The origin of wave-particle duality of matter revealed

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Abstract

The wave-particle duality is one of the most remarkable concepts in physics ever discovered. It is a central pillar upon which the entire theory of quantum mechanics is based. However the origin of the wave-particle duality is unrevealed yet and is generally taken as a postulate representing a fundamental fact of nature. Here we disclose the origin of this remarkable fact of nature. We show that the introduction of (fermionic or bosonic) exchange symmetry for the state describing a group of particles of matter would naturally lead those particles to demonstrate wave-like character from particle-like character. Thus the existence of (fermionic or bosonic) exchange symmetry among the particles of matter is absolutely necessary for their wave character to manifest thus shedding light on the microscopic origin of the peculiar quantum behavior of matter.

Keywords: wave character of matter, exchange symmetry, quantum superposition, double slit experiment

1 Introduction

The fundamental nature of light had been an important question in the time of Sir Isaac Newton. Newton proposed, in the year 1704, the corpuscular theory of light in which he argued the light to be composed of tiny particles called corpuscles [1]. According to his theory light consists of a stream of particles whose path is modified when it hits objects. Using this picture he explained various phenomena associated with light e.g. reflection, refraction etc. A contemporary proposal by Christian Huygens however claimed that light was actually made up of moving disturbances in its medium of propagation giving rise to the wave theory of light [2]. For around a century after Newton, the corpuscular theory of light was generally accepted as the nature of light however with the experiments of Thomas Young in the year 1801, Huygen's wave theory of light was vindicated [3]. At the start of the 20th century the quantum theory of light was initiated by Max Planck when he explained the radiation spectrum of a black body by assuming the quantized nature of the light emission from the black body [4]. This quantum theory of light was furthered strengthened by Albert Einstein in 1905 when he explained the photoelectric effect by assuming the quantized absorption of light by a metal [5]. Thus the light was argued to consist of both the wave and particle characteristics at the same time depending upon the experiments performed on them. In some experiments like diffraction, interference etc. light demonstrated a wave like behavior while in other experiments like the photoelectric effect it needed a particle like description. Such a dichotomy led to the birth of wave-particle duality of light.

Striking an analogy with the wave-particle duality of light, Louis de Broglie in 1924 postulated that just as the light contains dual character (wave and particle like) similarly even the matter contains a dual character of being simultaneous wave like and particle like [6]. He proposed a wave to be associated with a moving particle of matter of momentum 'p' with a wavelength $\lambda = h/p$ where h is the Planck constant, in analogy with the case of light. The light particles, i.e. photons, are known to propagate with the speed 'c' (=299792458 m/s). However de Broglie hypothesis was applicable to matter particles moving at non-relativistic speeds too. The hypothesis was later verified by a number of experiments which then became a fundamental fact of nature giving birth to quantum mechanics [7-10]. However, the applicability of the de Broglie theory to non-relativistic massive particles is curious.

The origin of this wave-particle duality of matter has remained elusive and has, so far, been accepted only as a postulate representing a fundamental fact of nature. In this paper we go a step ahead and elucidate the origin of this wave-particle duality of matter. We intend to disclose the microscopic mechanism for the formation of wave character from the particles of matter. We stress on the importance of the fermionic or bosonic exchange symmetry among the particles of matter as a necessary component for forming wave-like character from them. Quantitative estimations for the properties of quantum systems are well established via Schrödinger or Dirac formalisms. The unknown issues regarding quantum mechanics mainly arise from an interpretational point of view and would form the subject of this paper.

2 Results and Discussion

One of the most revealing experiments as far as the quantum properties of matter are concerned is the double slit experiment performed with electrons [11]. This experiment involves shining a beam of mono-energetic electrons upon two parallel, closely spaced (spacing d is of the order of the de Broglie wavelength of the electrons) narrow slits and measuring the electron pattern on a detector screen beyond the double slit. Surprisingly the electron pattern reveals interference fringes characteristic of the wave character for the incident electrons. The same experiment when repeated with reduced incident electron fluxes to an extent that only a single electron could pass through the apparatus at a time, surprisingly, reproduces the interference fringes like before, clearly revealing the wave phenomena to be associated with ‘individual’ electrons.

