Packaged entanglement states and particle teleportation

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The entanglement states of particles are now widely used in quantum communication. However, these entanglement states usually relate to only one of the particles' physical quantities. Here we theoretically show that there exists a packaged entanglement state which encapsulates all the necessary physical quantities for completely identifying the particles. We first show that a particle-antiparticle pair can form a packaged entanglement state in which the particles are indeterminate. Thereafter, we gave a possible experimental scheme for testing the packaged entanglement state. Finally, we proposed a protocol for teleporting a particle to an arbitrarily large distance using the packaged entanglement states. These packaged entanglement states could be useful for matter teleportation, medicine, remote control, and energy transfer.

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I. Introduction.— An entanglement state usually refers to the quantum state of a composite system which cannot be expressed as the direct product of the quantum states of its subsystems.[1–3] A common character of the entanglement states [4] is that a measurement on one of the particles will immediately change the state of other particles in the system via the "spooky action at a distance" [5] no matter how far these particles are spatially separated. The applications of the entanglement states in quantum communication [6] are now realized with photos [7, 8], electrons [9–11], atoms [12, 13], ions [14–16], and superconductors circuits [17, 18]. These entanglement states usually relate to one [7–18] or two [19] of the particle's physical quantities, such as spin, or polarization (of photons).

However, one may ask whether the particles can form a special entanglement state that packages in multiple physical quantities capable of completely identifying the particles? We shall call such a state as a "packaged entanglement state" with which one can teleport the entire particles instead of just one of its physical properties [20–22]. The packaged entanglement state may enable matter teleportation which is well described in science fiction [23]. In the matter teleportation process, every intrinsic properties of the particles need to be transmitted for the purpose of rematerialization. Therefore, the packaged entanglement state could be a promising candidate for accomplishing the mission.

On the other hand, it is known that many physical quantities (or freedoms) of the particles usually are independent, such as spin, baryon number, lepton number etc.[24, 25] But these physical quantities may be entangled together in the packaged entanglement state. Thus, it is highly desirable to carry out an in-depth investigation on the packed entanglement state and therefore answer at least the following questions: What interesting properties does this packaged entanglement state have? How to test it with experiments? Are there any other important applications for it?

In this paper, we first constructed the mathematical expression for the packaged entanglement state using the charge conjugation operator. Next, we proposed a possible experimental method for testing whether the particle-antiparticle is in the packaged entanglement state. Thirdly, we discussed how to teleport a particle to a place at a distance using the packaged entanglement state. Finally, we discussed the possible applications of the particle teleportation process in medicine, remote control, and targeted energy transfer.

II. Packaged Entanglement States of A Particle-antiparticle Pair.— For simplicity, we will choose a particle-antiparticle pair to study the packaged entanglement states and their applications. First, let us discuss how to construct the mathematical expression for the packaged entanglement state of the particle-antiparticle pair (with a zero total charge). This can be achieved by referring to the charge conjugation.

It is known that a particle and its antiparticle are symmetrical in the sense of charge, i.e., the charge of the particle and its antiparticle are equal in quantity but with opposite signs. For this reason, we shall call the charge as the gender of a particle. From particle physics [24–26] we know that a particle and its antiparticle can be interchanged by the charge conjugation operator C. Let us denote the quantum state of a fermion as $|f\rangle$ and its antiparticle as $|\bar{f}\rangle$. Thus, we have $C|f\rangle = |\bar{f}\rangle$ and $C|\bar{f}\rangle = |f\rangle$. Similarly, for a boson $|b\rangle$ and its antiparticle $|\bar{b}\rangle$, we have $C|b\rangle = |\bar{b}\rangle$ and $C|\bar{b}\rangle = |b\rangle$. This mutual transformation indicates that a particle and its antiparticle could be mixed together in the sense of wave function under certain conditions.

Although the name "charge conjugation" is used, it not only reverses the sign of the particle's electric charge (Q), but also reverses the sign of all other internal quantum numbers [24], i.e., baryon number (B), lepton number (L), isospin (I_3), charm (C), strangeness (S), topness (T), and bottomness (B'). All these internal quantum numbers are packaged together under the charge conjugation. Finally, it should be mentioned that the charge conjugation does not change the sign of mass, energy, momentum, and spin.

