Quantum Teleportation using Pseudorandom Number based Quantum Tricks

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Abstract

The present work purports to a radically new perspective towards exploiting the advantages of quantum computation using classical systems. Termed 'Quantum Tricks', the technique relies on replacing the inherent probabilistic nature of the qubit with a pseudorandom generator. Using this, extremely simple mathematical operations are developed to implement basic Pauli X, Y and Z gates, using FPGA. Following this, entanglement is implemented and a real life application of quantum computing, namely Quantum Teleportation is implemented using the 'QTrick Bits'. It is seen that the QTrick Bits are able to implement the teleportation with a remarkable degree of accuracy. Nonlinear characterization of the entangled states reveal the presence of chaos, thus providing subtle hints towards understanding the QTrick signal behavior. The extreme simplicity of the proposed design coupled with its efficacy in implementing quantum teleportation accurately form the highlights of the present work.

Keywords: Quantum Tricks, Pseudorandom Numbers, Entanglement, Quantum Teleportation, Chaos.

1. Introduction

The field of Quantum Computing has witnessed a phenomenal growth in recent years [1, 2]. This is partly due to the exciting prospects it provides to understanding the nature of various phenomena in quantum and particle physics, as has been proposed by Lloyd's Computational Universe Theories, and partially due to the exciting applications such as quantum teleportation and superdense coding offered in the Engineering domain [3, 4, 5, 6, 7]. At the heart of all these is the Quantum Bit, or 'qubit', an extension of the conventional bit with the added properties of quantum superposition and measurement-induced collapse [1]. The superposed states essentially enable qubits to perform the functionalities of many bits simultaneously. Conventionally, quantum computation has been implemented using quantum systems, such as superconductiing qubits, spintronics, nuclear magnetic resonance, or laser powered photonic devices [5, 6, 7, 8, 9, 10, 11].

However, the present work proposes a radically different way to achieve effects similar to quantum computation. The central idea is to use quantum tricks, using classical systems. Specifically, the present work purports to achieve the inherent probabilistic nature of a qubit using pseudorandom number generators [12]. Using this analogue, the basic Pauli Rotation Operators are explored. Following this, an important quantum computing phenomenon, entanglement is explored in the light of a real-life application, quantum teleportation [5]. It is observed that successful quantum teleportation is possible using quantum tricks, with very simple design. The entangled and teleported signals are studied using nonlinear characterization techniques and the presence of chaos is observed in the entangled signal. This observation suggests a novel perspective into quantum computation, and the possibility of explaining quantum phenomena and principles through nonlinear dynamics and chaos theory. The extreme simplicity of the pseudorandom number based quantum trick circuit combined with the accuracy seen in teleportation forms the novelty of the present work.

2. Qubits and 'Quantum Trick Pauli Gates'

The first step in quantum computation is the definition of the quantum bit, or qubit. The standard Bloch sphere representation of a qubit is given in Fig. (1) [1]. All points lying on the surface of the Bloch Sphere represent classical states of $|0>$ or $|1>$. All points not on the surface represent mixed states of $|0>$ and $|1>$, which can be represented as superposition states of $|0>$ and $|1>$. In accordance with normalization conditions, the superposition state is almost always seen as $1/\sqrt{2}[|0>+|1>]$ [1].

In order to create this scenario along with the inherent probabilistic nature of the qubit, the present work proposes the use of a pseudorandom number generator, generating random elements from the set [0,0.5,1] [12]. The occurrence of 0 and 1 are assumed as the states $|0>$ and $|1>$ respectively, and the occurrence of 0.5 is assumed as the superposition state $1/\sqrt{2}[|0>+|1>]$. The qubit analogues generated using the pseudorandom quantum trick shall hence be named 'QTrick-Bits'.