We, too, in our discussion will begin with the double slit experiment with electrons. In this case the incident electron beam is provided by an electron gun. Let us, for the sake of illustrating the origin of wave behavior from electrons, approximate the electron reservoir (infinitely many electrons) inside the electron gun to represent a gas of classical particles i.e. let us approximate every incident electron to be a classical particle. Since classical particles have well defined trajectories, we will associate every electron with a well defined trajectory for its travel through the double slit apparatus. Few electrons will have an overlap of the trajectory so there will be a statistical distribution of the number of electrons as a function of their trajectories. If we shine infinitely many electrons over the double slit, the predicted statistical distribution will be ultimately obtained. Now let us, for illustrative purpose, take an example of three distinct trajectories ‘A’, ‘B’ and ‘C’ (see Fig.1). Let us put an electron into each of these trajectories. Let us assume that the electron in ‘A’ is moving through the double slit at an instant of time. Now we introduce fermionic exchange symmetry among the three electrons (and subsequently among all electrons of the reservoir) and evaluate its consequences for the trajectory of the moving electron (see supplementary information section A for a more elaborate discussion). The introduction of fermionic exchange symmetry between the electrons occupying 'A' and 'B' will force the moving electron to pass through 'B' simultaneously with 'A' (and *vice versa*). Similarly, an exchange with the electron in 'C' will force the moving electron to simultaneously pass through 'C' along with 'A' and so on so forth. Thus the fermionic exchange symmetry among all the infinite electrons of the reservoir will force the electron in 'A' (and all other electrons too) to simultaneously pass through the trajectories of all other electrons of the reservoir giving rise to its (their) presence in an extended region of the space (a typical behavior expected from a wave). Since there are infinitely many electrons in the reservoir their trajectories will form a continuum inside the cross-section of the incident electron beam. Thus we see that the effect of the fermionic exchange symmetry is to smear the electron's probability distribution from a Dirac delta function (corresponding to a 'point' particle) to a 'wavefront' extending over the surface of the beam cross-section of the electron gun. For any overlap of trajectories the number of electrons possessing the fermionic exchange symmetry increases proportionately, leading to an increase of the amplitude of the 'wavefront' at that point consistent with the classical statistical distribution. Thus we appreciate the importance of the fermionic exchange symmetry in compressing the entire information of the classical statistical distribution for the electron beam inside one incident electron such that the single electron probability distribution in space resembles the classical statistical distribution. Thus we observe that the fermionic exchange symmetry leads to (i) the formation of a 'wavefront' of the probability distribution for the electron in space and (ii) the simultaneous propagation of all the electrons of the reservoir through the double slit. All of the electrons move through the double slit at once but partially such that their integrated probability flux equals the incident electron flux (see supplementary information section A).

Thus a well defined trajectory, a hallmark of classical behavior of the particles, is incompatible with the existence of the fermionic (or bosonic) exchange symmetry between those particles. Instead, as described above, the electron trajectory spreads over the region of the classical statistical distribution forming a 'wavefront' in space laying the groundwork for the formation of wave nature of electrons. However a wave has many other attributes like e.g. wavelength, phase etc. too. It remains a task to justify these attributes as arising because of the fermionic (or bosonic) exchange symmetry. The wavelength of a matter wave is given by the de Broglie formula. For justifying the applicability of the de Broglie formula to matter waves and to elucidate its origin from the fermionic (or bosonic) exchange symmetry among particles, we refer the reader to the supplementary information section B. The interesting issue is related to the phase of the matter wave. From elementary wave theory it is well known that a wave has both +ve and -ve phases corresponding to +ve and -ve displacements of a physical quantity about a reference value. The phase differences among superposing waves are responsible for generating the interference pattern which is the characteristic of their wave nature. In the case of the electron waves in the double slit experiment, we argue that the origin of different phases arise from the passage of the two ('partial') electrons either through same slit or through different slits. It is argued that these two different passages would contribute differently towards the interference pattern. The passage of the two electrons through the same slit would not contribute to the interference pattern while their passage through different slits would contribute to the interference pattern. This information is encoded (and distinguished) in the phase of the electron wave. Without loss of generality we can assume that the passage through different slits generates a +ve phase while the passage through the same slit generates a -ve phase. Since there are infinitely many electrons in the reservoir, for any arbitrary electron nominally passing through the upper slit, equal number of electrons passes through the upper slit and through the lower slit all of which have fermionic exchange symmetry with it. As a result the passage of the electron (nominally through the upper slit) would generate a wave of equal amplitude for both the phases at any arbitrary point 'P' on the other side of the double slit (in general, there will be a phase difference between both the phases reflecting the path length difference for the point 'P' from both the slits.). Thus we rationalize the emergence of two different phases in a matter wave from such an argument.

Following the origin of two different phases of a matter wave in a double slit experiment, a natural question arises as to how one explains the existence of two such phases in a matter wave propagating in free space where there is no such physical double slit arrangement present. In order to explain this we need to take recourse to the single slit diffraction experiment wherein a mono-energetic electron beam falls on a single slit and then gets diffracted (see Fig.2). This diffracted electron beam is collected on a screen kept after the single slit and the diffraction pattern is observed akin to the one observed when we shine photons, instead of electrons, on the single slit. The theoretical analysis of this diffraction experiment involves dividing the slit width (d) into two equal halves and treating them as harboring the continuum of double 'infinitesimally' wide slits arranged side by side along the slit width. These are not physical slits rather they are 'virtual' slits (Following Huygen's principle every point on the wavefront acts like a secondary source of light emitting spherical waves [2]. Thus every point along the slit width acts like a point source for the spherical wavefront. Using this concept we can hypothetically divide the slit width into a continuum of infinitesimally wide sections each of which can act like the ‘point’ source). Then the differences in the path lengths arising from these continuum ‘virtual’ double slits are calculated for any arbitrary point ‘P’ on the screen in order to calculate the diffraction pattern. Note that the point ‘P’ has a contribution from an equal length of the upper slit continuum and the lower slit continuum. Thus the wave at ‘P’ will contain both the phases having equal amplitudes except with a phase difference (corresponding to the path length difference for point ‘P’ from the upper and lower slit continuum) between both of them (see supplementary information section C). The observed diffraction pattern is a result of this phase difference. The free space can then be simulated by taking the limit $d\to\infty$. In this limit we recover the uniform intensity as expected for a wave moving in an isotropic space since the diffraction pattern vanishes. Thus we have explained qualitatively how the different attributes of a wave character emerge within particles when we switch on the fermionic (or bosonic) exchange symmetry among them.

Going back to the double slit experiment, an electron passing through the upper slit would then generate a secondary electron wave from the ‘point’ source of the upper slit and an electron passing through the lower slit would do the same from the lower slit. These secondary electron waves then interfere to generate an interference pattern marked by a complete destructive interference from waves of equal amplitudes with phase difference of ‘π’ among them.

