The origin of wave-particle duality of matter revealed

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Abstract

The wave-particle duality is one of the most remarkable concepts in physics ever discovered. It is a central pillar upon which the entire theory of quantum mechanics is based. However the origin of wave-particle duality is unrevealed yet and is generally taken as a postulate representing a fundamental fact of nature. In this paper we attempt to disclose the origin of this remarkable fact of nature. We argue that the introduction of exchange interaction among a group of particles would naturally lead them to demonstrate wave-like character from particle-like character. Thus the existence of exchange interaction among particles is an absolutely necessary criterion for quantum behavior to manifest thus shedding light on the microscopic origin of the peculiar quantum behavior of matter.

Introduction

The fundamental nature of light had been an important question in the time of Sir Isaac Newton. Newton proposed, in the year 1704, the corpuscular theory of light in which he argued the light to be composed of tiny particles called corpuscles [1]. According to his theory light consists of a stream of particles whose path is modified when it hits objects. Using this picture he explained various phenomena associated with light e.g. reflection, refraction etc. A contemporary proposal by Christian Huygens however claimed that light was actually made up of moving disturbances in its medium of propagation giving rise to the wave theory of light [2]. For around a century after Newton, the corpuscular theory of light was generally accepted as the nature of light however with the experiments of Thomas Young in the year 1801 [3], Huygen's wave theory of light was vindicated. At the start of the 20th century the quantum theory of light was initiated by Max Planck when he explained the radiation spectrum of a black body by assuming the quantized nature of the light emission from the black body [4]. This quantum theory of light was furthered strengthened by Albert Einstein in 1905 when he explained the photoelectric effect by assuming the quantized absorption of light by a metal [5]. Thus the light was argued to consist of both the wave and particle characteristics at the same time depending upon the experiments performed on them. In some experiments like diffraction, interference etc. light demonstrated a wave like behavior while in other experiments like the photoelectric effect it needed a particle like description. Such a dichotomy led to the birth of wave-particle duality of light.

Striking an analogy with the wave-particle duality of light, Louis de Broglie in 1924 [6] postulated that just as the light contains dual character (wave and particle like) similarly even the matter contains a dual character of being simultaneous wave like and particle like. He proposed a wave to be associated with a moving particle of matter of momentum 'p' with a wavelength $\lambda=h/p$ where h is the Planck constant, in analogy with the case of light. The light particles, i.e. photons, are known to propagate with the speed '*c*' (=299792458 m/s). However de Broglie hypothesis was applicable to matter particles moving at non-relativistic speeds too. The hypothesis was later verified by a number of experiments which then became a fundamental fact of nature giving birth to quantum mechanics [7-10]. However, the applicability of the de Broglie theory to non-relativistic massive particles is curious.

The origin of this wave-particle duality of matter has remained elusive and has, so far, been accepted only as a postulate representing a fundamental fact of nature. In this paper we go a step ahead and attempt to elucidate the origin of this wave-particle duality of matter. We intend to disclose the microscopic mechanism for the formation of wave character from particles. We stress on the importance of the exchange interaction among particles as a necessary component for forming wave-like character from particles. Quantitative estimations for the properties of quantum systems are well established via Schrödinger or Dirac formalisms. The unknown issues regarding quantum mechanics mainly arise from an interpretational point of view and would form the subject of this paper.

Results and discussion

One of the most revealing experiments as far as the quantum properties of matter are concerned is the double slit experiment performed with electrons [11]. This experiment involves shining a beam of mono-energetic electrons upon two parallel, closely spaced (spacing *d* is of the order of the de Broglie wavelength of the electrons) narrow slits and measuring the electron pattern on a detector screen beyond the double slit. Surprisingly the electron pattern reveals interference fringes characteristic of the wave character for the incident electrons. The same experiment when repeated with reduced incident electron fluxes to an extent that only a single electron could pass through the apparatus at a time, surprisingly, reproduces the interference fringes like before, clearly revealing the wave phenomena to be associated with 'individual' electrons.