It is known that not all particle (antiparticle) states, but only those neutral systems (with zero total charges) are the eigenstates of the charge conjugation operator C, such as γ, π^0, and a "particle-antiparticle" pair etc. It can be shown that [24], applying C to the bound state of a "particle-antiparticle" pair, one has

$$\begin{aligned} C\left|b\bar{b}\right\rangle &= (-1)^L \left|b\bar{b}\right\rangle, \quad \text{for bosons} \\ C\left|f\bar{f}\right\rangle &= (-1)^{L+S} \left|f\bar{f}\right\rangle, \quad \text{for fermions.} \end{aligned} \tag{1}$$

where L is the angular momentum quantum number and S is the total spin quantum number.

Eq.(1) shows that a "particle-antiparticle" pair, $\left|f\right\rangle\left|\bar{f}\right\rangle$ (or $\left|b\right\rangle\left|\bar{b}\right\rangle$), is the eigenstate of charge conjugation operator C. Apparently, the reversed configuration $\left|\bar{f}\right\rangle\left|f\right\rangle$ (or $\left|\bar{b}\right\rangle\left|b\right\rangle$) is also the eigenstate of C. Thus, the linear combination (superposition) of $\left|f\right\rangle\left|\bar{f}\right\rangle$ and $\left|\bar{f}\right\rangle\left|f\right\rangle$ (or $\left|b\right\rangle\left|\bar{b}\right\rangle$ and $\left|\bar{b}\right\rangle\left|b\right\rangle$) is also the eigenstates of C. More specifically, for a "particle-antiparticle" pair A and B, the following superposed states are the eigenstates of C,

$$\begin{aligned} \left|\Psi^b\right\rangle_{AB} &= \frac{1}{\sqrt{2}}\left(\left|b\right\rangle_A\left|\bar{b}\right\rangle_B + \left|\bar{b}\right\rangle_A\left|b\right\rangle_B\right), \quad \text{for bosons} \\ \left|\Psi^f\right\rangle_{AB} &= \frac{1}{\sqrt{2}}\left(\left|f\right\rangle_A\left|\bar{f}\right\rangle_B - \left|\bar{f}\right\rangle_A\left|f\right\rangle_B\right), \quad \text{for fermions.} \end{aligned} \tag{2}$$

The quantum states in Eq.(2) are entanglement states because they cannot be expressed as the direct product of the particle state and its antiparticle state.[1, 2] A fundamental character of the entanglement states in Eq.(2) is that each particle in the particle-antiparticle pair (A or B) is indeterminate. In other words, each particle in the pair is partially a particle and partially an antiparticle. Therefore, one cannot tell which is the particle and which is its antiparticle. When performing a measurement on A, it will collapse into either a particle, or an antiparticle. If A collapse into a particle, then B will collapse into an antiparticle. If A collapse into an antiparticle, then B will collapse into a particle.

On the other hand, one can easily show that [27] $C = C^\dagger$. This means that C is a Hermitian operator and is therefore an observable physical quantity. As the eigenstates of C, therefore, the entanglement states $\left|\Psi^b\right\rangle_{AB}$ and $\left|\Psi^f\right\rangle_{AB}$ in Eq.(2) must exist.

As mentioned before, the charge conjugation packages in a number of quantum numbers $(Q, B, L, I_3, C, S, T, B')$. Thus, the entanglement states constructed in Eq.(2) should involve all these quantum numbers. Due to this reason, we call the quantum states in Eq.(2) as packaged entanglement states. In addition, the physical quantities packaged in by the charge conjugation should entangle together in the packaged entanglement states. This feature can be embodied by rewriting Eq.(2). For example, the thermionic state can be formally rewritten as

$$\begin{aligned} \left|\Psi^f\right\rangle_{AB} =& \frac{1}{\sqrt{2}}\left(\left|Q, B, L, I_3, C, S, T, B'\right\rangle_A \left|-Q, -B, -L, -I_3, -C, -S, -T, -B'\right\rangle_B \right. \\ & \left. - \left|-Q, -B, -L, -I_3, -C, -S, -T, -B'\right\rangle_A \left|Q, B, L, I_3, C, S, T, B'\right\rangle_B\right). \end{aligned} \tag{3}$$

Eq.(3) shows that the total quantum states cannot be written as the product of the sub quantum states related to the individual quantum numbers (freedoms). The same is for the bosonic state $\left|\Psi^b\right\rangle_{AB}$. On the other hand, there are also other physical quantities, i.e., mass, energy, momentum, and spin, which are untouched by the charge conjugation. Thus, the sub quantum states related to these physical quantities could be factored out.