Following the origin of the wave nature of matter as arising due to the existence of the fermionic (or bosonic) exchange symmetry, a question arises whether wave theory could be applied to classical objects in everyday life like bat, bus, football etc. To date, it is generally believed that since all physical objects are made up of 'quantum' particles (like e.g. proton, neutrons, electrons etc.) the wave theory which is applicable to these quantum particles is naturally applicable even to such macroscopic objects but since their energy scales are much higher than those for the quantum particles, the quantum effects are not visible among them. Philosophical debates about the validity of quantum mechanics have occurred in the past, the famous one being the Schrödinger's cat paradox [12], which were often used to discredit quantum mechanics (or certain interpretations of quantum mechanics). Our position over this is that a paradox like the Schrödinger's cat paradox is non-existent since one cannot apply quantum mechanics to the two body system of a cat and a radioactive atom trigger since there is no such exchange symmetry between both of them. Thus the extrapolation that quantum mechanics would be naturally applicable to macroscopic objects is against our view. In our opinion quantum mechanics only applies to particles having fermionic (or bosonic) exchange symmetry among themselves (see supplementary information section D). In fact all the experimental evidences obtained so far concerning the observation of quantum behavior has always been obtained from such particles which is consistent with our viewpoint. And even for these cases it applies only under certain conditions where such an exchange symmetry is maintained. There are situations where the fermionic (or bosonic) exchange symmetry can be suppressed among the so-called identical particles via localization process [13] or via specific experimental techniques used [14]. In such cases the electron under study would fail to exhibit quantum behavior.

3 Conclusion

In summary, we highlight the origin of the wave theory of particles within the realm of quantum mechanics. We argue that the presence of fermionic (or bosonic) exchange symmetry among the particles of matter is indispensable for the manifestation of quantum behavior among them. The origin of their wave character is rationalized through the presence of such exchange symmetry among them. We justify different attributes of their wave character through such exchange symmetry. Finally, we argue that quantum mechanics is not applicable for everyday macroscopic objects due to the absence of fermionic (or bosonic) exchange symmetry among them but instead claim its applicability only for identical particles which possess such exchange symmetry among themselves.

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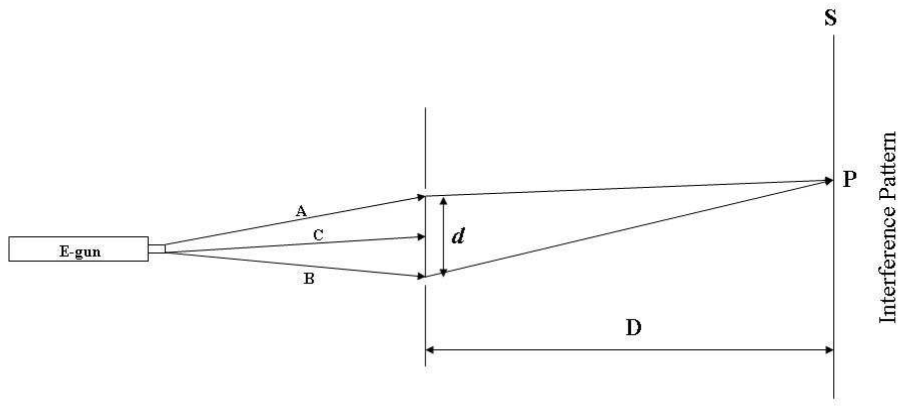

Fig.1. Schematic diagram for the double slit experiment with electrons: An electron gun shoots mono-energetic electrons at the double slit (width *d*) arrangement. Three electron trajectories 'A,' 'B' and 'C' are shown for illustration. Trajectory 'A' passes through upper slit, trajectory 'B' passes through lower slit and trajectory 'C' hits the barrier in between the double slit. The screen S records the interference pattern from electrons passing through the double slits.