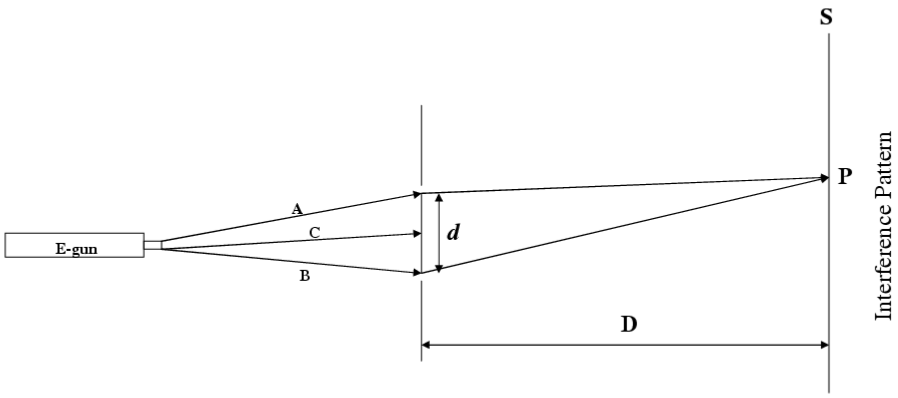

Fig.1. Schematic diagram for the double slit experiment with electrons. An electron gun shoots mono-energetic electrons at the double slit (width *d*) arrangement. Three electron trajectories A, B and C are shown for illustration. Electron A 'nominally' passes through upper slit, electron B 'nominally' passes through lower slit and electron C hits the barrier in between the double slit. The screen S records the interference pattern from electrons passing through the double slits.

We, too, in our discussion will begin with the double slit experiment with electrons. In this case the incident electron beam is provided by an electron gun. Let us approximate the electron reservoir (infinitely many electrons) inside the electron gun to represent a gas of classical particles i.e. let us approximate every incident electron to be a classical particle. Since classical particles have well defined trajectories, we will associate every electron with a well defined trajectory for its travel through the double slit apparatus. Thus we can label every electron by its distinct trajectory. Few electrons will have an overlap of the trajectory so there will be a statistical distribution of the number of electrons as a function of their trajectories. If we shine infinitely many electrons over the double slit, the predicted statistical distribution will be ultimately obtained. Now let us, for illustrative purpose, take an example of three electrons from the reservoir labeled by their distinct trajectories 'A', 'B' and 'C' (see Fig.1). Let us pass only a single electron through the double slit each time. Let us assume that electron 'A' is passing through the double slit. Now we introduce exchange interaction among the three electrons (and subsequently among all electrons of the reservoir) and evaluate its consequences concerning the trajectory of electron 'A' (see supplementary information section A). The introduction of exchange interaction between 'A' and 'B' will force the electron 'A' to occupy the state 'B' (and *vice versa*) or in other words electron 'A' will be forced to pass through the trajectory 'B' (simultaneously with its own trajectory 'A') *at the same time*. Similarly, an exchange with 'C' will force electron 'A' to simultaneously share the trajectory of 'C' and so on so forth. Thus the exchange interaction among all the *infinite* electrons of the reservoir will force electron 'A' (and all other electrons too) to *simultaneously* occupy the trajectories of all other electrons of the reservoir giving rise to its (their) presence in an extended region of the space (a typical behavior expected from a wave). Since there are infinitely many electrons in the reservoir their trajectories will form a continuum inside the crosssection of the incident electron beam. Thus we see that the effect of the exchange interaction is to smear the electron's probability distribution from a Dirac delta function (corresponding to a 'point' particle) to a 'wavefront' extending over the surface of the beam crosssection of the electron gun. For any overlap of trajectories the number of electrons undergoing the exchange interaction increases proportionately, leading to an increase of the amplitude of the 'wavefront' at that point consistent with the classical statistical distribution. *Thus we appreciate the importance of the exchange interaction in compressing the entire information of the classical statistical distribution for the electron beam inside one incident electron such that the single electron probability distribution in space resembles the classical statistical distribution.* Thus we observe that the exchange interaction leads to (i) the formation of a 'wavefront' of the probability distribution for the electron in space and (ii) the simultaneous propagation of all the electrons of the reservoir through the double slit. All the electrons move through the double slit at once but partially such that their integrated probability flux equals the incident electron flux.

