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In Searching of Trees Natural Vibration Frequency Based on Granular Particles Interactions and Vibration

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Inspiring by Leonardo's formula regarding cross section of trees from trunk to branches, Eloy (2011) found that Leonardo's exponent is between 1.8 and 2.3, instead of exact 2. In this work another aspect is investigated, tree branches natural frequency. It is already usual that in a nearly no-windy condition, some branches vibrated alternatively due to wind movement passing their leaves. This could explain why in a very windy situation a tree may survive, while a power line construction not. It could be addressed to their natural vibration frequency. A tree model based on granular particle interaction in Eloy's parameter ranges is developed. Vibration is induced to some parts to investigate which other part will be also vibrated. Few branches trees and many ones give different responses. From the results prevention may be designed to conserve our forestry resources.

Keywords: *cross section, granular model, natural frequency, tree, vibration.*

INTRODUCTION

Various branch diameters of a tree are response to winds from its surrounding and also other factors such as rains, snows, fruits, and upper parts of the tree, since a branch must support the stress induced by this loads (Eloy, 2011). How these stresses varied in time also gives different branch diameters variation or in general also morphology of the tree (Dassot *et al.*, 2015). Even in details, angle of individual leaf inclination is under influence of its weight and wind (Tadrist *et al.*, 2014), which also induces the density and strength of the wood (Fournier *et al.*, 2013). Observation of the angles (leaf angle distribution, LAD) is conducted photographically since LAD is important in plant productivity of field crops (Zou *et al.*, 2014). Further investigation on tree damage after a storm can be used to predict the wind velocity (Frelich and Ostuno, 2012). In this work interaction between particles are represented only using push-pull and bending springs, which is usual in molecule model (Allen, 2004).

MATERIALS AND METHODS

Model

Tree model is simplified into a two-dimension system where its solid beam and granular particle interaction (GPI) model is shown in Figure 1. In this GPI model the lower most particles is the largest trunk, where this particle position is fixed.

Figure 1. A two-dimension tree model: solid beam model (left) and granular particle interactions model (right).

Granular particle i with mass m_i and diameter D_i has position of $\vec{r}_i$ and it will have velocity of $\vec{v}_i$. If a tree is constructed of N granular particles, then its mass will be

$$M = \sum_{i=1}^{N} m_i \ . \tag{1}$$

Particle i and j are bound through pull-push spring force. On particle i due to existence of particle j the spring force is

$$\vec{S}_{ij} = -c_S \left(r_{ij} - l_{ij} \right) \hat{r}_{ij} \ , \tag{2}$$

with l_{ij} is normal distance between particle i and j, c_s is spring constant, and

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j \ , \tag{3}$$

$$r_{ij} = \left| \vec{r}_{ij} \right| = \sqrt{\vec{r}_{ij} \cdot \vec{r}_{ij}} \ , \tag{4}$$

and

$$\hat{r}_{ij} = \frac{\vec{r}_{ij}}{r_{ij}} \ , \tag{5}$$

are relative position of particle i from particle j, distance of particle i from particle j, and unit vector pointing to particle i from particle j, respectively. Illustration how Equation (2) can accommodate force in both direction (push and pull) is given in Figure 2.

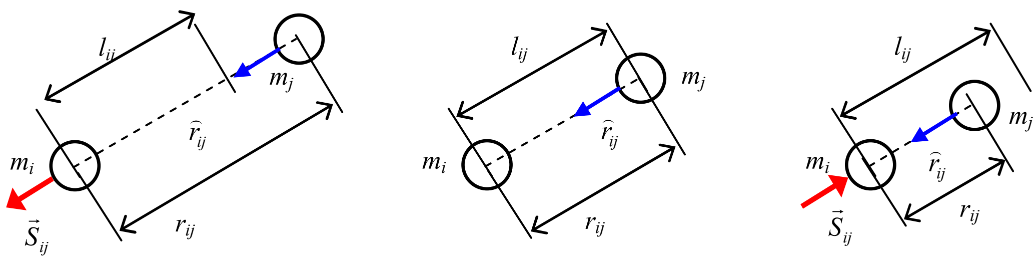


Figure 2. Spring force $\vec{S}_{ij}$ acting on particle i due to existence of particle j for condition of: $r_{ij} > l_{ij}$ (left), $r_{ij} = l_{ij}$ (center), and $r_{ij} < l_{ij}$ (right).