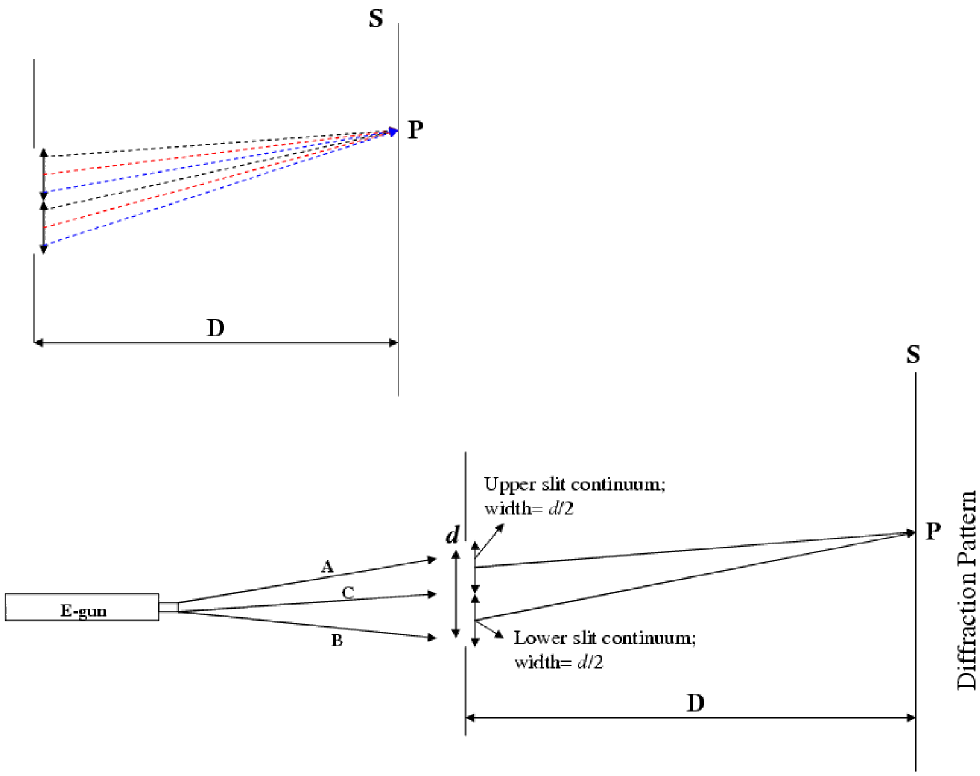

Fig.2. Schematic diagram for the single slit diffraction experiment with electrons: An electron gun shoots mono-energetic electrons at the single slit (width *d*) arrangement. Three electron trajectories A, B and C are shown for illustration. The screen S records the diffraction pattern from electrons passing through the single slit. The slit is hypothetically divided into two equal parts (for the diffraction analysis) into the upper slit continuum and lower slit continuum each containing a continuum of 'virtual' slits which act like sources for secondary electron wavefronts. Corresponding 'virtual' slits from the two continuums act like a pair of double slits that cause interference effects at 'P' (see the panel at top left. Such continuum pairs of double slits are depicted by different colors). The collective interference of all such pairs of 'virtual' double slits give rise to the diffraction pattern on S.

Supplementary Information

Section (A) Quantum superposition and the physical meaning of the fermionic (or bosonic) exchange symmetry

Consider two electrons '1' and '2' forming a singlet state. Then their wave function can be written as $|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2$. This state contains a linear combination of a two particle term and its particle exchanged counterpart. Note that in this state each of the electrons is in $\uparrow$ and $\downarrow$ spin states simultaneously. *Thus we clearly see that the fermionic exchange symmetry among electrons 'forces' an electron to be in multiple states simultaneously giving rise to a superposition of states.*

We will try to evaluate the consequences of this superposition (arising from the fermionic exchange symmetry) among the electrons inside the electron gun of the double slit experiment as described in the main text of the manuscript.

Discussion [1]

The classical state for the infinite number of 'classical' electrons (electrons '1', '2', '3', '4'…...etc. passing through the trajectories A, B, C, D……etc. respectively) of the electron gun can be represented by $(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto} \infty \text{ no. of electrons})$. When we switch on the fermionic exchange symmetry between electrons 1 and 2, the wave function for the infinite number of electrons would become:

$$\{(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) - (|B\rangle_1|A\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})\}$$

In this state electron '1' is passing through the trajectories A and B *at the same time* thus extending the distribution of its probability in space (along both the trajectories A and B). If now further we switch on the fermionic exchange symmetry among three electrons '1', '2' and '3' then the resultant state would be:

$$\begin{aligned}
&\{(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) - (|A\rangle_1|C\rangle_2|B\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})\\
&-(|C\rangle_1|B\rangle_2|A\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) + (|C\rangle_1|A\rangle_2|B\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})\\
&-(|B\rangle_1|A\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) + (|B\rangle_1|C\rangle_2|A\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})\}\\
&= \begin{vmatrix} |A\rangle_1 & |A\rangle_2 & |A\rangle_3 \\ |B\rangle_1 & |B\rangle_2 & |B\rangle_3 \\ |C\rangle_1 & |C\rangle_2 & |C\rangle_3 \end{vmatrix} \otimes (|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})
\end{aligned}$$

The resultant state is the tensor product of the Slater determinant for the three electrons ('1', '2' and '3') and a state for the remaining 'classical' electrons. One can see that in this state electron '1' is passing through the trajectories A, B and C *simultaneously*.

{Note: The above treatment, although demonstrated for electrons, applies, in principle, to any fermionic system and can easily be extended to bosons too. In fact double slit interference experiments have been performed for a number of fermions as well as bosons and interference phenomena has been observed for all of them. If we have a bosonic system then a particle exchange will not change the sign of the wavefunction. In that case the resultant state for the above case will become:

$$\begin{aligned}
&(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) + (|A\rangle_1|C\rangle_2|B\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})\\
&+(|C\rangle_1|B\rangle_2|A\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) + (|C\rangle_1|A\rangle_2|B\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})\\
&+(|B\rangle_1|A\rangle_2|C\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons}) + (|B\rangle_1|C\rangle_2|A\rangle_3|D\rangle_4 \text{..............upto } \infty \text{ no. of electrons})
\end{aligned}$$