Thus a well defined trajectory, a hallmark of classical behavior of the particles, is incompatible with the existence of exchange interaction between those particles. Instead, as described above, the electron trajectory spreads over the region of the classical statistical distribution forming a 'wavefront' in space laying the groundwork for the formation of wave nature of electrons. However a wave has many other attributes like e.g. wavelength, phase etc. too. It remains a task to justify these attributes as arising because of the exchange interaction. The wavelength of a matter wave is given by the de Broglie formula. For justifying the applicability of the de Broglie formula to matter waves and to elucidate its origin from the exchange interaction among particles, we refer the reader to the supplementary information section B. The interesting issue is related to the phase of the matter wave. From elementary wave theory it is well known that a wave has both +ve and -ve phases corresponding to +ve and -ve displacements of a physical quantity about a reference value. The phase differences among superposing waves are responsible for generating the interference pattern which is the characteristic of their wave nature. In the case of the electron waves in the double slit experiment, we argue that the origin of different phases arise from the passage of the two ('partial') electrons either through same slit or through different slits. It is argued that these two different passages would contribute differently towards the interference pattern. The passage of the two electrons through the same slit would not contribute to the interference pattern while their passage through different slits would contribute to the interference pattern. This information is encoded (and distinguished) in the phase of the electron wave. Without loss of generality we can assume that the passage through different slits generates a +ve phase while the passage through the same slit generates a -ve phase. Since there are infinitely many electrons in the reservoir, for an electron 'A' nominally passing through the upper slit, there are equal number of electrons passing through the upper slit and through the lower slit all of which have an exchange interaction with electron 'A'. As a result the passage of electron 'A' would generate a wave of equal amplitude for both the phases at any arbitrary point 'P' on the other side of the double slit (in general, there will be a phase difference between both the phases reflecting the path length difference for the point 'P' from both the slits.). Thus we rationalize the emergence of two different phases in a matter wave from such an argument.