Referring to Figure 1 normal distance between particle i and j should be

$$l_{ij} = D_i + D_j \tag{6}$$

for dense configuration of granular particles. In order three particles i, j, and k to have a stable arrangement an interaction in a form of bending torque must be introduced

$$\tau_{ijk} = -c_B \left(\theta_{ijk} - \lambda_{ijk} \right) , \tag{7}$$

where c_B is bending constant, θ_{ijk} is angle of the three particles, and λ_{ijk} is equilibrium angle.

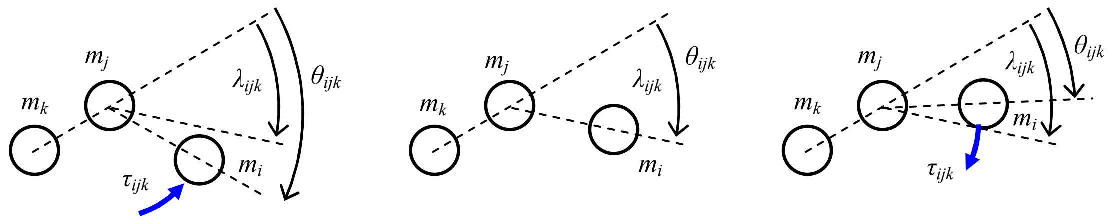


Figure 3. Bending torque τ_{ijk} acting on particle *i* due to existence of particles *j* and *k* for: $\theta_{ijk} > \lambda_{ijk}$ (left), $\theta_{ijk} = \lambda_{ijk}$ (center), and $\theta_{ijk} < \lambda_{ijk}$ (right).

All angles, including θ_{ijk} and λ_{ijk} are measured from the direction of $\hat{r}_{jk}$ in order to obey Equation (7), otherwise the minus sign should be dropped. Then, there will be also other interaction forces, not between particles but from the environment to the particles, i.e. gravitation and wind. The first force from the environment is simply

$$\vec{G}_i = m_i \vec{g}, \tag{8}$$

where $\vec{g} = -c_G \hat{e}_y$ is the gravity. And the second force will be

$$\vec{F}_i = -3\pi D \eta (\vec{v}_i - \vec{v}_w) = -c_F D (\vec{v}_i - \vec{v}_w) \tag{9}$$

with $\vec{v}_w$ is wind velocity. In general it can be assumed that $\vec{v}_w = \vec{v}_w(t)$, which can be alternated in time. Back to Equation (7), it does not yet represent the force acting on particle *i*. This means that the torque τ_{ijk} should be represented in term of force

$$\vec{B}_{ijk} = \frac{1}{r_{ij}} (\hat{r}_{ij} \times \vec{\tau}_{ijk}), \tag{10}$$

where direction of B_{ijk} can be found through triple vector product of $\vec{r}_{ij}$, $\vec{r}_{ij}$, and $\vec{B}_{ijk}$

$$\vec{r}_{ij} \times \vec{\tau}_{ijk} = \vec{r}_{ij} \times (\vec{r}_{ij} \times \vec{B}_{ijk}) = (\vec{r}_{ij} \cdot \vec{B}_{ijk}) \vec{r}_{ij} - (\vec{r}_{ij} \cdot \vec{r}_{ij}) \vec{B}_{ijk} = r_{ij}^2 \vec{B}_{ijk}. \tag{11}$$