}

Thus we see that by introducing the fermionic exchange symmetry among all the electrons of the electron gun we make electron '1' pass through the trajectories of all the electrons *simultaneously*. Since the choice of the electron is arbitrary therefore the conclusions drawn for electron '1' holds, in general, for every other electron also; that means every electron will pass through the trajectories of all the electrons simultaneously. Now if we assume electron ‘1’ to be moving through the double slit at a particular instant of time then it is ‘forced’ to move through the trajectories of all the electrons simultaneously thus creating a ‘wavefront’ in space. This wavefront extends over the crosssectional area of the incident electron beam. Since there are infinite number of electrons in the electron gun the crossectional distribution of their trajectories within the incident electron beam would form a continuum. Therefore this ‘wavefront’ is continuous across the crosssectional area of the incident electron beam. Thus we argue how a wavefront arises out of the gas of moving (infinite) classical particles upon introducing the fermionic exchange symmetry among them. At this stage the following picture emerges: We have the distribution of probability for every constituent electron (electron ‘1’ as well as other electrons) into each of the trajectories A, B, C, D etc. For moving electrons (e.g. electron ‘1’ in above case) the resulting wavefront is easy to imagine and is moving in space denoting the motion of the electron. For remaining electrons at rest (for whom the probability is distributed, too, among all the trajectories due to the exchange symmetry alike electron ‘1’) the ‘wavefront’ (‘wavefront’ here implies distribution of the electron across different trajectories) is hard to imagine since they are at rest but nevertheless it exists. Thus we argue how every constituent electron (moving as well as at rest) will form a ‘wavefront’ in space.

Discussion [2]

Furthermore, there is yet another aspect for the consequences of this fermionic exchange symmetry which needs to be highlighted as well.

Let us denote the different wavefronts by W_1, W_2, W_3, W_4,…. etc. These wavefronts can be thought of as different states available for the occupation of different electrons i.e. electron ‘1’, electron ‘2’, electron ‘3’, electron ‘4’,…… etc. Let us assume W_1, W_2, W_3, W_4,…. etc. to be occupied by electron ‘1’, electron ‘2’, electron ‘3’, electron ‘4’,…… etc. respectively. Then the many electron state for such a system can be written as $(|W_1\rangle_1|W_2\rangle_2|W_3\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons})$. Since we have assumed electron ‘1’ to be moving while the others are at rest therefore W_1 will denote a moving wavefront while W_2, W_3, W_4,…. etc. will denote wavefronts which are at rest. When there is a fermionic exchange symmetry between electron ‘1’ and ‘2’ then the many electron state can be written as

$$\{(|W_1\rangle_1|W_2\rangle_2|W_3\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons}) - (|W_2\rangle_1|W_1\rangle_2|W_3\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons})\}$$

In this state electron ‘1’ occupies the wavefronts W_1 (moving) and W_2 (at rest) *at the same time*. Thus we see that a part of electron ‘1’ is at rest and the remaining part is in motion simultaneously. Also we observe that the moving wavefront W_1 is simultaneously occupied by electrons ‘1’ and ‘2’ thus clearly showing that both the electrons are in a simultaneous state of motion. Thus the fermionic exchange symmetry between both the electrons gives rise to their simultaneous motion through the double slit. Similarly fermionic exchange symmetry between electrons ‘1’, ‘2’ and ‘3’ gives rise to the many electron state as

$$\begin{aligned}
&\{(|W_1\rangle_1|W_2\rangle_2|W_3\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons}) - (|W_1\rangle_1|W_3\rangle_2|W_2\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons}) \\
&-(|W_3\rangle_1|W_2\rangle_2|W_1\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons}) + (|W_3\rangle_1|W_1\rangle_2|W_2\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons}) \\
&-(|W_2\rangle_1|W_1\rangle_2|W_3\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons}) + (|W_2\rangle_1|W_3\rangle_2|W_1\rangle_3|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons})\} \\
&= \begin{vmatrix} |W_1\rangle_1 & |W_1\rangle_2 & |W_1\rangle_3 \\ |W_2\rangle_1 & |W_2\rangle_2 & |W_2\rangle_3 \\ |W_3\rangle_1 & |W_3\rangle_2 & |W_3\rangle_3 \end{vmatrix} \otimes (|W_4\rangle_4\ldots\ldots\text{upto } \infty \text{ no. of electrons})
\end{aligned}$$

The resultant state is the tensor product of the Slater determinant for the three electrons (‘1’, ‘2’ and ‘3’) and a state for the remaining electrons. One can see that in this state electron ‘1’ occupies the wavefronts W_1 (moving), W_2 (at rest) and W_3 (at rest) *simultaneously*. Here too the moving wavefront W_1 is occupied by all the three electrons (‘1’, ‘2’ and ‘3’) denoting the simultaneous motion of all the three electrons.

Thus we see that the fermionic exchange symmetry forces electron ‘1’ to be in motion and at rest at the same time. Since the choice of the electron is arbitrary the above conclusion holds in general for every other electron too. Thus we conclude that every electron is in the simultaneous state of motion and rest which runs into contradiction with our initial assumption about the motion of electron ‘1’ (and correspondingly about the motion of the remaining electrons too). Thus we see that the assumption that only a particular electron moves through the double slit at any time is incompatible with the existence of the fermionic exchange symmetry among the electrons. In fact we have already shown above that the fermionic exchange symmetry leads to the simultaneous motion of the concerned electrons. When we switch on the fermionic exchange symmetry among all the electrons then this leads to the simultaneous motion of all the electrons. The electrons move in such a way that their integrated probability flux matches the value set for the flux of the incident electron beam. This can happen only when all those electrons are moving *partially*. Thus we see that the introduction of the fermionic exchange symmetry among electrons (of the experimental apparatus) has two major consequences; (i) generation of an extended spatial distribution of the electron **-** wavefront formation (concluded from **Discussion [1]**) and (ii) the *simultaneous* motion of every constituent electron through the experimental apparatus *partially* at any instant of time (concluded from **Discussion [2]**).