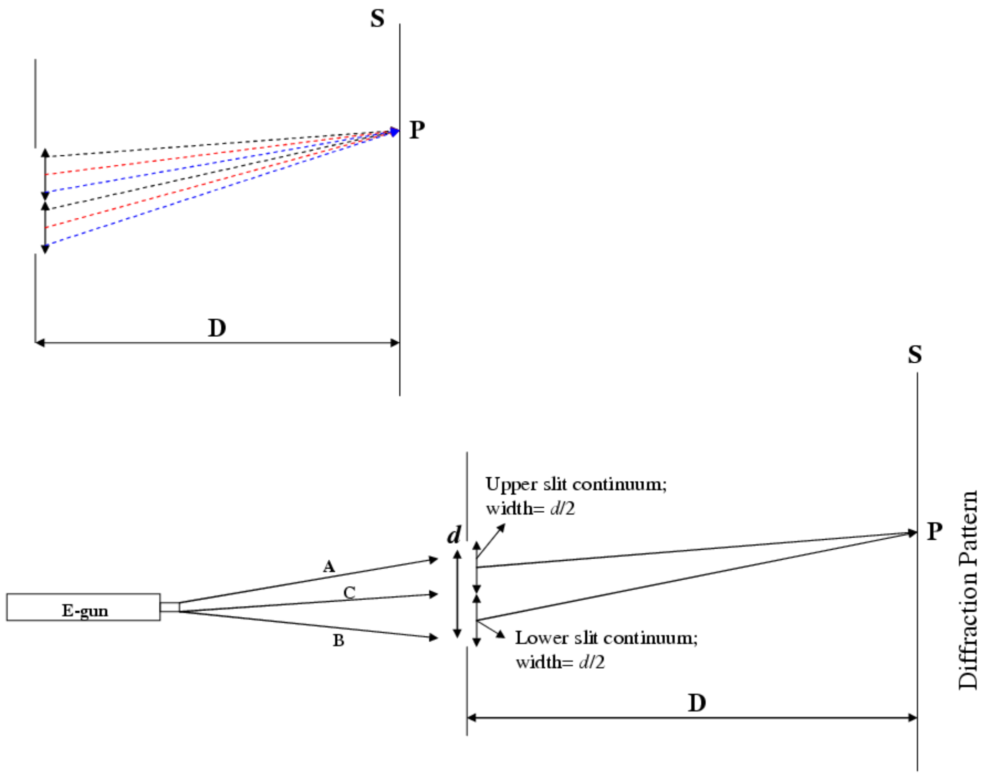

Fig.2. Schematic diagram for the single slit diffraction experiment with electrons. An electron gun shoots mono-energetic electrons at the single slit (width *d*) arrangement. Three electron trajectories A, B and C are shown for illustration. The screen S records the diffraction pattern from electrons passing through the single slit. The slit is hypothetically divided into two equal parts (for the diffraction analysis) into the upper slit continuum and lower slit continuum each containing a continuum of 'virtual' slits which act like sources for secondary electron wavefronts. Corresponding 'virtual' slits from the two continuums act like a pair of double slits that cause interference effects at 'P' (see the panel at top left. Such continuum pairs of double slits are depicted by different colors.). The collective interference of all such pairs of 'virtual' double slits give rise to the diffraction pattern on S.

Following the origin of two different phases of a matter wave in a double slit experiment, a natural question arises as to how one explains the existence of two such phases in a matter wave propagating in free space where there is no such physical double slit arrangement present. In order to explain this we need to take recourse to the single slit diffraction experiment wherein a mono-energetic electron beam falls on a single slit and then gets diffracted (see Fig.2). This diffracted electron beam is collected on a screen kept after the single slit and the diffraction pattern is observed akin to the one observed when we shine photons, instead of electrons, on the single slit. The theoretical analysis of this diffraction experiment involves dividing the slit width (d) into two equal halves and treating them as harboring the continuum of double 'infinitesimally' wide slits arranged side by side along the slit width. These are not physical slits rather they are 'virtual' slits {Following Huygen's principle every point on the wavefront acts like a secondary source of light emitting spherical waves [2]. Thus every point along the slit width acts like a point source for the spherical wavefront. Using this concept we can hypothetically divide the slit width into a continuum of infinitesimally wide sections each of which can act like the 'point' source.}. Then the differences in the path lengths arising from these continuum 'virtual' double slits are calculated for any arbitrary point 'P' on the screen in order to calculate the diffraction pattern. Note that the point 'P' has a contribution from an equal length of the upper slit continuum and the lower slit continuum. Thus the wave at 'P' will contain both the phases having equal amplitudes except with a phase difference (corresponding to the path length difference for point 'P' from the upper and lower slit continuum) between both of them (see supplementary information section C). The observed diffraction pattern is a result of this phase difference. The free space can then be simulated by taking the limit $d \to \infty$. In this limit we recover the uniform intensity as expected for a wave moving in an isotropic space since the diffraction pattern vanishes. Thus we have explained qualitatively how the different attributes of a wave character emerge within particles when we switch on the exchange interaction among them.

Going back to the double slit experiment, an electron passing through the upper slit would then generate a secondary electron wave from the 'point' source of the upper slit and an electron passing through the lower slit would do the same from the lower slit. These secondary electron waves then interfere to generate an interference pattern marked by a complete destructive interference from waves of equal amplitudes with phase difference of 'π' among them.