Cross product of $\vec{r}_{jk}$ and $\vec{r}_{ij}$ will give direction of θ_{ijk}

$$\hat{\theta}_{ijk} = \frac{\vec{r}_{jk} \times \vec{r}_{ij}}{|\vec{r}_{jk} \times \vec{r}_{ij}|}, \tag{12}$$

the value is given by

$$\cos\theta_{ijk} = \hat{r}_{jk} \cdot \hat{r}_{ij} \Rightarrow \theta_{ijk} \approx 2\sqrt{1 - \hat{r}_{jk} \cdot \hat{r}_{ij}} \tag{13}$$

for small angle approximation. Last term in Equation (13) holds if the tree is almost straight with $\lambda_{ijk} \approx 0$. Using this assumptions Equation (7) can be rewritten as

$$\vec{\tau}_{ijk} = -2c_B \sqrt{1 - \hat{r}_{jk} \cdot \hat{r}_{ij}}\, \hat{\theta}_{ijk}. \tag{14}$$

Substitution of Equation (12) into Equation (14) and then the result into Equation (10) will give

$$\vec{B}_{ijk} = -\left(\frac{2c_B \sqrt{1 - \hat{r}_{jk} \cdot \hat{r}_{ij}}}{r_{ij}} \right) \left[\frac{\hat{r}_{ij} \times (\vec{r}_{jk} \times \vec{r}_{ij})}{|\vec{r}_{jk} \times \vec{r}_{ij}|} \right]. \tag{15}$$

Triple vector product could simplify Equation (15) into

$$\vec{B}_{ijk} = -\left(2c_B r_{jk} \sqrt{1 - \hat{r}_{jk} \cdot \hat{r}_{ij}}\right) \left[\frac{\hat{r}_{jk} - (\hat{r}_{jk} \cdot \hat{r}_{ij}) \hat{r}_{ij}}{|\vec{r}_{jk} \times \vec{r}_{ij}|} \right]. \tag{16}$$

Figure 4 shows the relation between bending torque and bending force.

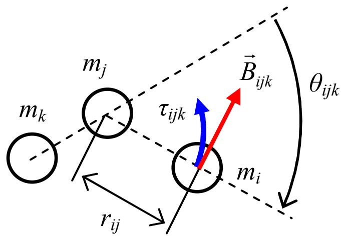


Figure 4. Relation between bending torque τ_{ijk} and its force $\vec{B}_{ijk}$ through distance r_{ij} and angle θ_{ijk}.

Simulation

Molecular dynamics method will used in this work for the simulation. Newton's second law of motion is used to get acceleration of each granular particle and for numerical integration the Euler algorithm is chosen. Particle acceleration is obtained through

$$\vec{a}_i(t) = \vec{g} + \frac{1}{m_i}\left(-c_F D[\vec{v}_i(t) - \vec{v}_w(t)] + \sum_{j\neq i, k\neq i} \left\{ \vec{S}_{ij}\left[\vec{r}_i(t), \vec{r}_j(t)\right] + \vec{B}_{ijk}\left[\vec{r}_i(t), \vec{r}_j(t), \vec{r}_k(t)\right] \right\} \right). \tag{17}$$

In general it can be said that $\vec{a}_i(t) = \vec{f}[\vec{g}, \vec{r}_{i-2}(t), \vec{r}_{i-1}(t), \vec{r}_i(t), \vec{r}_{i+1}(t), \vec{r}_{i+2}(t), \vec{v}_i(t), \vec{v}_w(t)]$. Push-pull spring force requires direct neighbor particles, while bending force requires one more neighbor beyond the direct ones. Velocity at time $t + \Delta t$ is given by

$$\vec{v}_i(t + \Delta t) = \vec{v}_i(t) + \vec{a}_i(t)\Delta t \tag{18}$$

and using improved Euler algorithm

$$\vec{r}_i(t + \Delta t) = \vec{r}_i(t) + \vec{v}_i(t + \Delta t)\Delta t \tag{19}$$

position at the same time can also be obtained. Not all particles are included in calculation of acceleration of particle i as given in Equation (17), but only related neighbors as shown in Figure 5.

