General relativity and the other gravity field equation and Solution

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ABSTRACT
In the general relativity theory, we find the other gravity field equation and the other solution. For example, the other solution treats in Schwarzschild solution, Reissner-Nodstrom solution. Hence, the uniqueness of GR solution is denied by numberless solutions.

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1. Introduction

In the general relativity theory, our article’s aim is that we find the other gravity field equation and the other solution.

First, the gravity potential $g_{\mu\nu}$ is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

(1)

In gravity potential $g_{\mu\nu}$, we introduce tensor $f_{\mu\nu}$ and scalar $K$.

$$f_{\mu\nu} = Kg_{\mu\nu} , \quad \frac{\partial K}{\partial x^\lambda} = 0$$

$$ds^2 = f_{\mu\nu}dx^\mu dx^\nu = Kg_{\mu\nu} \frac{\partial x^\mu}{\partial x^{\lambda\alpha}} \frac{\partial x^\nu}{\partial x^{\lambda\beta}} dx^{\lambda\alpha} dx^{\lambda\beta}$$

$$= Kg_{\alpha\beta} dx^{\lambda\alpha} dx^{\lambda\beta} = f_{\alpha\beta} dx^{\lambda\alpha} dx^{\lambda\beta}$$

$$g_{\alpha\beta} = g_{\mu\nu} \frac{\partial x^\mu}{\partial x^{\lambda\alpha}} \frac{\partial x^\nu}{\partial x^{\lambda\beta}} , \quad f_{\alpha\beta} = f_{\mu\nu} \frac{\partial x^\mu}{\partial x^{\lambda\alpha}} \frac{\partial x^\nu}{\partial x^{\lambda\beta}}$$

(2)

Inverse gravity potential $g^{\mu\nu}$.

$$f^{\mu\nu}f_{\mu\nu} = \delta^\mu_\nu = (\frac{1}{K} g^{\mu\nu})(Kg_{\mu\nu}) , \quad f^{\mu\nu} = \frac{1}{K} g^{\mu\nu}$$

(3)

In Christoffel symbol $\Gamma^{\rho}_{\mu\nu}$,

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} f^{\rho\lambda} \left( \frac{\partial f_{\mu\lambda}}{\partial x^{\nu}} + \frac{\partial f_{\nu\lambda}}{\partial x^{\mu}} - \frac{\partial f_{\mu\nu}}{\partial x^{\lambda}} \right)$$

$$= \frac{1}{2} \left( \frac{1}{K} g^{\rho\lambda} \right) \left( K \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} + K \frac{\partial g_{\nu\lambda}}{\partial x^{\mu}} - K \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} \right) = \Gamma^{\rho}_{\mu\nu}$$

(4)

Therefore, in the curvature tensor $R^\rho_{\mu\nu\lambda}$,

$$R^\rho_{\mu\nu\lambda} = \frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\rho}_{\mu\lambda}}{\partial x^{\nu}} + \Gamma^{\sigma}_{\mu\nu} \Gamma^{\rho}_{\sigma\lambda} - \Gamma^{\sigma}_{\mu\lambda} \Gamma^{\rho}_{\sigma\nu}$$

$$= \frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\rho}_{\mu\lambda}}{\partial x^{\nu}} + \Gamma^{\sigma}_{\mu\nu} \Gamma^{\rho}_{\sigma\lambda} - \Gamma^{\sigma}_{\mu\lambda} \Gamma^{\rho}_{\sigma\nu} = R^\rho_{\mu\nu\lambda}$$

(5)

In Ricci tensor $R_{\mu\nu}$,

$$R_{\mu\nu} = R^\rho_{\mu\rho\nu} = R^\rho_{\rho\mu\nu} = R_{\mu\nu}$$

(6)

In curvature scalar $R$
\[ R = f^\mu\nu R^\nu_{\mu\nu} = \frac{1}{K} g^\mu\nu R^\nu_{\mu\nu} = \frac{1}{K} R \]  
(7)

Hence, in the gravity field equation of Einstein,

\[ R^\mu_{\nu\mu} - \frac{1}{2} f^\mu_{\nu\mu} R^\nu_{\mu\nu} = R^\mu_{\nu\mu} - \frac{1}{2} K g^\mu_{\nu\mu} \left( \frac{1}{K} R \right) \]

\[ = R^\mu_{\nu\mu} - \frac{1}{2} K g^\mu_{\nu\mu} R = -\frac{8\pi G}{c^4} T^\mu_{\nu\mu} \]  
(8)