Following the origin of the wave nature of matter as arising due to the existence of the exchange interaction, a question arises whether wave theory could be applied to classical objects in everyday life like bat, bus, football etc. To date, it is generally believed that since all physical objects are made up of 'quantum' particles (like e.g. proton, neutrons, electrons etc.) the wave theory which is applicable to these quantum particles is naturally applicable even to such macroscopic objects but since their energy scales are much higher than those for the quantum particles, the quantum effects are not visible among them. Philosophical debates about the validity of quantum mechanics have occurred in the past, the famous one being the Schrödinger's cat paradox [12], which were often used to discredit quantum mechanics (or certain interpretations of quantum mechanics). Our position over this is that a paradox like the Schrödinger's cat paradox is non-existent since one cannot apply quantum mechanics to the two body system of a cat and a radioactive atom trigger since there is no exchange interaction between both of them. Thus the extrapolation that quantum mechanics would be naturally applicable to macroscopic objects is against our view.

In our opinion quantum mechanics only applies to particles having an exchange interaction among themselves (in fact all the experimental evidence obtained so far concerning the observation of quantum behavior has always been obtained from such particles which is consistent with our viewpoint) and even for these cases it applies only under certain conditions where such exchange interaction is maintained. There are situations where the exchange interaction can be suppressed among the so-called identical particles via localization process [13] or via specific experimental techniques used [14]. In such cases the electron under study would fail to exhibit quantum behavior.

Summary

In summary, we highlight the origin of the wave theory of particles within the realm of quantum mechanics. We argue that the presence of exchange interaction among particles is indispensable for the manifestation of quantum behavior among them. The origin of their wave character is rationalized through the presence of exchange interaction among them. We justify different attributes of the wave character of the particles through the exchange interaction. Finally, we argue that quantum mechanics is not applicable for everyday macroscopic objects due to the absence of exchange interaction among them and instead claim its applicability only for identical, indistinguishable particles which possess exchange interaction among themselves.

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Supplementary

Information

A) Quantum superposition and the physical meaning of the exchange interaction

Consider two electrons '1' and '2' forming a singlet state. Then their wave function can be written as $|\uparrow\rangle_1|\downarrow\rangle_2-|\downarrow\rangle_1|\uparrow\rangle_2$. This state contains a linear combination of a two particle term and its particle exchanged counterpart. Note that in this state each of the electrons is in $\uparrow$ and $\downarrow$ spin states simultaneously. *Thus we clearly see that the exchange interaction among electrons 'forces' an electron to be in multiple states simultaneously giving rise to a superposition of states.*

We will try to evaluate the consequences of this superposition (arising from the exchange interaction) among the electrons inside the electron gun of the double slit experiment as described in the main text of the manuscript. The classical state for the infinite number of 'classical' electrons (electrons '1', '2', '3', '4'......etc. passing through the trajectories A, B, C, D……etc. respectively) of the electron gun can be represented by $(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons})$. When we switch on the exchange interaction between electrons 1 and 2, the wave function for the infinite number of electrons would become:

$$\{(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons}) - (|B\rangle_1|A\rangle_2|C\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons})\}$$

In this state electron '1' is passing through the trajectories A and B *at the same time* thus extending the distribution of its probability in space (along both the trajectories A and B). If now further we switch on the exchange interaction among three electrons '1', '2' and '3' then the resultant state would be:

$$\begin{aligned}
&\{(|A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons}) - (|A\rangle_1|C\rangle_2|B\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons}) \\
&-(|C\rangle_1|B\rangle_2|A\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons}) + (|C\rangle_1|A\rangle_2|B\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons}) \\
&-(|B\rangle_1|A\rangle_2|C\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons}) + (|B\rangle_1|C\rangle_2|A\rangle_3|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons})\} \\
&= \begin{vmatrix} |A\rangle_1 & |A\rangle_2 & |A\rangle_3 \\ |B\rangle_1 & |B\rangle_2 & |B\rangle_3 \\ |C\rangle_1 & |C\rangle_2 & |C\rangle_3 \end{vmatrix} \otimes (|D\rangle_4 \ldots\ldots \text{upto } \infty \text{ no. of electrons})
\end{aligned}$$