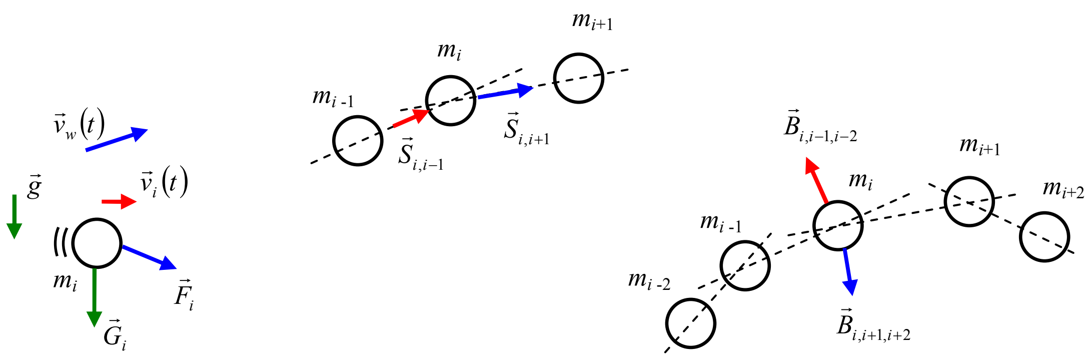


Figure 5. Neighbor particles included in calculation of: gravitation and wind forces (left), push-pull spring force (center) and bending force (right).

Another boundary condition also must be used, i.e. for lower most grain it should be not integrated since its position remains the same and its velocity is always zero.

Nomenclature

Next step is how to label the particles if there is a branch. Nomenclature in chemistry in naming structure of organic compound (Petrucci *et al.*, 2007) can be used with modification in this work. Figure 6 shows some structure for $N = 10$ or decatree. Notice that particle index begins with 0 instead of 1 as in organic molecules.

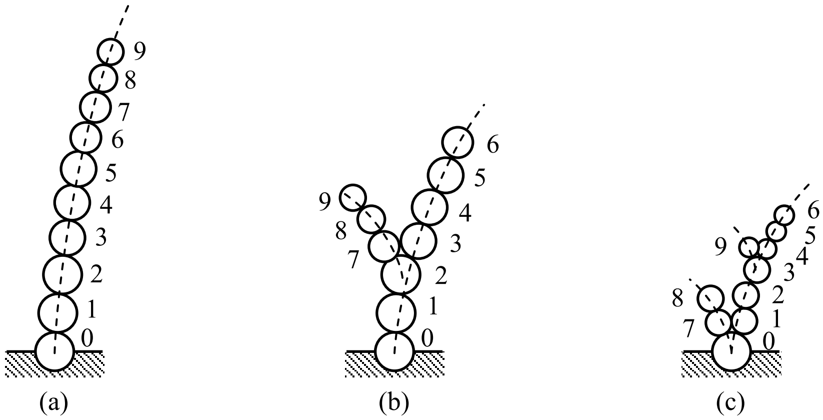


Figure 6. Some structures for a decatree: (a) decatree, (b) 2-propabranch-heptatree, and (c) 0-ethabranch-3-methabranch-heptatree; and numbering the particles.

Each particle has x_i and y_i positions, mass m_i, and diameter D_i, each pair of two particles has push-pull spring constant c_s, each pair of three particles has bending constant c_B. Input file can be designed as follow

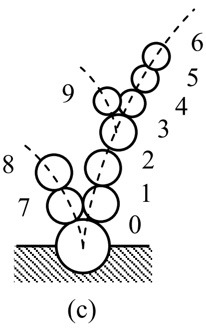
```
NAME    0-ethbranch-3-methbranch-hepttree

NPAR    10
0       ..      ..      ..      ..
..
9       ..      ..      ..      ..

CS      1E3
CS-PAIRS        9
0       1
1       2
2       3
3       4
4       5
5       6
0       7
7       8
3       9

CB      4E3
CB-PAIRS        9
1       0       7
2       1       0
3       2       1
4       3       2
5       4       3
6       5       4
7       0       1
8       7       0
9       2       1

CF      0.5
CG      10
```



The input file will be used as input and also output from the program.

