p \) shell, 3 different spectral lines, with reference to \( d \) shell, 5 different spectral lines can be expected. Similarly with reference to \( f \) shell, 7 different spectral lines can be expected.

F) If so, it is also possible to assume that, probability of finding any one \( s \) shell is \( \left( \frac{1}{n^2} \right) \). Based on this
proposition, from relation (18), discrete potential energy of \( s \) shell in hydrogen atom can be expressed as follows.

\[
E_{pot} \approx -\left( \frac{1}{n^2} \right) \left( \frac{e^2}{4\pi\varepsilon_0 G_m^2} \right) \left( \frac{e^2 e^2}{c^2} \right)
\]

(23)

where \( \left( \frac{e^2}{4\pi\varepsilon_0 G_m^2} \right) \) represents a force ratio and \( n^2 \) represents the total number of \( s \) shells corresponding to \( n^2 \) principal quantum shell. Thinking in this way, orbiting radius of \( n^2 \) \( s \) shells can be expressed as

\[
a_n \approx n^2 \left( \frac{4\pi\varepsilon_0 G_m^2}{e^2 m_e^2} \right) \left( \frac{G_m m_p}{c^2} \right)
\]

(24)

Clearly speaking, \( a_n \) represents the orbiting radius of \( n^2 \) \( s \) shells. In this way, the long standing concept of 1:4:9:16 etc. can be understood in a more meaningful approach.

\( s \) shell’s discrete kinetic energy can be expressed as follows.

\[
E_{kin} \approx \frac{1}{2} E_{pot} \approx \frac{1}{2n^2} \left( \frac{e^2}{4\pi\varepsilon_0 G_m^2} \right) \left( \frac{e^2 e^2}{c^2} \right)
\]

(25)

Discrete total energy of one \( s \) shell can be expressed as follows.

\[
E_{tot} \approx -\frac{1}{2n^2} \left( \frac{e^2}{4\pi\varepsilon_0 G_m^2} \right) \left( \frac{e^2 e^2}{c^2} \right)
\]

(26)

Here it may be noted that, in the hydrogen atom, there exists only one electron. Hence relation (26) can be considered as a representation of the total energy of electron. Comparing this relation (26) with Bohr’s theory of hydrogen atom, relation (9) can be obtained. Now the famous expression of integral nature of angular momentum can be expressed as:

\[
h \approx n \left( \frac{G_m m_p}{c} \right) \left( \frac{G_m m_p}{c} \right)
\]

(27)

Here, \( n \approx \sqrt{n^2} \) represents the number of \( s \) shells. The emitted energy can be expressed as follows.

\[
E_{emis} \approx \left| \left( \frac{e^2}{4\pi\varepsilon_0 G_m^2} \right) \left( \frac{e^2 e^2}{c^2} \right) \right| \left( \frac{1}{2n^2} - \frac{1}{2n^2} \right)
\]

(28)

where \( 2n_1^2 > 2n_1^2 \)

Relation (24) seems to be more meaningful than relation (27). The central idea of this section can be stated as follows.

1. Within the atom, electronic arrangement is ‘systematic’.
2. In \( n^2 \) principal quantum shell, there is a scope for the existence of \( n^2 \) number of (currently believed) \( s \)-shells.
3. 3 number of \( s \)-shells can be collectively called as one ‘p-shell’. Similarly 5 number of \( s \)-shells can be collectively called as one ‘d-shell’ and so on.

6. TO FIT THE NEWTONIAN GRAVITATIONAL CONSTANT

According to J.E. Brandenburg,

\[
\alpha \left( \frac{e^2}{4\pi\varepsilon_0 G_m^2} \right) \approx \text{exp} \left( \frac{m_p}{m_e} \right)^2
\]

(29)

This is a very interesting relation. J.E. Brandenburg first published its original form in 1992 [12]. This result can also be seen in his recently published paper [13].