Finally, we would like to mention that Eq.(2) is valid for both elementary particles and composite particles by referring to the charge conjugation. For composite particles, Eq.(2) can be further expressed by their component particles. For example, a proton is a composite particle made of three quarks [24, 25], i.e., $\left|p\right\rangle = \left|uud\right\rangle$, where u is up quark and d is down quark. Thus, the packaged entanglement state of a $p-\bar{p}$ pair can be written as

$$\begin{aligned} \left|\Psi^p\right\rangle_{AB} &= \frac{1}{\sqrt{2}}\left(\left|p\right\rangle_A\left|\bar{p}\right\rangle_B - \left|\bar{p}\right\rangle_A\left|p\right\rangle_B\right) \\ &= \frac{1}{\sqrt{2}}\left(\left|uud\right\rangle_A\left|\bar{u}\bar{u}\bar{d}\right\rangle_B - \left|\bar{u}\bar{u}\bar{d}\right\rangle_A\left|uud\right\rangle_B\right) \end{aligned} \tag{4}$$

III. Possible Verification Scheme.— Although the experimental method for creating the particle-antiparticle pairs in the packaged entanglement state of Eq.(2) is unavailable at present, we can still use the following experimental scheme to test whether a particle-antiparticle pair is in the packaged entanglement state.

Eq.(2) shows that each particle (A or B) in the entanglement states is a mixture of a particle and an antiparticle. When A (or B) encounters an external particle X (from a particle source), the particle-antiparticle annihilation phenomenon [28, 29] will force A (or B) to collapse into a particle conjugating to particle X (with every internal quantum number of A opposite to that of X) and then annihilate each other. More specifically, if X is a particle, then A will collapse into an antiparticle; if X is an antiparticle, then A will collapse into a particle. This indicates that each particle in the packaged entanglement state can annihilate with both a particle and an antiparticle. This property could be used to test the existence of the packaged entanglement states.[30]

Suppose that Alice has invented an experimental method [1] to create the particle-antiparticle pairs in the packaged entanglement states of Eq.(2). Now Alice use this method to create a particle-antiparticle pair and then send one of the particles (particle A) to annihilate with an external similar particle X (see Fig. 1). Repeating this procedure for a number of times, Alice then starts to calculate the particle annihilating rate.

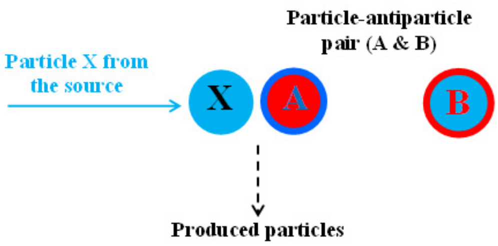

FIG. 1: (Color online) Schematic diagram for testing the packaged entanglement state of a particle-antiparticle pair. If the annihilation rate is 1, then the pairs are in the packaged entanglement states. If the annihilation rate is 0.5, then the pairs are in the normal states.

(a) If the particle-antiparticle pair is in the packaged entanglement state, then the annihilation rate would be 1. This is because each particle in the pair is indeterminate, therefore, it can certainly annihilate with the external particle X (it doesn't matter whether X is a particle or an antiparticle).

(b) If the pair is in a normal state, then the annihilation rate would be 0.5. This is because each particle in the pairs is determinate, therefore, the probability for particle A to be a particle or an antiparticle is 0.5, respectively. Thus, only half of the particles A can annihilate with the external particles X.

Finally, Alice can tell whether the particle-antiparticle pairs are in the packaged entanglement states by checking the annihilation rate.

IV. Particle Teleportation.— We have constructed the mathematical expression for the packaged entanglement states. We shall now use the packaged entanglement state and particle-antiparticle annihilation phenomenon to perform a possible particle teleportation (see Fig. 2). This is the foundation for matter teleportation. Here the particle teleportation does not mean to transmit a particle to the receiver (from Alice to Bob), but only transmit the packaged quantum information carried by the particle to the receiver.[23] We will choose a fermionic system ($|f\rangle$ and $|\bar{f}\rangle$) to carry out the study. For the convenience of discussion, our teleportation protocol is divided into 5 steps.