RESULTS AND DISCUSSION

Configurations and parameters

Three different configurations are tested for drawing only with $N = 30$ as given in Figure 7.

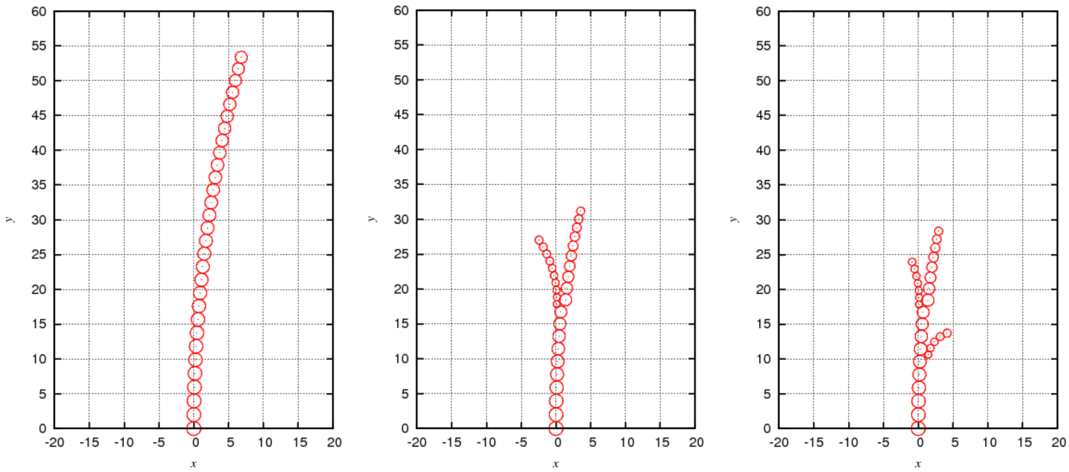


Figure 7. Three configurations are tested for drawing only with N = 30: triacontatree (left), 9-decabranch-icosetree (center), and 5-pentabranch-9-heptabranch-octadecatree (right).

Following parameters are used in the simulation as shown in Table 1 if not otherwise stated.

Table 1. Simulation parameters.

Parameters	Value
N	5, 9
c_S	10^5
c_B	10^4
c_F	10
c_G	1
Δt, T_{data}	10^{-4}, 10^{-2}
v_w	1, 2, 4, 8, 16, 32, 64
m, l	1, 1
λ_{ijk}	0.0873 - 0.3927

Two configurations are investigated in this work, the first is pentatree and the second is 1-propabranch-hexatree as illustrated in Figure 8.

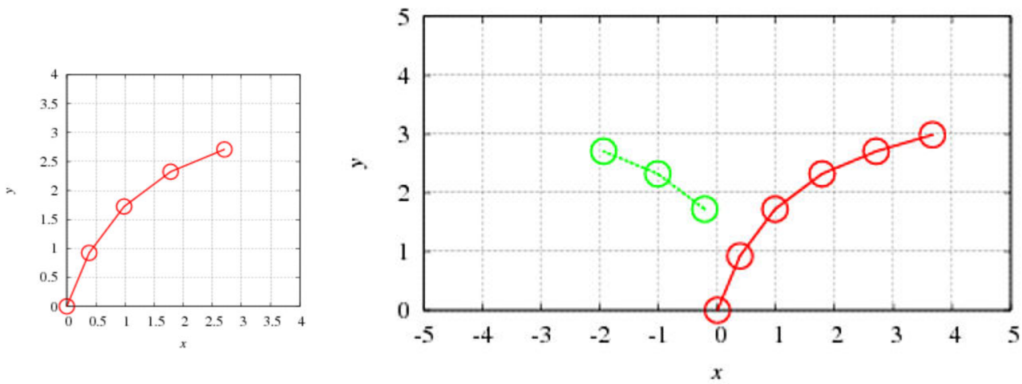


Figure 8. Investigated configurations in this work: pentatree (left) and 1-propabranch-hexatree (right).