The resultant state is the tensor product of the Slater determinant for the three electrons ('1', '2' and '3') and a state for the remaining 'classical' electrons. One can see that in this state electron '1' is passing through the trajectories A, B and C *simultaneously*.

Thus we see that by introducing the exchange interaction among all the electrons of the electron gun we make electron '1' pass through the trajectories of all the electrons *simultaneously*. Since the choice of the electron is arbitrary therefore the conclusions drawn for electron '1' holds, in general, for every other electron also; that means every electron will pass through the trajectories of all the electrons simultaneously. Now if we assume electron '1' to be moving through the double slit at a particular instant of time then it is 'forced' to move through the trajectories of all the electrons simultaneously thus creating a 'wavefront' in space. This wavefront extends over the crosssectional area of the incident electron beam. Since there are infinite number of electrons in the electron gun the crossectional distribution of their trajectories within the incident electron beam would form a continuum. Therefore this 'wavefront' is continuous across the crosssectional area of the incident electron beam. Thus we argue how a wavefront arises out of the gas of moving (infinite) classical particles upon introducing the exchange interaction among them. At this stage the following picture emerges: We have the distribution of probability for every constituent electron (electron '1' as well as other electrons) into each of the trajectories A, B, C, D etc. For moving electrons (e.g. electron '1' in above case) the resulting wavefront is easy to imagine and is moving in space denoting the motion of the electron. For remaining electrons at rest (for whom the probability is distributed, too, among all the trajectories due to the exchange interaction alike electron '1') the 'wavefront' ('wavefront' here implies distribution of the electron across different trajectories) is hard to imagine since they are at rest but nevertheless it exists. Thus we argue how every constituent electron (moving as well as at rest) will form a 'wavefront' in space. Furthermore, there is yet another aspect for the consequences of this exchange interaction which needs to be highlighted as well. Assuming that electron '1' (or wavefront '1') moves through the double slit a particular time constrains the remaining electrons (or 'wavefronts' for the remaining electrons) to remain at rest at that time. Then the introduction of the exchange interaction between electron 1 (or wavefront '1') and the remaining electrons (or remaining wavefronts) will put part of electron 1 to rest (and, *vice versa*, parts of the remaining electrons will be forced to move as well) which runs into contradiction with our initial assumption about the motion of electron '1' (and correspondingly about the motion of the remaining electrons too). Thus we see that the assumption that only a particular electron moves through the double slit at any time is incompatible with the existence of the exchange interaction among the electrons. In fact a careful/deeper thought reveals that all the electrons are 'forced' to move through the double slit *at the same time* such that their integrated probability flux matches the value set for the flux of the incident electron beam. This notion of simultaneous motion of all the electrons is fully compatible with the existence of the exchange interaction among them. Thus we see that the introduction of the exchange interaction among electrons (of the experimental apparatus) has two major consequences; (i) generation of an extended spatial distribution of the electron - wavefront formation and (ii) the *simultaneous* motion of every constituent electron through the experimental apparatus *partially* at any instant of time.