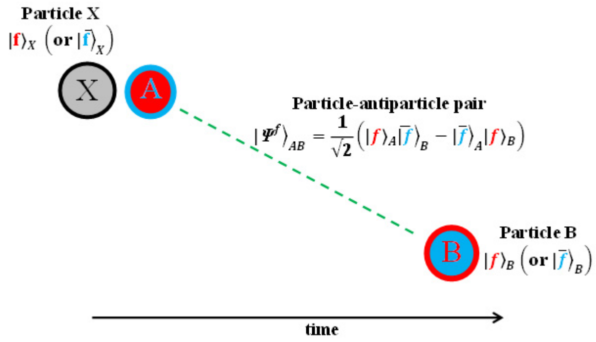

FIG. 2: (Color online) Schematic diagram for particle teleportation using the packaged entanglement state of a fermionic particle-antiparticle pair $\left|\Psi^f\right\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|f\rangle_A \left|\bar{f}\right\rangle_B - \left|\bar{f}\right\rangle_A |f\rangle_B\right)$.

(1) Encoding. Consider that Alice has a particle X (or a sequence of particles) want to teleport to Bob. The particle X can be either a particle or an antiparticle. Without loosing generality, let us write out the state of particle X as

$$|\phi\rangle_X = \alpha\,|f\rangle_X + \beta\left|\bar{f}\right\rangle_X . \tag{5}$$

If X is a particle, then we have $\alpha = 1$ and $\beta = 0$. If X is an antiparticle, then we have $\alpha = 0$ and $\beta = 1$.

(2) Quantum channel creation. To send the information stored on particle X to Bob, Alice needs a quantum channel, i.e., a particle-antiparticle pair in the packaged entanglement state as shown in Eq.(2). For a fermionic system, we have $\left|\Psi^f\right\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|f\rangle_A \left|\bar{f}\right\rangle_B - \left|\bar{f}\right\rangle_A |f\rangle_B\right)$.

After the entangled particle-antiparticle pair is created, one of it (particle A) is sent to Alice and another one (particle B) is sent to Bob. Before Alice carry out any further operation, the complete state of the three particles (X, A, B) is

$$\begin{aligned}\left|\Psi^f\right\rangle_{XAB} =& |\phi\rangle_X \left|\Psi^f\right\rangle_{AB} \\ =& \frac{\alpha}{\sqrt{2}}\left(|f\rangle_X |f\rangle_A \left|\bar{f}\right\rangle_B - |f\rangle_X \left|\bar{f}\right\rangle_A |f\rangle_B\right) \\ &+ \frac{\beta}{\sqrt{2}}\left(\left|\bar{f}\right\rangle_X |f\rangle_A \left|\bar{f}\right\rangle_B - \left|\bar{f}\right\rangle_X \left|\bar{f}\right\rangle_A |f\rangle_B\right)\end{aligned} \tag{6}$$

(3) Sending. With the quantum channel, Alice can send out her information stored on particle X by letting particle X to annihilate with particle A (dematerialization in matter teleportation). Due to the particle-antiparticle annihilation phenomenon, A will collapse into a particle conjugating to X and then they will annihilate each other. Thus, the $\left|\Psi^f\right\rangle_{XAB}$ in Eq.(6) will collapse into an expression only with terms $|f\rangle_X \left|\bar{f}\right\rangle_A$ and $\left|\bar{f}\right\rangle_X |f\rangle_A$, i.e.,

$$\begin{aligned}\left|\Psi^f\right\rangle'_{XAB} &= -\alpha\left(|f\rangle_X \left|\bar{f}\right\rangle_A\right)|f\rangle_B + \beta\left(\left|\bar{f}\right\rangle_X |f\rangle_A\right)\left|\bar{f}\right\rangle_B \\ &= -\alpha\,|Ps\rangle_{XA}\,|f\rangle_B + \beta\,|Ps\rangle_{XA}\left|\bar{f}\right\rangle_B .\end{aligned} \tag{7}$$

where $|Ps\rangle_{XA}$ is the particles produced by the $|f\rangle_X \left|\bar{f}\right\rangle_A$ (or $\left|\bar{f}\right\rangle_X |f\rangle_A$) annihilation [28].

(4) Receiving. Bob can receive the packaged information from Alice because Bob's particle B becomes related to particle X after Alice sent out her information by annihilating X and A. More specifically, if X is a particle, i.e., $|\phi\rangle_X = |f\rangle_X$, then Eq.(7) becomes

$$\left|\Psi^f\right\rangle'_{XAB} = -\,|Ps\rangle_{XA}\,|f\rangle_B\,, \tag{8}$$

and B become a particle identical to X (with a negative sign); if X is an antiparticle, i.e., $|\phi\rangle_X = \left|\bar{f}\right\rangle_X$, then Eq.(7) becomes

$$\left|\Psi^f\right\rangle'_{XAB} = |Ps\rangle_{XA}\left|\bar{f}\right\rangle_B\,, \tag{9}$$

and B become an antiparticle identical to X.