Relaxation process

Each configuration is relaxed with certain value of λ_{ijk} as listed in Table 1. Unfortunately not every value of λ_{ijk} can give a stable relaxed configuration, since it depends also on number of particle N. Pentatree configuration has N = 5, while 1-propabranch-hexatree has N = 9. Smaller values of λ_{ijk} give more stable configuration. Samples of stable and unstable configurations are given in Figures 9 and 10, respectively. For tree with branch, e.g. the 1-propabranch-hexatree, each branch could have different stability as shown in Figure 11.

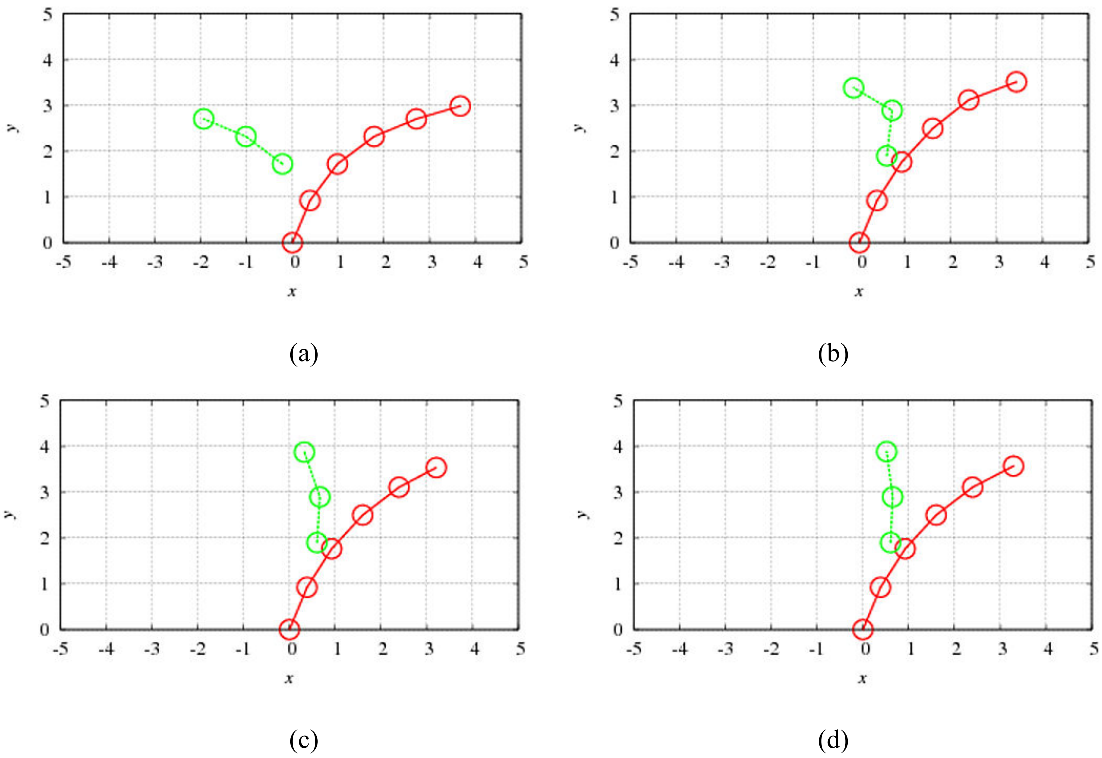


Figure 9. Stable configuration of 1-propabranch-hexatree for $\lambda_{ijk} = 0.0873$ at time t: (a) 0, (b) 100, (c) 125, and (d) 150.