In Newtonian approximation, Energy-momentum tensor \( T^\mu_{\nu\mu} \) is

\[ \nabla^2 f_{00} = \nabla^2 K g_{00} \approx -\frac{8\pi G}{c^4} K T_{00} = -\frac{8\pi G}{c^4} T^1_{00} \]  
(9)

\[ \rho c^2 = T_{00}, \quad K \rho c^2 = T^1_{00} \]

\[ T^1_{\mu\nu} = K T^\mu_{\nu\mu} \]  
(10)

Einstein’s gravity field equation is

\[ R^\mu_{\nu\mu} - \frac{1}{2} f^\mu_{\nu\mu} R^\nu_{\mu\nu} = R^\mu_{\nu\mu} - \frac{1}{2} g^\mu_{\nu\mu} R = -\frac{8\pi G}{c^4} T^\mu_{\nu\mu} = -\frac{8\pi G}{c^4} \frac{1}{K} T^\mu_{\nu\mu} \]  
(11)

Therefore, tensor \( f^\mu_{\nu\mu} \) satisfy the gravity field equation of Einstein.

\[ f^\mu_{\nu\mu} \left( R^\nu_{\mu\nu} - \frac{1}{2} f^\mu_{\nu\mu} R^\nu_{\mu\nu} \right) = \frac{1}{K} g^\mu_{\nu\mu} \left( R^\nu_{\mu\nu} - \frac{1}{2} g^\mu_{\nu\mu} R \right) = -\frac{8\pi G}{c^4} \frac{1}{K} g^\mu_{\nu\mu} T^\mu_{\nu\mu} = -\frac{8\pi G}{c^4} \frac{1}{K} T^\lambda_{\nu\mu} \]

\[ = \frac{8\pi G}{c^4} f^\mu_{\nu\mu} \frac{T^\mu_{\nu\mu}}{K} = -\frac{8\pi G}{c^4} \frac{1}{K} T^\lambda_{\nu\mu} \]

\[ \rightarrow - R = \frac{1}{K} R = -\frac{8\pi G}{c^4} \frac{1}{K} T^\lambda_{\nu\mu} = -\frac{8\pi G}{c^4} \frac{1}{K} T^\lambda_{\nu\mu} , \]

\[ T^{\lambda}_{\nu\mu} = T^{\lambda}_{\nu\mu} \]

\[ R^\mu_{\nu\mu} = R^\mu_{\nu\mu} = \frac{8\pi G}{c^4} \left( T^\mu_{\nu\mu} - \frac{1}{2} g^\mu_{\nu\mu} T^\lambda_{\nu\mu} \right) = -\frac{8\pi G}{c^4} \left( \frac{1}{K} - \frac{1}{2} \frac{f^\mu_{\nu\mu}}{K} T^\lambda_{\nu\mu} \right) \]  
(12)

2. The other solution in Schwarzschild solution, Reissner-Nordstrom solution

\[ f_{\mu\nu} = K g_{\mu\nu} , \quad \frac{\partial K}{\partial x^k} = 0 \]

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad ds^2 = f_{\mu\nu} dx^\mu dx^\nu \]  
(13)
For example, \( K \) is
\[
K = 1 + n_2 \exp(-n_1 \frac{hc}{GM^2}) \quad \text{or} \quad K = [1 + n_2 \exp(-n_1 \frac{hc}{GM^2}) \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})]
\]

\( n_1, m_1 > 0, \quad n_1, n_2, m_1, m_2 \) is number

(14)

Schwarzschild solution (vacuum solution) is
\[
ds^2 = -c^2 \left(1 - \frac{2GM}{rc^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\]

(15)

The new solution is
\[
ds^2 = f_{\mu\nu} dx^\mu dx^\nu = Kg_{\mu\nu} dx^\mu dx^\nu = K \cdot ds^2
\]
\[
= [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \left[-c^2 \left(1 - \frac{2GM}{rc^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2\right]
\]
\[
K = 1 + n_2 \exp(-n_1 \frac{hc}{GM^2})
\]

(16)

Reissner-Nordstrom solution is
\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu
\]
\[
= -c^2 \left(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\]

(17)

The new solution is
\[
ds^2 = f_{\mu\nu} dx^\mu dx^\nu = Kg_{\mu\nu} dx^\mu dx^\nu = K \cdot ds^2
\]
\[
= [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})] \cdot [-c^2 \left(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}}
\]
\[
+ r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2\], \quad K = [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})]
\]

(18)

3. Conclusion
We find the other solution in the General relativity theory. Hence, the uniqueness of GR solution is denied by numberless solutions.
Reference