Bell's theorem is silly, false, misleading, interesting

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1 October 2015: A reply to the challenge, "What's your problem with Bell's theorem?"
(For me, a problem is a deviation from an expectation.)

1 Bell's theorem

#1.0. This reply is based on commonsense local-realism (CLR): (i) taking realism to be the view that external reality exists and has definite properties, my analysis is bound by the union of commonsense local-causality (no causal influence propagates superluminally) and commonsense physical-realism (some physical properties change interactively). (ii) all my results accord with a CLR interpretation of quantum mechanics. (iii) no step in my analysis is negated by experiment. (iv) I will show that a naive error in Bell (1964) infects Bell's later work as well as the CHSH (1969) inequality and related derivatives, etc. Taking maths to be the best logic, let's see.

$$. A^{\pm} \equiv \pm 1 = A_i(\mathbf{a}, \lambda_i) \Leftarrow D(\mathbf{a}) \leftarrow p(\lambda_i) \leftarrow S_{EPRB} \rightarrow p(\lambda_i') \rightarrow D(\mathbf{b}) \Rightarrow B_i(\mathbf{b}, \lambda_i') = \pm 1 \equiv B^{\pm}. \quad (1)$$

$$\textbf{Given } A(\mathbf{a}, \lambda_i) = \pm 1 \equiv A^{\pm},\ B(\mathbf{b}, \lambda_i') = \pm 1 \equiv B^{\pm},\ \lambda_i + \lambda_i' = 0, \int \rho(\lambda) d\lambda = 1, \quad (2)$$

$$\textbf{then } \left\langle AB \,|\, Q_{\frac{1}{2}} \right\rangle = \langle AB \,|\, EPRB \rangle \equiv \frac{1}{n} \sum_{i=1}^{n} A(\mathbf{a}, \lambda_i) B(\mathbf{b}, \lambda_i') = -\frac{1}{n} \sum_{i=1}^{n} A(\mathbf{a}, \lambda_i) A(\mathbf{b}, \lambda_i) \neq -\mathbf{a}.\mathbf{b}. \quad (3)$$

#1.1. In my terms — given Bell (1964) and EPRB defined by (1)-(2) — (3) is Bell's theorem. I take it that Bell (2004:65) agrees. Then, consistent with Bell (1964:195), the 'additional variables' [λ] are taken to be EPR-motivated beables that will restore causality and locality to quantum theory. (NB: λ are not EPR 'elements of physical reality' since I reject them and Bell places no such restriction upon the 'additional variables'.) Thus: S_{EPRB} delivers EPRB-correlated particle-pairs; $Q_{\frac{1}{2}}$ denotes the EPRB experiment; Bell's $P(\mathbf{a}, \mathbf{b})$ notation is replaced by $\langle AB \mid Q_{\frac{1}{2}} \rangle$; $\langle AB \rangle$ is used when the conditioning experiment is clear.

#1.2. Detectors D are polarizer-analyzers. The principal-axis of Alice's dichotomic linear-polarizer is oriented $\mathbf{a}$ in 3-space, Bob's $\mathbf{b}$. When clarity requires, beables in Bob's locale are identified by primes ($'$). Thus $\{p(\lambda_i),\ p(\lambda_i') \mid Q_{\frac{1}{2}}, \mathbf{a}, \mathbf{b};\ i = 1, 2, ..., n\}$ is the set built from n random particle-pairs that deliver EPRB results $A^{\pm}$ and $B^{\pm}$ at detectors $D(\mathbf{a})$ and $D(\mathbf{b})$ respectively. n provides an adequate accuracy. Given CLR's compatibility with Einstein-locality and Bell's initial (1964:195) locality (local particle/detector interactions alone yield local results): A_i is determined by $\mathbf{a}$ and λ_i alone, B_i by $\mathbf{b}$ and λ_i' alone. (It is this *local* dynamic that Bell later, with his theorem, rejects. I do not.)