The fermionic (or bosonic) exchange symmetry of the wave function is *not just a mathematical constraint required by the theory (quantum field theory) but on a physical level it causes both the particles to swap their states throughout their journey through an experiment/measurement*. This has not been mentioned explicitly in the previous literature hence it requires a clarification. This fact is very counterintuitive since we usually assume that any single electron would quietly pass through the experimental apparatus contributing to the measurement but on the contrary it is in constant state of a swap between the two states. A consequence of this exchange is that at any instant of time all the electrons are *simultaneously but partially* passing through the experimental apparatus such that the integrated electron flux matches the value set forth for the incident electron flux within the instrument. Thus the quantum behavior is completely manifested within such an experiment/measurement since all electrons remain 'indistinguishable' ('indistinguishable' because the measurement is not specifically contributed by few electrons more than others. No electron is preferred over others during the measurement. In fact, all the electrons contribute equally to the measurement at the same time. Note that indistinguishability among particles is a NECESSARY criterion for quantum mechanics to be applicable for them.) during the course of the experiment/measurement. Exceptions to this are obtained when the fermionic exchange symmetry of the electron under study is suppressed, either due to the electron state being localized owing to the electrostatic crystal lattice potential/electron correlations (ref. arXiv:1409.7156 or viXra:1511.0040) which does not allow its fermionic exchange symmetry with the mobile conduction electrons to fully develop or by specifically 'looking' at a single electron within an experiment via measuring its single particle property (which naturally 'forces' all other electrons to stay out from the experiment/measurement) (ref. J. Phys.: Cond. Matter **25**, 382205 (2013)). Under such situations the 'distinguished' electron under study would not display quantum behavior.

Section (B) Justifying de Broglie's hypothesis to matter waves

Louis de Broglie's hypothesis claimed the same equation to be valid for calculating the wavelength of matter waves as it is for the wave length of the photon i.e. $\lambda=h/p$ where h is the Planck's constant and p is the momentum of the photon. In de Broglie's hypothesis p becomes the relativistic momentum of a massive particle. This hypothesis has now become an experimentally validated fact. But the basic issue remains as how to justify the de Broglie hypothesis to matter waves even if the particles are moving at non-relativistic speeds. We present our viewpoint over its explanation.

We argue that the fermionic (or bosonic) exchange symmetry among massive particles giving rise to the wave nature of the particles, originates from the exchange of mediating particles among the massive particles. These mediating particles propagate at the speed of light *c* irrespective of the speed of the motion of the massive particles and carry a momentum p with them which is the same as the momentum of the massive particles. The existence of these exchange mediating particles is crucial for forming the wave character out of these massive particles; as a result all the attributes corresponding to their wave character arise from these exchange mediating particles. Since the exchange mediating particles propagate at *c* (just like photons) the expression for the wavelength of photons is equally valid for them. Therefore the de Broglie's formula for the wavelength of matter waves remains the same as for the wavelength of photons even in case of the non-relativistic motion of the massive particles. We propose a new interpretation for the de Broglie formula in case of massive particles:

λ=h/p , where h is the Planck's constant and p is relativistic momentum of the exchange mediating particle.

An immediate consequence of this idea is that the fermionic (or bosonic) exchange symmetry induced correlations are not instantaneously propagating in space but travel with the speed of light *c*. But for most practical purposes when the distances involved are very small (e.g. typical distances within a laboratory experimental setup ~ few meters) the fermionic (or bosonic) exchange symmetry induced correlations can be *assumed* to be practically instantaneous.

Section (C) Rationalizing the amplitude/phase content of a matter wave

The results of the single slit diffraction experiment with electrons that we present in our manuscript can be easily analyzed within the Fraunhofer's diffraction theory assuming a simplified picture of a plane, monochromatic wavefront of electrons falling on a single slit of width *d* and the diffracted intensity falling on a screen S kept at a distance 'D' much larger than *d*.

We divide the wavefront passing through the slit into two equal halves. The upper half represents upper slit continuum and the lower half represents the lower slit continuum. These sections of the incident wavefront will independently superpose and produce a resultant wavefront at any arbitrary point 'P' on the screen. Our goal is to find out and compare the amplitude and phase of the two superposed wavefronts at 'P'.

Note that in the Fraunhofer's theory of diffraction (ref. http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinint.html#c2) the total phase angle δ (phase difference between the secondary waves emanating from the top and bottom of the slit and arriving at 'P' at same time) is related to the deviation angle θ (angle subtended by point 'P' at the slit) from the optic axis and is given by

$$\delta = \frac{2\pi d \sin\theta}{\lambda} \text{ ; } \lambda \rightarrow \text{de Broglie wavelength of the electron wave}$$

When treating upper and lower slit continuum separately (whose slit width is *d*/2) the total phase angle for upper and lower slit continuum will be

$$\delta = \frac{2\pi d \sin\theta}{2\lambda} = \frac{\pi d \sin\theta}{\lambda}$$

This angle is the same for both of them since θ remains practically unchanged for both of them following our assumption of D>>*d* within the Fraunhofer's diffraction theory.

If A_0 is the amplitude of the incident electron wavefront then the resultant amplitude from the upper (A_{upper}) and lower (A_{lower}) slit continuum (formed by a vector summation of individual amplitude elements in them) at 'P' would be given by;

$$A_{upper} = 2\frac{A_0}{\delta}\sin\frac{\delta}{2} = A_{lower} = A$$, which is same for upper and lower slit continuum.

