General relativity and the other Solution

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ABSTRACT

In the general relativity theory, we find the other solution.For example, the other solution treats in Schwarzschild solution, Reissner-Nodstrom solution. Hence, the uniqueness of GR solution is denied by numberless solutions.

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1. Introduction

In the general relativity theory, our article's aim is that we find the other solution.

First, the gravity potential $g_{\mu\upsilon}$ is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon \tag{1}$$

In gravity potential $g_{\mu\upsilon}$, we introduce tensor $f_{\mu\upsilon}$ and scalar K.

$$f_{\mu\upsilon} = K g_{\mu\upsilon} \quad , \quad \frac{\partial K}{\partial x^\lambda} = 0$$

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} dx'^\alpha \, dx'^\beta$$

$$= K g'_{\alpha\beta} \, dx'^\alpha \, dx'^\beta = f'_{\alpha\beta} \, dx'^\alpha \, dx'^\beta$$

$$g'_{\alpha\beta} = g_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} \quad , \quad f'_{\alpha\beta} = f_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} \tag{2}$$

Inverse gravity potential $g^{\mu\upsilon}$,

$$f^{\mu\upsilon} f_{\mu\upsilon} = \delta_\mu{}^\upsilon = (\frac{1}{K} g^{\mu\upsilon})(K g_{\mu\upsilon}), \; f^{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} \tag{3}$$

In Christoffel symbol $\Gamma^\rho{}_{\mu\upsilon}$,

$$\Gamma'^\rho{}_{\mu\upsilon} = \frac{1}{2} f^{\rho\lambda} (\frac{\partial f_{\mu\lambda}}{\partial x^\upsilon} + \frac{\partial f_{\upsilon\lambda}}{\partial x^\mu} - \frac{\partial f_{\mu\upsilon}}{\partial x^\lambda})$$

$$= \frac{1}{2} (\frac{1}{K} g^{\rho\lambda})(K \frac{\partial g_{\mu\lambda}}{\partial x^\upsilon} + K \frac{\partial g_{\upsilon\lambda}}{\partial x^\mu} - K \frac{\partial g_{\mu\upsilon}}{\partial x^\lambda}) = \Gamma^\rho{}_{\mu\upsilon} \tag{4}$$

Therefore, in the curvature tensor $R^\rho{}_{\mu\upsilon\lambda}$,

$$R'^\rho{}_{\mu\upsilon\lambda} = \frac{\partial \Gamma'^\rho{}_{\mu\upsilon}}{\partial x^\lambda} - \frac{\partial \Gamma'^\rho{}_{\mu\lambda}}{\partial x^\upsilon} + \Gamma'^\sigma{}_{\mu\upsilon} \Gamma'^\rho{}_{\sigma\lambda} - \Gamma'^\sigma{}_{\mu\lambda} \Gamma'^\rho{}_{\sigma\upsilon}$$

$$= \frac{\partial \Gamma^\rho{}_{\mu\upsilon}}{\partial x^\lambda} - \frac{\partial \Gamma^\rho{}_{\mu\lambda}}{\partial x^\upsilon} + \Gamma^\sigma{}_{\mu\upsilon} \Gamma^\rho{}_{\sigma\lambda} - \Gamma^\sigma{}_{\mu\lambda} \Gamma^\rho{}_{\sigma\upsilon} = R^\rho{}_{\mu\upsilon\lambda} \tag{5}$$

In Ricci tensor $R_{\mu\upsilon}$,

$$R'_{\mu\upsilon} = R'^\rho{}_{\mu\rho\upsilon} = R^\rho{}_{\mu\rho\upsilon} = R_{\mu\upsilon} \tag{6}$$

In curvature scalar R

$$R' = f^{\mu\upsilon} R'_{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} R_{\mu\upsilon} = \frac{1}{K} R \tag{7}$$

Hence, in the gravity field equation of Einstein,

$$R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R' = R_{\mu\upsilon} - \frac{1}{2} K g_{\mu\upsilon} (\frac{1}{K} R)$$

$$= R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon},$$

Of course $T'_{\mu\upsilon} = T_{\mu\upsilon}$ (8)

Therefore, tensor $f_{\mu\upsilon}$ satisfy the gravity field equation of Einstein.

If $T_{\mu\upsilon} = 0$,

$$R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R' = R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = 0$$

$$f^{\mu\upsilon} [R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R'] = K g^{\mu\upsilon} [R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R] = 0$$

$$\rightarrow \ -R' = -KR = 0 \ \rightarrow \quad R'_{\mu\upsilon} = R_{\mu\upsilon} = 0 \qquad (9)$$

2. The other solution in Schwarzschild solution, Reissner-Nodstrom solution

$$f_{\mu\upsilon} = K g_{\mu\upsilon} \ , \quad \frac{\partial K}{\partial x^{\lambda}} = 0$$

$$ds^2 = g_{\mu\upsilon} dx^{\mu} dx^{\upsilon}, \ ds'^2 = f_{\mu\upsilon} dx^{\mu} dx^{\upsilon} \qquad (10)$$

For example, K is

$$K = 1 + n_2 \exp(-n_1 \frac{hc}{GM^2}) \ \text{or} \ K = 1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})$$

$n_1, \ m_1 > 0, \ n_1, \ n_2, m_1, m_2$ is number (11)

Schwarzschild solution(vaccum solution) is

$$ds^2 = -c^2 (1 - \frac{2GM}{rc^2}) dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \qquad (12)$$

The new solution is

$$ds'^2 = f_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = K g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = K \cdot ds^2$$

$$= [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})]\,[-c^2(1 - \frac{2GM}{rc^2})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

$$= [-c^2(1 - \frac{2GM}{rc^2})dt'^2 + \frac{dr'^2}{1 - \frac{2GM}{rc^2}} + r'^2\, d\theta^2 + r'^2 \sin^2\theta d\phi^2]$$

$$t' = t[1 + n_2 \exp(-n_1 \frac{hc}{GM^2})]^{\frac{1}{2}}, r' = r[1 + n_2 \exp(-n_1 \frac{hc}{GM^2})]^{\frac{1}{2}}$$

(13)

Reissner-Nodstrom solution is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon$$

$$= -c^2(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \quad (14)$$

The new solution is

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})] \cdot [-c^2(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}}$$

$$+ r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad , \quad K = [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})]$$

(15)

3. Conclusion

We find the other solution in the General relativity theory. Hence, the uniqueness of GR solution is denied by numberless solutions.

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