#1.3. Based on Bell (1964): it's a matter of indifference whether λ denotes a single variable or a set, or whether the variables are discrete or continuous. Though λ is little studied by others, I associate spin s with discrete λ. Thus, in (2), my pristine λ_i and λ_i' are correlated by the conservation of angular momentum. Then, as local beables, my λs are random unit-vectors in 3-space with a uniform distribution. It's thus probability zero that any two particle-pairs are the same. Hence probability zero that $\lambda_i = \lambda_j$ or that $\lambda_i = \lambda_{n+i}$ in general.

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#1.4. Of course, were we conducting classical tests on classical objects, then $\lambda_i = \lambda_{n+i}$ would be possible under naive-realism. But neither my local-realism nor the EPRB experiment is constrained by such limiting classicality. Instead, both Bell and I are bound by the search for 'additional variables' [λ] that will 'restore causality and locality to quantum theory', after Bell (1964:195). Thus, given Bell's later rejection of such variables and to be clear regarding Bell's theorem: Proving the validity of λ in (1)-(2), I will refute Bell's mathematical version (3) and his (to me, unrelated) textual version:

> "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz-invariant," Bell (1964:199).

#1.5. I will also refute related naive claims. Including Bell's (1990:5), 'I cannot say that action at a distance is required in physics. But I can say that you cannot get away with no action at a distance.' Goldstein *et al.* (2011), 'experiments establish that our world is non-local.' Maudlin (2014), 'Non-locality is here to stay.' So a central focus here is on experiments and the principle of locality.

> "The paradox of Einstein, Podolsky and Rosen [EPR (1935)] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables [λ]. These additional variables were to restore to the theory causality and locality [Einstein (1949:85); see #3.1 below]. In this note that idea will be formulated mathematically [under EPRB] and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality [...] that creates the essential difficulty," Bell (1964:195). NB: λ is not required to be (and cannot be) an EPR 'element of physical reality'. Sharing Einstein's dissatisfaction with EPR, I reject such elements.

#1.6. The excision [...] reads: 'or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past.' But I take CLR, defined in #1.0 and (1)-(2), to be: (i) broader than Bell's narrow 'precision' here. (ii) as one with Einstein-locality and therefore beyond objection on that score; see Peres (1995:163). (iii) mathematically clean; refuting Bell's *locally causal theorizing/factorizing* – eg, Bell (2004:54, eq. 2) – at #4.8-11.

#1.7. In short: Taking care with nature, I hold a consequence of realism to be this: 'at all times, the set of beables possessed by a system fully determines all relevant probabilities,' after Gisin (2014:1). But I will refute claims like this:

> "... for a realistic theory to predict the violation of some Bell inequalities, the theory must incorporate some form of nonlocality," Gisin (2014:4).

2 Bell's theorem is silly

#2.1. So. To prove his inequality (3) – 'the main result will now be proved' – Bell (1964:197) goes beyond (1)-(3) and invokes $\mathbf{c}$ (a third unit-vector) in three unnumbered equations following his 1964:(14). Number them (14a)-(14c). Then Bell employs a *naively-realistic restriction* to write (14b) = (14a). To see this, let an EPRB experiment distribute $2n$ random particle-pairs equally (for convenience in presentation) over randomized detector settings, such that the particle-sets (see #1.2) are $\{p(\lambda_i),\ p(\lambda'_i) \mid Q_{\frac{1}{2}}, \mathbf{a}, \mathbf{b};\ i = 1, 2, ..., n\}$ and $\{p(\lambda_{n+i}),\ p(\lambda'_{n+i}) \mid Q_{\frac{1}{2}}, \mathbf{a}, \mathbf{c};\ i = 1, 2, ..., n\}$. Then:

$$\text{Bell's (14a)} = \langle AB\rangle - \langle AC\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i) - A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i})] \tag{4}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)[A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) - 1]. \tag{5}$$

#2.2. (5) is the discrete form of Bell's (14a). And I accept Bell's (14b) = (14c). However, doubting that Bell's (14b) = (14a) is valid under CLR and EPRB, I have: (5) = (14a) $\stackrel{?}{=}$ (14b) = (14c).