However there is a phase difference between both these amplitudes as a result of the vector summation. This phase difference is equal to δ. Following the law for summation of vectors, the amplitude of the summed vector A_{sum} is related to the resultant amplitudes from the individual elements (i.e. A_{upper} and A_{lower}) as;

$$A_{sum}^2 = A_{upper}^2 + A_{lower}^2 - 2A_{upper}.A_{lower}.\cos(\pi-\delta) = A^2 + A^2 - 2A.A.\cos(\pi-\delta) = 2A^2(1+\cos\delta)$$

Now for destructive interference we have $A_{sum}=0$. This can happen when A=0 or when $(1+\cos\delta)=0$. The latter happens when $\delta = p\pi$ when p is odd integer. After plugging in the expression for A the former can written as;

$A=2\frac{A_0}{\delta}\sin\frac{\delta}{2}=0 \Rightarrow \sin\frac{\delta}{2}=0 \Rightarrow \delta=2n\pi$, where n is any integer ($\neq$0).
(Note that $A_0\neq0$ since we have a finite incident wavefront).

Combining both these results we get the following conditions for destructive interference;

$\delta=m\pi$, where m is any integer ($\neq$0).

Therefore, $\delta=m\pi=\frac{\pi d\sin\theta}{\lambda} \Rightarrow d\sin\theta=m\lambda$ which is well known criterion for the destructive interference in a diffraction experiment performed on a single slit of width *d* within Fraunhofer's diffraction theory.

When simulating the free space within Fraunhofer's theory, it is possible to increase the slit width to a finite value much larger than λ and also to keep the distance D much larger than *d* in order to still remain within the Fraunhofer limit. We can see that qualitatively we still maintain the theoretical results as we had derived for a case where *d* was comparable to λ except that the diffraction pattern shrinks progressively with such an increase of *d* (implying a reduction of obstacles in the path of the electron waves). So to a certain accuracy we are able qualitatively verify the consequences of electron waves moving in free space within Fraunhofer's theory. In the limit $d\rightarrow\infty$ we fully recover the uniform intensity in space expected for a wave moving in an isotropic space however the Fraunhofer's theory cannot be applied in this limit. For a more general treatment Fresnel's theory of diffraction may be applied.

From an incident wavefront arising due to the motion of massive particles we have, therefore, rationalized the existence of two different phases of the matter waves having equal amplitudes (with a phase difference) at any arbitrary point 'P' in space (within Fraunhofer's limit). The phase difference varies across the space and is responsible for the generation of interference effects within the matter waves giving rise to the diffraction pattern. We are thus successful in justifying the wave character arising out of a beam of classical particles upon introducing fermionic (or bosonic) exchange symmetry among them. Thus we elucidate, qualitatively, the origin of the wave character of matter.

Section (D) Origin of the quantum behavior of a single electron

Even for a single electron eigenvalue problem, say for example hydrogen atom problem solved using the Schrödinger's equation, we do find that the single electron displays quantum behavior i.e. possessing a spatially extended wavefunction, energy quantization etc. even though we do not 'apparently' have any so-called 'electron reservoir' with whom it would be subjected to particle exchanges analogous to that mentioned in the case of double slit interference experiments with electrons. This might raise a lot of doubt about how the wave behavior emerges for the single electron in the absence of any exchanges with other electrons. To answer this we argue that the vacuum surrounding the said electron is constantly under the influence of fluctuations in energy leading to the formation of short lived 'virtual' electron-positron pairs due to Heisenberg's uncertainty principle. This fluctuation of the vacuum and its effect under the action of the electromagnetic field of the electron is a well established fact and is known to give rise to vacuum polarization (ref. https://en.wikipedia.org/wiki/Vacuum_polarization). This 'sea' of 'virtual' electron-positron pairs gives rise to the screening effect in the presence of an electromagnetic field thereby modifying the magnitude of the original electromagnetic field analogous to what happens to a dielectric when placed in an external electric field. The 'virtual' electrons thus generated due to these fluctuations form the 'electron reservoir' (note that this electron reservoir extends all throughout the space till infinity) and participate in exchanges with the said electron for the sake of producing the wave (or quantum) behavior of that electron.

However the probability of the exchange with these 'virtual' electrons largely depends upon the bound/unbound nature of the said electron in space. This can be illustrated clearly while critically analyzing the time evolution of the narrow wave packet in space which is a well known result from quantum mechanics. When, supposedly, an electron is kept 'intentionally' localized at a particular point in space then its wave function must be described as that of a narrow wave packet centered at that point in space (ideally it should be Dirac delta function at that point in space). Let us assume that such a scenario exists till time *t*=0. At *t*=0 we 'release' (set free) the electron and allow its wavefunction to evolve with time. Thus for times $t > 0$, the dynamics of the electron is dictated by the time dependent Schrödinger's equation. A well known result is that the wave packet, which was localized at the site of the electron earlier, gradually spreads in space with time eventually occupying the whole of infinite space as $t \to \infty$. More importantly, the wavepacket tends to flatten out with time eventually becoming completely flat at $t \to \infty$ when we tend to have identical amplitude for the wavefunction at every point in the space denoting uniform probability of the existence of the electron in space. Thus at $t < 0$, even though the whole of the infinite space was always filled with the sea of virtual electron-positron pairs, there was no exchange happening between the localized electron and the virtual electrons of the sea. For if, on the contrary, the exchange was to happen then the wavefunction for the localized electron would remain finite (i.e. non-zero) throughout the space (even at $t < 0$) as a result of this exchange. Hence, logically, we must conclude from here that the localized state of the electron in space must be devoid of its fermionic exchange symmetry with that of the other electrons. Such an idea was introduced earlier in arXiv:1409.7156. Our aforementioned discussion lends very *strong* support to the idea introduced in arXiv:1409.7156.