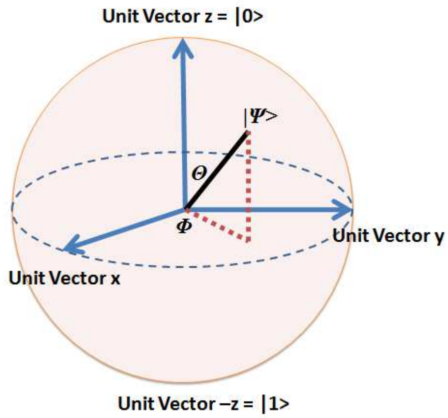

Figure 1: The Bloch Sphere Representation of a Qubit

The next step is to explore the implementation of basic operators acting on QTrick-Bits. It is known that almost any operator in quantum computing can be replaced as suitable combinations of the basic Pauli Gates, of which there are three - X, Y and Z, corresponding to the three rotation operators.

The Pauli X Gate is given by the following matrix, and is similar in function to a conventional NOT Gate [1].

$$[X] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{1}$$

It essentially interchanges $|0>$ and $|1>$ states, and for this reason is called the Bit-Flip operator. Denoting the QTrick Bit by Q, the Pauli X Gate is mathematically implemented as $X = 1 - Q$.

The Pauli Z Gate is given by the following matrix [1]:

$$[Z] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

This operator leaves $|0>$ unchanged while shifting $|1>$ by a phase of 180 degrees and for this reason is called the Phase-Flip operator. Mathematically, for a QTrick Bit, Pauli Z Gate is $Z = -Q$.

The Pauli Y Gate combines the functionalities of both the X and Z Gates, and is given by the following matrix [1]:

$$[Y] = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{3}$$

The most notable feature of the Pauli Y Gate is its ability to transform between real and imaginary spaces. According to the commutivity principle, the Y Gate can be constructed as a combination of the X and Z gates in this fashion: $ZX = iY$ [1].

Based on the above mentioned mathematical relations, the Pauli X, Y and Z Gates are implemented using Altera Cyclone II 2C20 for a QTrick Bit Q, generated using a pseudorandom number generator for 1000 samples, each of which represents a qubit sampled at that instant. The results for the three Pauli Gates are shown in Fig. (2).

3. QTrick Entanglement and Teleportation

Having studied the basic Pauli gates, the next step is to explore the phenomenon of Entanglement, which occurs when the phase relationship between two Qubits forces the observer to describe one particle in terms of another [13, 14, 15, 16]. In other words, any measurement made on one particle instantaneously affects the other. This is what Einstein famously described as spooky action at a distance [4]. The most celebrated way of generating entanglement is by creating the Bell states. The Bell states for a two qubit system are given as follows: $|B00 >= 0.707(|00 > +|11 >)$, $|B01 >= 0.707(|01 > +|10 >)$, $|B10 >= 0.707(|00 > -|11 >)$ and $|B11 >= 0.707(|01 > -|10 >)$ [1]. The generation of the Bell states can be explained using a combination of the Hadamard and the CNOT gates. The Hadamard Gate is essentially a quantum interference gate, capable of transforming pure states into superposed states and vice versa. The matrix formulation of a Hadamard Gate is as follows [1]:

$$[H] = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{4}$$

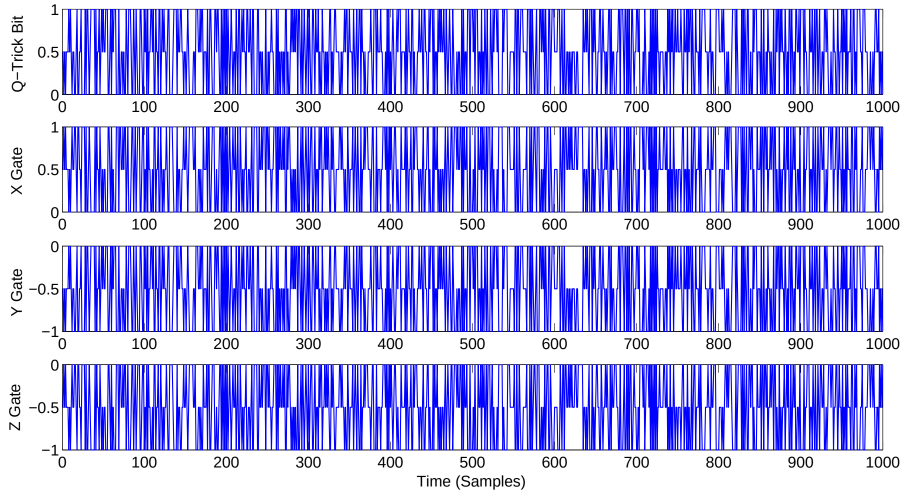

Figure 2: Pauli Gate Implementations for Q-Trick Bits

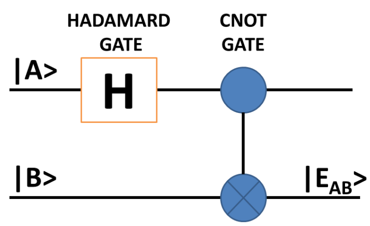

Figure 3: Schematic of Entanglement Operation

For QTrick bits, Hadamard Gate is represented as $H = -0.5 \times [Q - 1]$ The CNOT Gate is a two qubit gate, which performs an X Gate operation on the second qubit (called the target) only when the first qubit (called the control) is at a $|1>$ state. The matrix representation of the CNOT Gate is as follows [1]:

$$[CNOT] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{5}$$

For two QTrick Bits QC (control) and QT (Taget), $CNOT = QC - QT$. Based on these two gates, the creation of Bell Entangled States is represented as a schematic in Fig. (3).

Quantum teleportation is defined as transmitting a quantum state from one place to another without that state traversing the space in between [9]. One bit quantum teleportation can be done using appropriate combinations of entanglement, CNOT, Hadamard, measurement and the Pauli X gates [1]. The measurement operation cases a qubit to collapse into a $|0>$ or $|1>$ state, and for QTrick Bits, the measurement is achieved by the Thresholding operation.

The schematic of a One bit Quantum Teleportation is illustrated in Fig. (4).

As per the mathematical formulations of Pauli Gates, Hadamard and CNOT Gates mentioned above, the Quantum Teleportation is implemented in FPGA level with 2 FPGA's playing the roles of 'Alice' and 'Bob', and the results obtained are as shown in Fig. (5).

From the plots, it is seen that the implementation of quantum teleportation using QTrick Bits is able to accurately reproduce the transmitted message on the receiver side, and the extreme simplicity of design is an added advantage.

4. Nonlinear Characterization of QTrick Entanglement

Owing to the simplicity of design, a nonlinear characterization of the QTrick bits could help understand the properties of the mysterious phenomenon of entanglement. Taking cue from this, the waveforms obtained in the Fig. (5) are