$$\text{That is, based on Bell's (14b)-(14c)-(15):}\ \ \langle BC\rangle \equiv -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_{2n+i})A(\mathbf{c},\lambda_{2n+i}) \tag{6}$$

$$\stackrel{?}{=} -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i});\ \text{from (5)} = \text{from Bell's (14a).} \tag{7}$$

#2.3. Alas, to remove the **?** from (7) and justify his (14b) = (14a), Bell requires the (under EPRB) probability zero relation $\lambda_i = \lambda_{n+i}$ (see #1.2-3). So here are two genuine EPRB-based inequalities:

$$\text{Bell 1964: (14b)} \neq \text{Bell 1964: (14a).} \tag{8}$$

$$-\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) \neq -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_{2n+i})A(\mathbf{c},\lambda_{2n+i}) = \langle BC\rangle\,. \tag{9}$$

#2.4. Then. Since Bell's (14b) = (14a) is false under EPRB, (8) is the source of the false inequality in (3). Thus the source of Bell's theorem has nothing to do with locality, separability, spooky-action, etc. Rather: Bell's theorem arises from the use of naive-realism in the context of EPRB! An error exposed by (9). An error at the heart of all Bellian/CHSH thinking known to me – eg, Peres (1995:162-165), Mermin (2005)/Gill, Hensen *et al.* (2015) – as we'll see. Here's Bell (2004:147), for example:

> "To explain this dénouement [eg, in Bell 1964:(14)-(15); the subject of (4)-(9) above] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."
>
> And here's d'Espagnat (1979:166), recast for EPRB: 'One can infer that in *every particle-pair*, one particle has the property A^+ and the other has the property A^-, one has property B^+ and one B^-, and one has property C^+ and one C^-. Such conclusions require a subtle ... extension of the meaning assigned to our notation A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis A would give the definite result A^+, then that particle is said to have the property $A+$. In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times. ... This view is contrary to the conventional interpretation of quantum mechanics.'

#2.5. Now, as stated above at #1.4: were we conducting classical tests on classical objects, then $\lambda_i = \lambda_{n+i}$ would be possible; ie, no longer probability zero. And the Bell-d'Espagnat explanation would then hold routinely (not profoundly). But here's Bell (in 1987) joining the founding fathers and me against his view in #2.4:

> "... the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'. It seems to me that full appreciation of this would have aborted most of the 'impossibility proofs' [like Bell's theorem], and most of 'quantum logic'," Bell (2004: xi-xii).

#2.6. Agreeing, especially re my insertion [...], I re-read Bell (2004). Then, based on Bell's thoroughgoing (1964-1990) naive-realism in the context of EPRB-style experiments, and given its continuing consequences – in CHSH, Peres, Mermin, Gisin, etc – I again conclude: Bell's theorem is silly, and all Bellians are being rather silly! A conclusion (and phrasing) foreshadowed in Bell's later thinking:

> "Now, it's my feeling that all this action at a distance and no action at a distance business will go the same way [eg, as the ether]. But someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. *Maybe someone will just point out that we were being rather silly*, and it won't lead to a big new development. But anyway, I believe the questions will be resolved," Bell (1990:9) with added emphasis.

#2.7. I agree. However. Given this particular Bellian problem, a loophole remains: the naive-realism associated with λ in Bell's proof – when it's not in his premises or the related experiment – is no proof that Bell's theorem is false. So – recalling my commitment to Bell (1964:195) and Einstein: λs are additional variables that restore causality and locality to quantum theory – I provide that proof next.

3 Bell's theorem is false

#3.1. I now prove Bell's theorem false and thereby refute a common Bellian implication:

> "Einstein argued that the EPR correlations could be made intelligible only by completing the quantum mechanical account in a classical way. But detailed analysis shows that any classical account of these correlations has to contain just such a 'spooky action at a distance' [Einstein in Born (1971:158)] as Einstein could not believe in. [For Einstein believed]:
>
> > 'But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former,' Einstein (1949:85).
>
> If nature follows quantum mechanics in these correlations, then Einstein's conception of the world is untenable," Bell (2004:86).

#3.2. Now Bell's premise is true: nature does indeed follow quantum mechanics in EPRB correlations. But Bell's conclusion is false: as will be seen by my CLR completion of the quantum mechanical account. So. In the face of unknowns like λ_i, I begin with classical probability theory (the science of logical inference) and a thought-experiment Q_s.