At any instant of time the two electron state for the localized electron and a virtual electron can be represented as $|\uparrow\rangle_{\text{loc}}|\downarrow\rangle_{\text{vir}} - e^{i\Theta}|\downarrow\rangle_{\text{loc}}|\uparrow\rangle_{\text{vir}}$, where $|\uparrow\rangle_{\text{loc}}$ and $|\downarrow\rangle_{\text{vir}}$ denote localized and virtual electron state respectively (Note that the state $|\uparrow\rangle_{\text{loc}}$ represents a classical state for the localized electron corresponding to its point particle like description prior to switching on the exchange mechanism between the two electrons while $|\downarrow\rangle_{\text{vir}}$ represents a *constant* 'field' occupying the whole of infinite space arising due to the 'sea' of virtual electrons. Such a 'field' description for virtual electrons is a distinct facet of quantum field theory. We are indeed finding out the origin of the wave behavior of an electron. Hence the 'ingredients' that form such a wave behavior, e.g. the localized electron in this case, should be treated as a classical particle initially which later on conspire to create the wave behavior via the exchange mechanism. Moreover one can see that only when $\Theta = 0$ we have the antisymmetry of the two electron wavefunction while at other values of Θ the antisymmetry is 'partially' realized. At the extreme value of $\Theta = \pi/2$ we have a 'mixed' exchange symmetry i.e. the wavefunction of the two electron systems becomes $|\uparrow\rangle_{\text{loc}}|\downarrow\rangle_{\text{vir}} - i|\downarrow\rangle_{\text{loc}}|\uparrow\rangle_{\text{vir}}$ which has an exchange symmetry in between fermionic and bosonic exchange symmetry. Please see arXiv:1409.7156_supp. info. for details) and Θ is function of spatial coordinates and time i.e. $\Theta = \Theta(x,y,z,t)$ or $\Theta = \Theta(r,\theta,\phi,t)$ depending upon whether we describe the space in terms of Cartesian or spherical polar coordinates respectively. For the above case at $t<0$ we have the situation (if we assume the localized electron is situated exactly at the origin i.e. the Dirac delta wavefunction for the electron at the origin) that $\Theta = 0$ for $(x,y,z)=(0,0,0)$ and $\Theta = \pi/2$ otherwise. So there is a discontinuity in the value of Θ at the origin due to the existence of Dirac delta type of the wavefunction for describing the electron. When the electron has been set free at $t=0$ then for the later times ($t>0$) the wave packet starts 'spreading' within the whole of space via the exchange mechanism. In that case Θ becomes a continuous function of its arguments all throughout the space.

The 'spreading' of the wave packet can be understood as the evolution of the function Θ in space with time. Such an evolution is highly influenced by the dynamics of the aforementioned 'screening effect' (due to the 'sea' of virtual electron-positron pairs) for the electromagnetic field of the localized electron in space. Eventually at $t \to \infty$, when the wave packet has been fully flattened out, Θ becomes 0 everywhere in space.

The above case was illustrated for an 'intentionally' localized electron at a point in space. The same holds true in case of the localization of the bound electrons in space due to strong electrostatic fields etc. from the nuclei of the atoms. For example, the electronic orbitals of the Hydrogen atom are examples of bound states which lead to a certain degree of 'localization' of the electron in space in that although the electronic orbitals individually extend to infinity in space however the amplitudes of their wavefunctions are not uniform across the space. Instead the amplitudes are seen to peak in certain regions of space while they diminish far away from it thus giving rise to the tendency of the electron to stay in those regions of space preferentially (orbital formation). This preference is a consequence of the localization of the bound electron in space. For such a case of a bound electron, its exchange with the sea of 'virtual' electrons is not fully realized (i.e. $\Theta(x,y,z,t) \neq 0$ for every point in space at any time) since the bound electron is not able to extend all throughout the space with equal 'ease' to participate in the exchanges. The probability of the exchange is maximum (i.e. $\Theta(x,y,z,t) \to 0$) at distances close to the bound electron and vanishes far away (i.e. $\Theta(x,y,z,t) \to \pi/2$) from it. Hence the wavefunction for the bound electron has large amplitude in regions close to itself and vanishes at infinity. E.g. all of the electronic wavefunctions for the Hydrogen atom vanish at infinity since the strong attractive interaction between the Hydrogen nucleus and the electron creates a bound state for the electron thus partly suppressing its exchange with the infinite sea of virtual electrons. On the contrary, an unbound electron can fully demonstrate its exchange mechanism throughout space due to the absence of any restrictions on its location as such the wavefunction for an unbound electron is not localized in space rather it extends uniformly across the space denoting a uniform exchange with the infinite sea of virtual electrons. The details regarding the variation of intensity and directional dependence (for states with angular momentum $l \neq 0$) of the spatial profile of the wavefunction for the electron in Hydrogen atom can only be understood after taking into consideration the aforementioned screening effect (i.e. the estimation of $\Theta(x,y,z,t)$ in space can only be done after studying the effect of the aforementioned screening on the electromagnetic field of the said electron). Thus in short we have attempted to rationalize our idea that particle exchanges occurring between the said electron with the 'virtual' electrons arisen from the vaccuum fluctuations give rise to the formation of the wave/quantum behavior exhibited by the said electron.