We would like to highlight here that the exchange interaction actually leads to an exchange between the particles. By this we mean that the exchange symmetry of the wave function is *not just a mathematical constraint required by the theory (quantum field theory etc.) but on a physical level it certainly causes both the particles to swap their states throughout their journey through an experiment/measurement*. This has not been mentioned explicitly in the previous literature hence it requires a clarification. This fact is very counterintuitive since we usually assume that any single electron would quietly pass through the experimental apparatus contributing to the measurement but on the contrary it is in constant state of a swap between the two states. A consequence of this exchange is that at any instant of time all the electrons are *simultaneously but partially* passing through the experimental apparatus such that the integrated electron flux matches the value set forth for the incident electron flux within the instrument. Thus the quantum behavior is completely manifested within such an experiment/measurement since all electrons remain 'indistinguishable' ('indistinguishable' because the measurement is not specifically contributed by few electrons more than others. No electron is preferred over others during the measurement. In fact, all the electrons contribute equally to the measurement at the same time. Note that indistinguishability among particles is a NECESSARY criterion for quantum mechanics to be applicable for them.) during the course of the experiment/measurement. Exceptions to this are obtained when the exchange interaction of the electron under study is suppressed, either due to the electron state being localized owing to the electrostatic crystal lattice potential/electron correlations (ref. arXiv:1409.7156; see http://vixra.org/abs/1511.0040 for the latest update of arXiv:1409.7156) which does not allow its exchange interaction with the mobile conduction electrons to fully develop or by specifically 'looking' at a single electron within an experiment via measuring its single particle property (which naturally 'forces' all other electrons to stay out from the experiment/measurement) (ref. J. Phys.: Cond. Matter **25**, 382205 (2013)). Under such situations the 'distinguished' electron under study would not display quantum behavior.

B) Justifying de Broglie's hypothesis to matter waves

Louis de Broglie's hypothesis claimed the same equation to be valid for calculating the wavelength of matter waves as it is for the wave length of the photon i.e. $\lambda = h/p$ where h is the Planck's constant and p is the momentum of the photon. In de Broglie's hypothesis p becomes the relativistic momentum of a massive particle. This hypothesis has now become an experimentally validated fact. But the basic issue remains as how to justify the de Broglie hypothesis to matter waves even if the particles are moving at non-relativistic speeds. We present our viewpoint over its explanation.

We argue that the exchange interaction among massive particles giving rise to the wave nature of the particles, originates from the exchange of mediating particles among the massive particles. These mediating particles propagate at the speed of light c irrespective of the speed of motion of the massive particles and carry a momentum p with them which is the same as the momentum of the massive particles. The existence of these exchange mediating particles is crucial for forming the wave character out of these massive particles; as a result all the attributes corresponding to their wave character arise from these exchange mediating particles. Since the exchange mediating particles propagate at c (just like photons) the expression for the wavelength of photons is equally valid for them. Therefore the de Broglie's formula for the wavelength of matter waves remains the same as for the wavelength of photons even in case of the non-relativistic motion of the massive particles. We propose a new interpretation for the de Broglie formula in case of massive particles:

$\lambda = h/p$, where h is the Planck's constant and p is relativistic momentum of the exchange mediating particle.

An immediate consequence of this idea is that the exchange interaction is not instantaneously propagating in space but travels with the speed of light c. But for most practical purposes when the distances involved are very small (e.g. typical distances within a laboratory experimental setup ~ few meters) the exchange interaction can be *assumed* to be practically instantaneous.

C) Rationalizing the amplitude/phase content of a matter wave

The results of the single slit diffraction experiment with electrons that we present in our manuscript can be easily analyzed within the Fraunhofer's diffraction theory assuming a simplified picture of a plane, monochromatic wavefront of electrons falling on a single slit of width *d* and the diffracted intensity falling on a screen S kept at a distance 'D' much larger than *d*.

We divide the wavefront passing through the slit into two equal halves. The upper half represents upper slit continuum and the lower half represents the lower slit continuum. These sections of the incident wavefront will independently superpose and produce a resultant wavefront at any arbitrary point 'P' on the screen. Our goal is to find out and compare the amplitude and phase of the two superposed wavefronts at 'P'.