In this section, the authors considered and slightly modified the RHS of above relation (29) and developed another useful relation with reference to the proposed strong interaction elementary charge. Considering the rest masses of electron, proton and neutron and considering the proposed strong interaction elementary charge, gravitational constant can be fitted accurately with the following semi empirical expression.

\[
G \approx \text{exp} \left( \frac{m_p}{m_e} \right)^2 \frac{m_p}{m_e} \left( \frac{e^2}{4\pi\varepsilon_0 m_p^2} \right)
\]

\[
\approx 6.673004423 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}
\]

where

\[
\{ e \approx 4.720586603 \times 10^{-19} \text{ C}, \varepsilon_0 \approx 8.854187817 \times 10^{-19} \text{ F/m}, \}
\]

\[
\{ m_n \approx 1.67492716 \times 10^{-27} \text{ kg}, m_p \approx 1.67262158 \times 10^{-27} \text{ kg} \}
\]

\[
\{ m_p \approx 9.10938188 \times 10^{-31} \text{ kg} \}
\]

This obtained value can be compared with other experimental values and recommended values [14-18]. It may be noted that, fitting the gravitational constant with elementary physical constants is a very challenging issue. With the coincidence of above relations, it is possible to interconnect the three gravitational constants in a unified manner. The authors are working in this new direction.

7. TO UNDERSTAND THE REST MASS OF PROTON AND TO FIT THE UP AND DOWN QUARK REST MASSES

Let,

\[
m_p \equiv \beta \sqrt{\frac{e^2}{4\pi\varepsilon_0 G_s}} \equiv \beta \times 137.5796592 \text{ MeV/c}^2
\]

(32)
\[
\beta \equiv \sqrt{\frac{4\pi\alpha G m_e^2}{c^2}} \approx 6.819845328
\]

Using this number, miracles can be done like down and up quark mass ratio can be fitted and proton-electron mass ratio can be fitted.

If \( \gamma \equiv (m_d/m_u) \) represents the ratio of down and up quark rest masses [11]. It is noticed that,

\[
\gamma^2 + \gamma - \beta \equiv 0
\]

\[
\gamma \approx -1 + \sqrt{1 + (4\beta)} \approx 2.158918075
\]

By considering a factor 2, it is noticed that,

\[
\begin{align*}
\frac{m_d}{m_u} & \equiv 2\gamma^2 - 2m_u \approx 4.672 \text{ MeV}/c^2 \\
\frac{m_u}{m_d} & \equiv 2\gamma^2 + 2m_u \approx 2.1641 \text{ MeV}/c^2
\end{align*}
\]

These values can be compared with the recommended values [11] of 4.76 MeV/c^2 and 2.15 MeV/c^2 respectively. Proceeding further,

A) Neutron – proton mass difference can be interlinked with down and up quark masses with the following relation.

\[
\frac{(m_d - m_u)^2}{(m_u - m_p)^2} \approx \frac{1}{\sqrt{\alpha_e}} - 1
\]

B) Proton-electron mass ratio can be approximated with the following relation.

\[
\frac{m_p}{m_e} \approx \beta^4 - \beta^3 \approx 1846.014
\]

8. TO UNDERSTAND THE REST MASS OF ELECTRON AND TO FIT THE MUON AND TAU REST MASSES

Let,

\[
m_e \approx \eta \sqrt{\frac{e^2}{4\pi\alpha G}} \approx \eta \times 0.001746 \text{ MeV}/c^2
\]

\[
\eta \approx \sqrt{\frac{4\pi\alpha G m_e^2}{e^2}} \approx 292.2328
\]

If so, muon and tau rest masses can be fitted in the following way.

\[
m_{\mu}c^2 \approx \left\{ m_e c^2 \right\}^{1/3} + \left\{ \left[ n^2 \eta \right]^n \right\}^{1/3}
\]

where

\[
\begin{align*}
E_e & \approx \alpha_e \sqrt{\frac{e^2 c^4}{4\pi\alpha G}} \approx 15.8483 \text{ MeV} \\
E_\mu & \approx \alpha_e \sqrt{\frac{e^2 c^4}{4\pi\alpha G}} \approx 917.635 \text{ MeV}
\end{align*}
\]

If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. By maximizing \( B(A,Z) \) with respect to \( Z \), we find the number of protons \( Z \) of the stable nucleus of atomic weight \( A \) as,
\[ Z \approx \frac{A}{2 + (a_r/2a_e)A^{2/3}} \]  

(42)

This is roughly \( A/2 \) for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of \( Z \) back into \( B \) one obtains the binding energy as a function of the atomic weight, \( B(A) \). Maximizing \( B(A)/A \) with respect to \( A \) gives the nucleus which is most strongly bound or most stable.