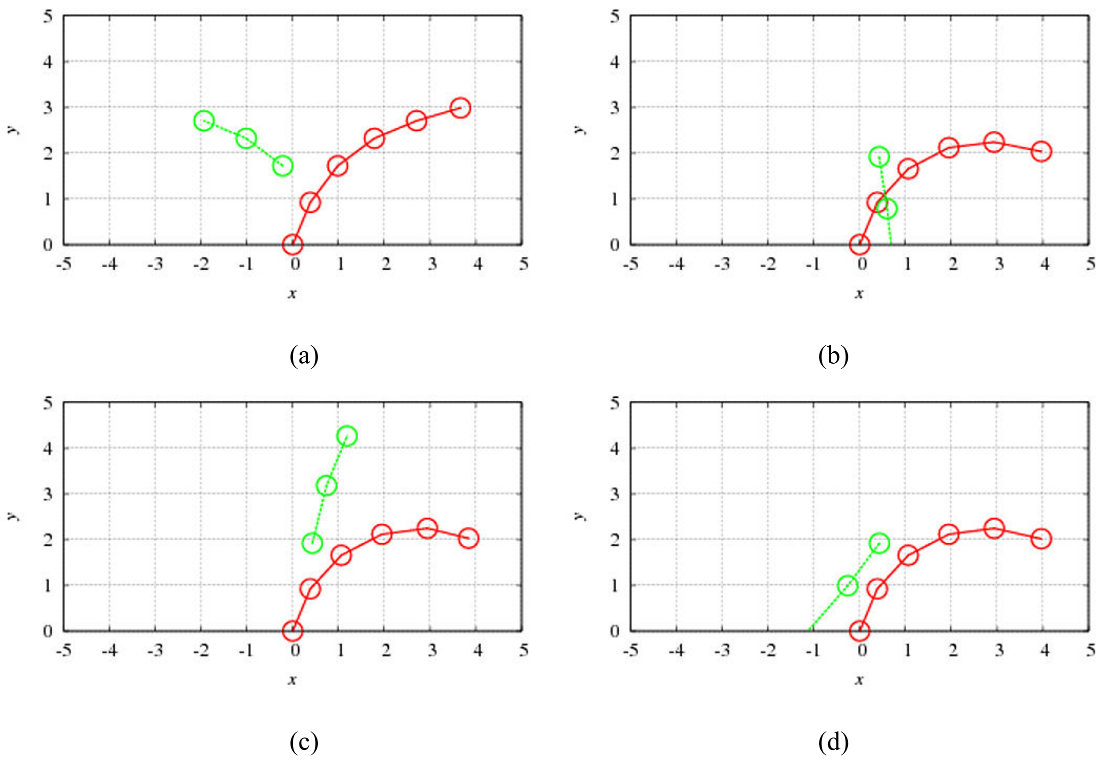


Figure 10. Unstable configuration of 1-propabranch-hexatree for $\lambda_{ijk} = 0.1745$ at time t: (a) 0, (b) 100, (c) 125, and (d) 150.