#3.3. Based on particles with spin $s = \frac{1}{2}$ or 1, Q_s is designed to take me from old certainties to new. For me, old certainties are provided and confirmed by experiments – eg, Aspect's (2002) experiment (denoted Q_1, the subscript indicating the related spin s) – as I seek new certainties re $Q_{\frac{1}{2}}$ (EPRB). Thus. As in Aspect (2002) and EPRB, the expectation for Q_s is:

$$\langle AB \mid Q_s \rangle \equiv P(A^+B^+ \mid Q_s) - P(A^+B^- \mid Q_s) - P(A^-B^+ \mid Q_s) + P(A^-B^- \mid Q_s) \tag{10}$$

$$= 4P(A^+B^+|Q_s) - 1 \ (11.1) = 4P(A^+|Q_s)P(B^+|Q_sA^+) - 1 \ (11.2) = 2P(B^+|Q_sA^+) - 1 \ (11.3). \tag{11}$$

#3.4. To be clear on a crucial point in the context of Bell's theorem: consistent with CLR (#1.0), no causal influences are invoked, required or implied in (10)-(11). Then, identifying the sub-equalities in (11) as (11.1)-(11.3):

#3.5. Given (10): (11.1) follows via the symmetry of the Q_s-state; ie,

$$P(A^+B^+ \mid Q_s) = P(A^-B^- \mid Q_s);\ P(A^+B^- \mid Q_s) = P(A^-B^+ \mid Q_s). \tag{12}$$

#3.6. Given (11.1): (11.2) can never be false in classical probability theory.

#3.7. Given (11.2): (11.3) follows via $P(A^+|Q_s) = P(B^+|Q_s) = \frac{1}{2}$: since λ is a random variable.

#3.8. Given (11.3) – with CLR, λ and quantum theory – Q_s delivers:

$$\langle AB\,|Q_s\rangle = 2P(B^+|Q_sA^+) - 1 = \cos 2s(\pi \pm (\mathbf{a}, \mathbf{b})), \tag{13}$$

$$\therefore P(B^+|Q_sA^+) = \tfrac{1}{2}(1 + \cos 2s(\pi \pm (\mathbf{a}, \mathbf{b}))) : \tag{14}$$

$$P(B^+|Q_1A^+) = \cos^2(\mathbf{a}, \mathbf{b}) \rightarrow \textbf{in agreement with Aspect's (2002) experiment}. \tag{15}$$

$$P(B^+|Q_{\frac{1}{2}}A^+) = \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}) \rightarrow \textbf{a prediction for EPRB, the experiment in Bell (1964)}. \tag{16}$$

#3.9. To be clear: with certainty, (16) will be adequately confirmed under EPRB, just as (15) is adequately confirmed under Aspect (2002). So, with certainty, (16) leads to (3) being corrected to:

$$\left\langle AB\,|Q_{\frac{1}{2}}\right\rangle \equiv \langle AB\,|EPRB\rangle = \text{Bell's (1964) } P(\mathbf{a}, \mathbf{b}) = -\mathbf{a}.\mathbf{b}; \tag{17}$$

ie, an equality replaces Bell's inequality in (3); with (17) confirmed by substituting $s = \frac{1}{2}$ in (13).

#3.10. Supporting Einstein's argument for completing the quantum mechanical account of EPRB correlations in a classical way, I conclude: Bell's theorem is false. A result that delivers: (i) Bell's (2004:167) hope for a simple constructive model of reality based on local causality. (ii) Bell's (1990:10) expectation that relativity and quantum mechanics would one day be reconciled (see #5.1).

4 Bell's theorem is misleading

#4.1. Much research on Bell's theorem follows Bell's naive-realism (#2.4) into error. Examples to be addressed here (some in passing) include: CHSH. Peres (1995) and the related Mermin (2005) with Gill. Goldstein *et al.* (2011), 'In light of Bell's theorem, [many] experiments ... establish that our world is non-local. This conclusion is very surprising, since non-locality is normally taken to be prohibited by the theory of relativity.' Maudlin (2014), 'Non-locality is here to stay ... the world we live in is non-local.' Hensen *et al.* (2015). Even 't Hooft (2014): whose author 'did not refute Bell's theorem but by-passed it by accepting superdeterminism,' after G 't Hooft (2014, pers. comm., 1 July).