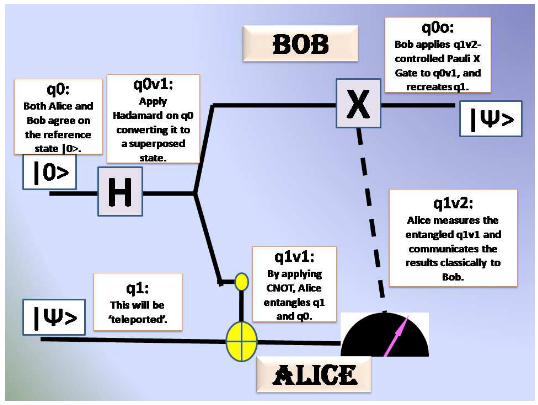

Figure 4: Schematic of One Bit Quantum Teleportation

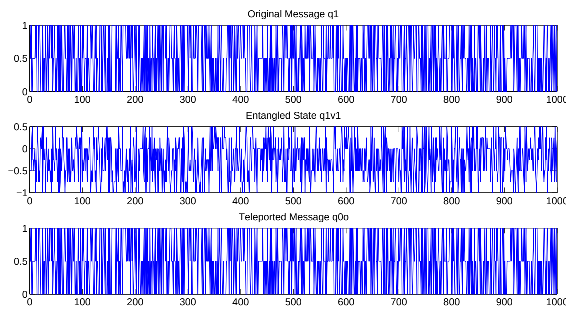

Figure 5: Implementation of Quantum Teleportation using QTrick Bits

analyzed. First, the histograms of the original QTrick bit'q1' and the entangled QTrick bit 'q1v1' are shown in Fig. (6). As can be seen, the entangled state shows a much wider distribution of states between $|0>$ and $|1>$ than the raw states.

An important tool in nonlinear characterization is the phase portrait, which plots the time derivative of a signal (dV/dt) as a function of the signal (V) illustrating the dynamics of the signal in the phase space and describing the stability aspects of the chaotic system behavior, qualitatively serving as a tool to assess various chaotic parameters such as sensitivity and ergodicity [17]. The phase portraits of the Qtrick Bits 'q1' and 'q1v1' before and after entanglement are shown in Fig. (7)

It is clear from the plots that the phase portraits of the entangled state are much more ornamental and intricate, suggesting higher ergodicity and chaoticity.

Finally, the most apt measure to quantitatively ascertain the presence of and to characterize chaos in the generated output is the Largest Lyapunov Exponent (LLE), a measure of a system's sensitive dependence on initial conditions [18, 19]. In the present work, Rosenstein's algorithm is used to compute the Lyapunov Exponents λ_i from the voltage waveform, where the sensitive dependence is characterized by the divergence samples $d_j(i)$ between nearest trajectories represented by j given as follows, C_j being a normalization constant [19]:

$$d_j(i) = C_j e^{\lambda_i (i\delta t)} \tag{6}$$

It is observed that the LLE for the QTrick Bit 'q1' is -0.7304 whereas the corresponding value for the entangled

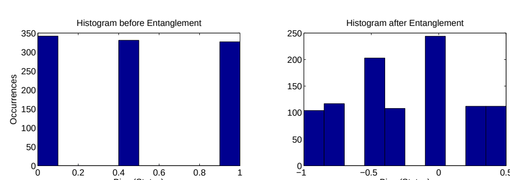

Figure 6: Histograms of QTrick Bits before and after Entanglement

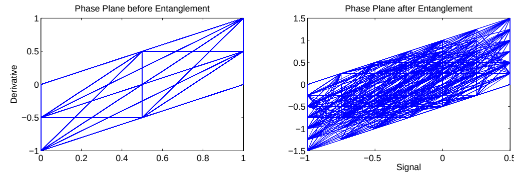

Figure 7: Phase Portraits of QTrick Bits before and after Entanglement

'q1v1' is 1.3324. The positive value of LLE obtained is a clear indication that the process of entanglement indeed induces chaotic nature in the qubits [20].

5. Conclusion

A novel approach to exploit the advantages of quantum computation, using Quantum Tricks is proposed. Specifically, the inherent randomness of a qubit is achieved with a pseudorandom number generator, implemented using FPGA. Basic Pauli gates using this formulation are explored, following which Entanglement is studied. Finally, quantum teleportation is implemented using QTrick bits and it is observed that the quantum tricks proposed in the present work are able to accurately perform the teleportation. Nonlinear Characterization of the entangled qubits using standard measures such as Lyapunov exponents and phase portraits confirm the fact that the operation of entanglement induces chaotic nature in the QTrick bits. The extreme simplicity of the proposed design, combined with the efficiency of implementation of real life phenomena such as quantum teleportation form the highlights of the present work. It is beleived that this will usher in a new era of 'Affordable Quantum Computation'.

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