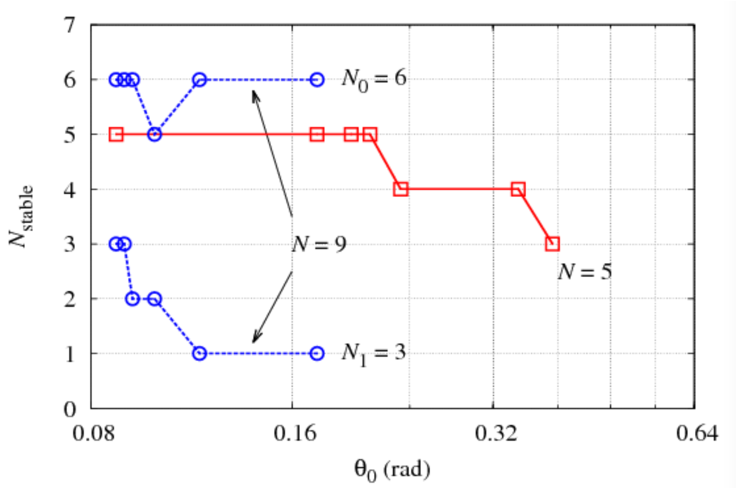


Figure 11. Investigated configurations in this work and influence of λ_{ijk} or θ_0 for: pentatree (left) and 1-propabranch-hexatree (right).

Wind influence

Configuration shown in Figure 10 is the point near $N_1 = 3$ in Figure 11, where only one particle is stable in the 1 branch. The main trunk is indicated with $N_0 = 6$, which is stable. After get stable configuration a wind with velocity from 1 to 64 as listed in Table 1 is given to the system. By assuming that only far end particle is vibrated, then only this particle is observed.

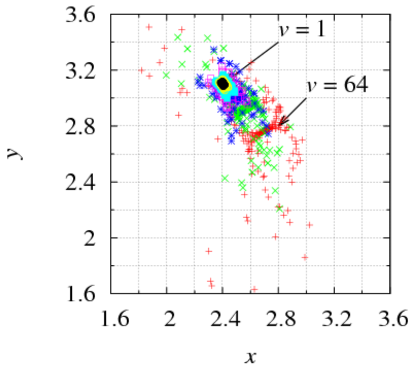


Figure 12. Position of last particle ($i = 4$) in the pentatree for different value of wind velocity v.

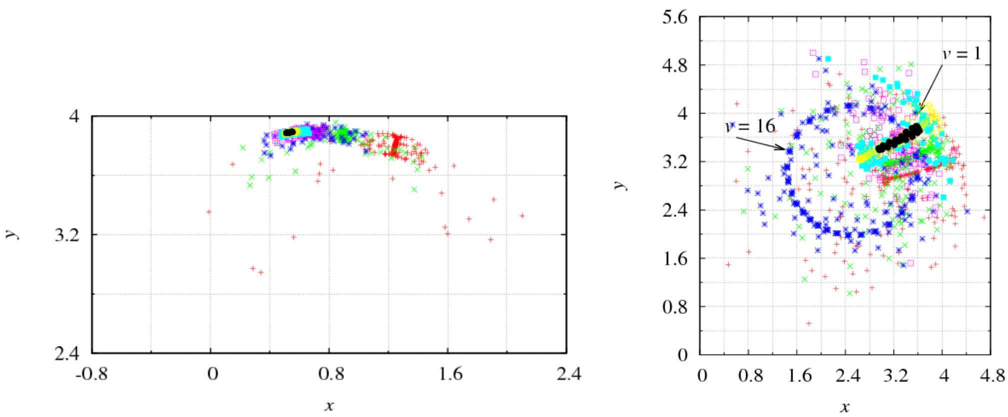


Figure 13. Position of last particle ($i = 8$, left and $i = 5$, right) in the 1-propabranch-hexatree for different value of wind velocity v.

It can be seen from Figure 12 that the particle vibrate with higher amplitude for high wind velocity, but a periodic vibration is not observed. And for the 1-propabranch-hexatree position of particle in trunk and the first branch is given in Figure 13. Even the configuration is already stable but due to wind velocity through Equation (9), it can be unstable or at least it vibrates. The circular particle path in Figure 13 (left) is about two times of l, which gives information that the last two particles become unstable instead only the last particles.

SUMMARY

A tree model based on granular particle has been constructed. Relaxation process small value of λ_{ijk} in order to give stable tree configuration. Wind with higher velocity will turn the stable configuration into unstable one, while small wind only vibrates the last particle slightly.

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