#4.2. But few err more than Bell when he is misled to 'another theorem'. So I now address Bell's *local inequality theorem*: foreshadowed by Bell in 1987 (see Bell 2004: xii) and delivered in 1990 (Bell 2004: Ch. 24). Significantly, as will be shown, the theorem relies on falsely factoring a probability distribution to deliver the (naive, as will be shown) CHSH inequality (see #4.12-14).

#4.3. According to classical probability theory (#3.6): since (11.1) is true, (11.2) never can be false. Yet Bell repeatedly and mistakenly rejects such expressions, even in his final essay: see Bell (2004:243) and the move there from his (9) to his (10); ie, Bell equates *causal independence* to *statistical independence* as a consequence of *local causality*. A view most gardeners with adjoining crops reject. For, in keeping with CLR (#1.0): *correlated causes*, not *direct causation*, link *causally independent results* (no mutual influence) like those in (1)-(2) to local-realistic correlations like those in (13)-(16).

#4.4. Indeed: given (11.1), the *slightest correlation* calls forth that never-can-be-false (11.2). And Bell recognizes the centrality of *correlation* (which is by no means slight) in EPRB:

> Recasting Bell (2004:208) in line with (1)-(3): "There are no 'messages' in one system from the other. The inexplicable [sic] correlations of quantum mechanics do not give rise to signalling between noninteracting systems. Of course, however, there may be correlations (eg, those of EPRB) and if something about the second system is given (eg, that it is the other side of an EPRB setup) and something about the overall state (eg, that it is the EPRB singlet state) then inferences from events in one system [eg, A^+ from Alice's detector] to events in the other [eg, B^+ from Bob's detector] are possible."

#4.5. So. Putting it plainly: in EPRB, under classical probability theory, the correlation between A^+ and B^+ *demands* (11.3). And in this way the following issue is resolved.

> "One general issue raised by the debates over locality is to understand the connection between stochastic independence (probabilities multiply) [ie, $P(XY) = P(X)P(Y)$] and genuine physical independence (no mutual influence) [ie, there is no mutual influence between $A_i^+(\mathbf{a}, \lambda_i)$ and $B_i^+(\mathbf{b}, \lambda_i')$]. It is the latter that is at issue in 'locality,' but it is the former that goes proxy for it in the Bell-like calculations. We need to press harder and deeper in our analysis here," Arthur Fine, in Schlosshauer (2011:45).

#4.6. Our pressing, thus far, proves the following (contra Bell): when outcomes are correlated as in Q_s, stochastic independence is no proxy for local-causality. So we now press on to finality via Q_c, a classical thought-experiment in which *now-polarized* particles are pair-wise correlated by $\phi_i + \phi_i' = 0$. That is, following Bell's (2004:166) dictum – 'always test your general reasoning against simple models' – Q_c is (with certainty) a classical locally-causal experiment with causally-independent outcomes. [NB: The Q_s-state, invariant under rotations in 3-space, breaches the CHSH inequality. The Q_c-state, with its reduced correlation (invariant under rotations about the line of flight only), does not.]

#4.7. To convert Q_s to Q_c we sandwich the Q_s source between two yoked single-channel linear-polarizers. The polarizers are so coupled that, at all times: their principal-axes are parallel to each other while their common stepped rotation is constrained to be orthogonal to the line of flight of each particle-pair. Thus aligned, the polarizers step randomly (in unison) about the line of flight to orientation ϕ_i (in 2-space) for the i-th test. So, similar to (10)-(11):

$$\langle AB \mid Q_c\rangle \equiv P(A^+B^+ \mid Q_c) - P(A^+B^- \mid Q_c) - P(A^-B^+ \mid Q_c) + P(A^-B^- \mid Q_c) \tag{18}$$

$$= 4P(A^+B^+|Q_c) - 1 \text{ (19.1)} = 4P(A^+|Q_c)P(B^+|Q_cA^+) - 1 \text{ (19.2)} = 2P(B^+|Q_cA^+) - 1 \text{ (19.3).} \tag{19}$$