### 9.1 UNDERSTANDING PROTON – NEUTRON STABILITY

Proton-neutron stability relation can be expressed as follows [19]. With this expression, stable nucleon number can be directly estimated by considering the proton number.

\[
A_p \approx 2Z + \left( \frac{G_m m_e}{\hbar c} \right) (2Z)^2
\]

\[
\approx 2Z + \left( \frac{e_p}{m_p} \right) \left( \frac{e_e}{m_e} \right) (2Z)^2 \approx 2Z + \left( 0.00642 + Z \right)^2
\]

(43)

It may be noted that,

\[
\left( \frac{G_m m_e}{\hbar c} \right) \approx \left( \frac{e_p}{m_p} \right) \left( \frac{e_e}{m_e} \right) \approx \frac{1}{623.053} \approx 0.001605 \approx k \text{ (say)}
\]

If \( Z = 92 \), obtained \( A_p \approx 238.17 \) and its actual stable mass number is 238. Considering even-odd corrections, naturally occurring stable atomic nuclides can also be fitted with this relation. In addition, super heavy stable atomic nuclides can also be predicted. See columns 1 and 2 of table-1.

### 9.2 UNDERSTANDING NUCLEAR BINDING ENERGY

#### Step-1: To find the characteristic binding energy potential

Individual self potential energy of the strongly interacting proton can be fitted as follows. It may be noted that, this is a very great support for the proposed existence of ‘strong interaction elementary charge’ and the authors sincerely request the science community to review this section in a true scientific spirit.

\[
E_{pot} \approx -\frac{3}{5} \left( \frac{e_p^2}{4\pi\varepsilon_0 R_p} \right)
\]

\[
\approx -\frac{3}{5} \left( \frac{e_p^2}{4\pi\varepsilon_0 \left( \sqrt{2G_m m_p/c^2} \right)} \right)
\]

\[
\approx -8.56 \text{ MeV to } -8.92 \text{ MeV}
\]

(45)

where, \( R_p \equiv (0.841 \text{ to } 0.8763) \text{ fm} \)

It is noticed that, \( \left( 8.92^2 \times 8.56 \right)^{1/2} \approx 8.8 \text{ MeV} \) and corresponding RMS radius of proton is 0.853 fm.

Very interesting observation is that, 8.8 MeV can be considered as the maximum binding energy per nucleon.

#### Step-2 To fit the energy coefficients of semi empirical mass formula

Semi empirically, it is noticed that,

\[
a_p \approx \sqrt{\alpha} \times 8.8 \text{ MeV} \approx 0.752 \text{ MeV}
\]

where \( \alpha \approx (1/137.036) \) is the fine structure ratio.

\[
\left( a_s, a_p \right) \equiv 2 \left( 8.8 \pm 0.752 \right) \text{ MeV}
\]

\[
\approx (19.1 \text{ MeV, } 16.1 \text{ MeV})
\]

(46)

\[
a_s \approx \frac{8}{3} \times 8.8 \approx 23.47 \text{ MeV}
\]

(47)

\[
a_p \approx \frac{1}{2} a_s \approx \frac{4}{3} \times 8.8 \approx 11.73 \text{ MeV}
\]

(48)

\[
a_s + a_p \equiv a_s + a_p \equiv 3 a_p \approx \frac{3}{2} a_s
\]

(49)

\[
4 \times 8.8 \text{ MeV} \equiv 35.2 \text{ MeV}
\]

(50)

With these proposed energy coefficients, liquid drop binding energy [20,21,22] can be estimated with the following standard relation.

\[
B = a_s A - a_s A^{2/3} - a_s \frac{Z-1}{A} - a_s \frac{A-2Z}{A} \pm a_p \frac{A}{\sqrt{A}}
\]

(51)