(5) Decoding. After received Alice's packaged information, Bob needs to decode it (rematerialization in matter teleportation). Eq.(8) and Eq.(9) show that Bob's particle B is identical to particle X. This means that Bob can successfully decode the packaged information (carried by particle X) sent to him by Alice.

Similarly, one can repeat the above particle teleportation process using the packaged entanglement state of bosons, $\left|\Psi^b\right\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|b\rangle_A \left|\bar{b}\right\rangle_B + \left|\bar{b}\right\rangle_A |b\rangle_B\right)$. Eq.(7) becomes

$$\left|\Psi^b\right\rangle'_{XAB} = \alpha\,|Ps\rangle_{XA}\,|b\rangle_B + \beta\,|Ps\rangle_{XA}\left|\bar{b}\right\rangle_B . \tag{10}$$

If $|\phi\rangle_X = |b\rangle_X$, then $\left|\Psi^b\right\rangle'_{XAB} = |Ps\rangle_{XA}\,|b\rangle_B$. If $|\phi\rangle_X = \left|\bar{b}\right\rangle_X$, then $\left|\Psi^b\right\rangle'_{XAB} = |Ps\rangle_{XA}\left|\bar{b}\right\rangle_B$.

V. Discussion.— In early quantum teleportation protocols [6, 31], Alice sends out the information of particle X by performing a Bell measurement on particles X and A. This measurement has four possible results. Thereafter, Alice needs a classical channel to inform Bob about her results of the Bell measurement. In the present particle teleportation protocol, however, Alice sent out the information of X by annihilating X with A. Alice's experimental result is fixed by the particle-antiparticle annihilation phenomenon. Thus, Bob's result has a fixed relationship with that of X and he can decode the information directly. The classical channel between Alice and Bob is then removed.

In the packaged entanglement states, every additive quantum number is symmetrical (with opposite sign). Thus, the particle teleportation process satisfies a number of conservation principles, i.e., charge (Q) conservation, baryon number (B) conservation, lepton number (L) conservation, isospin (I_3) conservation, charm (C) conservation, strangeness (S) conservation, topness (T) conservation, and bottomness (B') conservation. In addition, it also satisfies the principles of linear momentum and total energy conservation, and angular momentum conservation.

In the particle teleportation process, Alice can control the gender of Bob's particle B at a distance, or control particle B to be a particle or an antiparticle. This means that Alice can control particle B whether to annihilate or not to annihilate with its environment particles at will. This property may be found important applications in medicine, such as positron emission tomography (PET) [32], positron annihilation spectroscopy (PAS), and the band structure measurements in solid state physics as well. It could also be applied in remote control. Furthermore, the particle-antiparticle annihilation process can release a large amount of energy. Thus, the particle teleportation process could be used to transport energy.

VI. Conclusion.— The properties of packaged entanglement state and its application in matter teleportation is studied. It is shown that the packaged entanglement state of a particle-antiparticle pair are also the eigenstates of the charge conjugation operator. The species of the particles in the packaged entanglement state are indeterminate, but a mixture of a particle and an antiparticle. An experimental scheme for confirming the existence of the packaged entanglement state based on the particle-antiparticle annihilation was also proposed. A protocol for particle teleportation using the packaged entanglement state is also proposed. Different to early studies, the particle teleportation protocol introduced here does not need a classical channel due to the particle-antiparticle annihilation phenomenon.

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[27] Let us denote a particle (a fermion or boson) as $|P\rangle$ and its antiparticle as $|\bar{P}\rangle$. Apply the charge conjugation operator C to $|P\rangle$ twice, we have $C^2|P\rangle = C|\bar{P}\rangle = |P\rangle$. This gives $C^2 = 1$. On the other hand, the normalization condition $\langle P|C^\dagger C|P\rangle = \langle \bar{P}|\bar{P}\rangle = 1$ gives $C^\dagger C = 1$. Thus, we have $C = C^\dagger$.

[28] When a low-energy elementary particle-antiparticle pair annihilates, say an electron annihilates with a positron, they can only produce two or more photons. But when a composite particle-antiparticle pair annihilates, say a proton annihilates with an antiproton, they will produce multiple particles, such as photons, electrons, positrons, and neutrinos.

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