#4.8. Then, consistent with local causality and causal independence (ie, no mutual influence):

$$P(A^+|Q_c, s, \mathbf{a}, \phi) = \tfrac{1}{2\pi}\int d\phi \cos^2 s(\mathbf{a}, \phi) = P(B^+ \mid Q_c, s, \mathbf{b}, \phi') = \tfrac{1}{2\pi}\int d\phi \cos^2 s(\mathbf{b}, \phi') = \tfrac{1}{2}. \tag{20}$$

$$\therefore \tfrac{1}{4} = P(A^+ \mid Q_c)P(B^+ \mid Q_c) \neq P(A^+B^+ \mid Q_c) = \tfrac{1}{2\pi}\int d\phi \cos^2 s(\mathbf{a}, \phi)\cos^2 s(\mathbf{b}, \phi') \tag{21}$$

$$= \tfrac{1}{8}(2 + \cos 2s(\pi \pm (\mathbf{a}, \mathbf{b}))) = P(A^+ \mid Q_c)P(B^+ \mid Q_cA^+). \tag{22}$$

#4.9. Thus. Comparing LHS (21) with (22), we refute Bellian factorizations as follows. Recasting Bell (2014:243) in terms of #4.8 for easier understanding:

> "Factorization – like $P(A^+|Q_c)P(B^+|Q_c)$ in LHS (21) – is often taken as the starting point of the analysis. I [John Bell] prefer to see it not as the *formulation* of 'local causality', but as a consequence thereof."

#4.10. However. From (20), A^+ and B^+ are (clearly) locally-casual and causally-independent. So the expression $P(A^+B^+ \mid Q_c)$ in (21) is (clearly) an *unfactored locally-causal formulation*. Alas, for Bell's new theorem, failure follows: for the combination of factorization and stochastic independence that Bell seeks is *an impossibilit*y and not in any way *a consequence* of $P(A^+B^+ \mid Q_c)$. That is:

#4.11. The expression $P(A^+ \mid Q_c)P(B^+ \mid Q_c)$ in LHS (21) is refuted by the factorization in (22): for (22) flows directly from that *unfactored locally-causal formulation* $P(A^+B^+ \mid Q_c)$ in (21). Thus, confirming Bell's (2004:239) 'utmost suspicion' regarding his own work toward a locally causal theory:

Bell threw the baby out with the bathwater.

#4.12. Moreover, despite Bell's factorization being rejected as above, further confirmatory trouble follows. For, per Bell (2004: xii): via his *local inequality theorem*, the CHSH (1969) inequality is obtained. Which is to be expected: CHSH's first equation is infected by the same bug (naive-realism) that delivers Bell's first false theorem (3); cf, #2.1-4. With yet more trouble.

#4.13. Let Q_x be any experiment that satisfies CHSH; eg, non-destructive tests on stable classical objects like Bertlmann's socks. Then, after Peres (1995:164, eq. 6.29), the jth pair yields

$$A_jB_j + B_jC_j + C_jD_j - D_jA_j \equiv \pm 2. \tag{23}$$

$$\therefore\ S(Q_x) \equiv |\langle AB|Q_x\rangle + \langle BC|Q_x\rangle + \langle CD|Q_x\rangle - \langle DA|Q_x\rangle| \leq 2. \tag{24}$$

#4.14. Yet, seemingly unaware of the Bell/naive-realism/CHSH relationship: Bellians – eg, Peres (1995), Mermin (2005)/Gill, Hensen *et al.* (2015) – routinely compare the naive Q_x-based CHSH inequality with Q_s-style experiments that deliver telling CLR-based results like this:

$$S(Q_s) = |\frac{1}{n}\sum_{j=1}^{n}[A_jB_j + B_{n+j}C_{n+j} + C_{2n+j}D_{2n+j} - A_{3n+j}D_{3n+j}]| \leq 2\sqrt{2}; \tag{25}$$

four randomized particle-sets being generated (after #1.2, #2.1); no two particle-pairs the same.