Table-1: Estimated stable mass number and liquid drop binding energy

<table>
<thead>
<tr>
<th>Proton number</th>
<th>Estimated Stable mass number</th>
<th>Estimated liquid drop binding energy(MeV)</th>
<th>Reference liquid drop binding energy[20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>56</td>
<td>488.93</td>
<td>484.37</td>
</tr>
<tr>
<td>50</td>
<td>116</td>
<td>984.48</td>
<td>983.81</td>
</tr>
<tr>
<td>82</td>
<td>207</td>
<td>1609.0</td>
<td>1612.22</td>
</tr>
<tr>
<td>92</td>
<td>238</td>
<td>1795.0</td>
<td>1795.64</td>
</tr>
</tbody>
</table>
Step: 3 To fit the binding energy of medium and heavy stable atomic nuclides of A>56

Proceeding further it is also possible to show that, at the stable mass numbers of A>56

\[ a_x A_x - a_x A_x - \alpha_x \frac{Z(Z-1)}{A_x} \approx A_x 8.8 \text{ MeV} \quad (52) \]

Proceeding further, for medium and heavy stable atomic nuclides, it is possible to show that,

\[ B_{A_x} \equiv \left( A_x \ast 8.8 \text{ MeV} \right) - \left( k^2 A_x \ast 8.8 \text{ MeV} \right) \equiv \left( 1 - k^2 A_x \right) A_x \ast 8.8 \text{ MeV} \quad (53) \]

Here \( A_x \ast 8.8 \text{ MeV} \) can be called as the first term and \( \left( k^2 A_x \right) 8.8 \text{ MeV} \) can be called as the second term. Another interesting observation is that, second term can be compared with the currently believed ‘Asymmetry energy term’. The difference is that, for mass numbers A>200, magnitude of the proposed second term is gradually crossing the currently believed asymmetry energy term. In addition, the factor \( 1 - k^2 A_x \) can be called as the ‘binding energy reduction factor’.

Step: 4 To fit the binding energy of light stable atomic nuclides of A<56

For light stable atomic nuclides, A<56, binding energy can be fitted with the following relation.

\[ B_{A_x} \equiv \left( \frac{A_x}{56} \right)^{1/2} A_x \ast 8.8 \text{ MeV} - \left( \frac{A_x}{56} \right)^{1/2} \left( k^2 A_x \right) 8.8 \text{ MeV} \quad (54) \]

Here \( \left( \frac{A_x}{56} \right)^{1/2} A_x \ast 8.8 \text{ MeV} \) can be called as the first term and \( \left( \frac{A_x}{56} \right)^{1/2} \left( k^2 A_x \right) 8.8 \text{ MeV} \) can be called as the second term. See the following table-2.

<table>
<thead>
<tr>
<th>Proton number</th>
<th>Estimated stable mass number</th>
<th>Estimated total binding energy (MeV)</th>
<th>Reference total binding energy [22]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>28.25</td>
<td>28.296</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>126.76</td>
<td>127.619</td>
</tr>
<tr>
<td>14</td>
<td>29</td>
<td>241.06</td>
<td>245.011</td>
</tr>
</tbody>
</table>

It may be noted recently Ghahramany et al developed an integrated model for estimating and understanding nuclear binding energy [22]. The authors are working on these interesting relations (42) to (54) to understand the binding energy above and below the stable atomic mass numbers [24].

10. CONCLUSION

Juan M. Maldacena says [1]: “Our present world picture is based on two theories: the Standard Model of particle physics and general relativity, the theory of gravity. These two theories have scored astonishing successes. It is therefore quite striking when one learns that this picture of the laws of physics is inconsistent. The inconsistency comes from taking a part of theory, the Standard Model, as a quantum theory and the other, gravity, as a classical theory”.

It may be noted that, String theory [23] was first studied in the late 1960s as a theory of the strong nuclear force, before being abandoned in favor of quantum chromodynamics. Subsequently, it was realized that the very properties that made string theory unsuitable as a theory of nuclear physics made it a promising candidate for a quantum theory of gravity. Even though, string theory could not provide any clue for understanding the observed elementary particle mass spectrum and atomic and nuclear structures in terms of gravity. In this context, qualitatively and quantitatively, from the above concepts and relations, the authors would like to stress the following points.

A) The proposed three assumptions can be given some priority at fundamental level.
B) Quantum constants can be assumed to be secondary physical constants.
C) Discrete nature of orbiting electron can be better understood with ‘systematic arrangement’ of \( n^2 \) number of s-shells.
D) With further research and analysis, basics of final unification can be explored [24].

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