Note that in the Fraunhofer's theory of diffraction (ref. http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinint.html#c2) the total phase angle δ (phase difference between the secondary waves emanating from the top and bottom of the slit and arriving at 'P' at same time) is related to the deviation angle θ (angle subtended by point 'P' at the slit) from the optic axis and is given by

$$\delta = \frac{2\pi d \sin\theta}{\lambda} \; ; \lambda \rightarrow \text{de Broglie wavelength of the electron wave}$$

When treating upper and lower slit continuum separately (whose slit width is *d*/2) the total phase angle for upper and lower slit continuum will be

$$\delta = \frac{2\pi d \sin\theta}{2\lambda} = \frac{\pi d \sin\theta}{\lambda}$$

This angle is the same for both of them since θ remains practically unchanged for both of them following our assumption of D>>*d* within the Fraunhofer's diffraction theory.

If A_0 is the amplitude of the incident electron wavefront then the resultant amplitude from the upper (A_{upper}) and lower (A_{lower}) slit continuum (formed by a vector summation of individual amplitude elements in them) at 'P' would be given by;

$$A_{upper} = 2\frac{A_0}{\delta}\sin\frac{\delta}{2} = A_{lower} = A \text{, which is same for upper and lower slit continuum.}$$

However there is a phase difference between both these amplitudes as a result of the vector summation. This phase difference is equal to δ. Following the law for summation of vectors, the amplitude of the summed vector A_{sum} is related to the resultant amplitudes from the individual elements (i.e. A_{upper} and A_{lower}) as;

$$A_{sum}{}^2 = A_{upper}{}^2 + A_{lower}{}^2 - 2A_{upper}.A_{lower}.\cos(\pi-\delta) = A^2 + A^2 - 2A.A.\cos(\pi-\delta) = 2A^2(1+\cos\delta)$$

Now for destructive interference we have $A_{sum}=0$. This can happen when A=0 or when $(1+\cos\delta)=0$. The latter happens when $\delta=p\pi$ when p is odd integer. After plugging in the expression for A the former can written as;

$$A=2\frac{A_0}{\delta}\sin\frac{\delta}{2}=0 \Rightarrow \sin\frac{\delta}{2}=0 \Rightarrow \delta=2n\pi$$, where n is any integer ($\neq 0$).

(Note that $A_0\neq 0$ since we have a finite incident wavefront).

Combining both these results we get the following conditions for destructive interference;

$\delta=m\pi$, where m is any integer ($\neq 0$).

Therefore, $\delta=m\pi=\frac{\pi d\sin\theta}{\lambda} \Rightarrow d\sin\theta=m\lambda$ which is well known criterion for the destructive interference in a diffraction experiment performed on a single slit of width *d* within Fraunhofer's diffraction theory.

When simulating the free space within Fraunhofer's theory, it is possible to increase the slit width to a finite value much larger than λ and also to keep the distance D much larger than *d* in order to still remain within the Fraunhofer limit. We can see that qualitatively we still maintain the theoretical results as we had derived for a case where *d* was comparable to λ except that the diffraction pattern shrinks progressively with such an increase of *d* (implying a reduction of obstacles in the path of the electron waves). So to a certain accuracy we are able qualitatively verify the consequences of electron waves moving in free space within Fraunhofer's theory. In the limit $d\rightarrow\infty$ we fully recover the uniform intensity in space expected for a wave moving in an isotropic space however the Fraunhofer's theory cannot be applied in this limit. For a more general treatment Fresnel's theory of diffraction may be applied.

From an incident wavefront arising due to the motion of massive particles we have, therefore, rationalized the existence of two different phases of the matter waves having equal amplitudes (with a phase difference) at any arbitrary point 'P' in space (within Fraunhofer's limit). The phase difference varies across the space and is responsible for the generation of interference effects within the matter waves giving rise to the diffraction pattern. We are thus successful in justifying the wave character arising out of a beam of classical particles upon introducing exchange interaction among them. Thus we elucidate, qualitatively, the origin of the wave character of matter.