#4.15. I take the lesson to be this. Bell's misleading (3): (i) led CHSH to their naively-restricted inequality. (ii) encouraged Bell to seek an alternate route to the CHSH result via his erroneous *local inequality theorem*. (iii) stoked Bell's ambivalence re action at a distance; eg, from Bell (1990):

> "... I cannot say that action at a distance is required in physics. But I can say that you cannot get away with no action at a distance. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly. Well, that's just the fact of the situation; the Einstein program fails, that's too bad for Einstein, but should we worry about that?" (pp.5-6). "And it might be that we have to learn to accept not so much action at a distance, but [the] inadequacy of no action at a distance," (p.6). "And that is the dilemma. We are led by analyzing this situation to admit that in somehow distant things are connected, or at least not disconnected," (p.7). "I don't know any conception of locality which works with quantum mechanics. So I think we're stuck with nonlocality," (p.12). "There is no energy transfer and there is no information transfer either. That's why I am always embarrassed by the word action, and so I step back from asserting that there is action at a distance, and I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is action at a distance," (p.13).

#4.16. Given the extent of Bellian research – eg, Bell, CHSH, Peres, Mermin, Goldstein *et al.* (2011), Maudlin (2014), 't Hooft (2014), etc: all influenced by that underlying naive-realism, yet none corrected in the face of many experimental refutations – I rest my case that Bell's theorem (3) is misleading.

5 Conclusion

#5.1. Given #2.6, #3.10, #4.16 – and reviewing the textual form of Bell's theorem in #1.4 (evidently reflecting Bell's de Broglie-Bohm sympathies) – I conclude as I began in 1989: Bell's theorem is silly, false, misleading and the way is open for a CLR quantum theory. For this next is true:

> The real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former (Einstein 1949): under Q_*, correlated tests $D(\mathbf{a})$ and $D(\mathbf{b})$ on correlated systems like S_1 and S_2 – eg, $p(\lambda_i)$ and $p(\lambda'_i)$ in Q_s or $p(\phi_i)$ and $p(\phi'_i)$ in Q_c – yield correlated results $A^\pm$ and $B^\pm$ without mystery (after Watson 1989).

#5.2. The polarizer orientations **a** and **b** are in 3-space under Q_s and in 2-space (orthogonal to the line of flight) under Q_c. Some understanding of those $A^\pm$ and $B^\pm$ correlations (and the 'collapse' of the wave-function in quantum theory to yield A^+) thus flows from (14) and (22) with (20):

$$P(B^+ \mid Q_sA^+) = \tfrac{1}{2}(1 + \cos 2s(\pi \pm (\mathbf{a}, \mathbf{b}))). \; P(B^+|Q_cA^+) = \tfrac{1}{2}(1 + \tfrac{1}{2}\cos 2s(\pi \pm (\mathbf{a}, \mathbf{b}))). \tag{26}$$

$$P(B^+ \mid Q_sA^+) - P(B^+|Q_cA^+) = \tfrac{1}{4}\cos 2s(\pi \pm (\mathbf{a}, \mathbf{b})). \tag{27}$$

#5.3. The debate about defective premises in Bell's theorem is resolved satisfactorily: there are none. For the departure from CLR (#1.0) arises from the bowels of Bell's naive-realistic analysis (see #2.3-4); and not otherwise. Thus local-realism – so poorly defined in the Bellian literature; so little attention paid to the nature of λ; and notwithstanding 't Hooft, Aspect, Bell, CHSH, d'Espagnat, Gisin, Goldstein *et al*, Hensen *et al*, Maudlin, Mermin/Gill, Peres – survives as commonsense local-realism.

#5.4. As to Bell's theorem being interesting; that topic awaits another day. In the meantime, and relatedly, Hanson (2015) makes interesting reading as we look forward to Q_s-style tests being routinely compared to the predictions of a commonsense local-realistic quantum theory; absent CHSH.

#5.5. It's a pleasure to again thank Michel Fodje for many helpful exchanges.

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