

# Secure Solitary Wavelets using Simple X-Band Microwave Circuitry

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## Abstract

The operation of various contemporary communication systems such as WiMax in the Microwave X-Band, coupled with the information explosion in recent times has enforced a high pressure in the capacity and security handling aspects, where it would be convenient to provide these facilities at the physical level itself. The present work proposed a radically new solution to this issue, by developing a Secure Solitary Wavelet. Specifically, a Gunn Oscillator Microwave source is fed to a microwave transistor, and by using a slave low frequency signal, the nature of the output is tuned and the envelope is observed to be chaotic, and is characterized using Kolmogorov Entropy and Lyapunov Exponents. This envelope is modulated onto a hyperbolic secant function to create a Secure Solitary Wavelet. It is seen that the solitary nature induces extreme compactness and smoothness as seen by the vanishing higher moments, while the chaotic envelope is controlled by the slave frequency acting as a key and providing a layer of security to the wavelet. A Secure Solitary Wavelet based Orthogonal Frequency Division Multiplexing (OFDM) is proposed and compared with conventional OFDM in terms of performance assessment. The extreme simplicity of the proposed design coupled with the compactness and security of the Secure Solitary Wavelet form the highlights of the present work.

*Keywords:* X-Band, Gunn Diode Source, Frequency controlled Chaos, Secure Solitary Wavelets

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## 1. Introduction

It is a well known fact that most of the recent wireless Communication Protocols developed in recent years including WiMax and other systems based on Orthogonal Frequency Division Multiplexing (OFDM), all use the X-Band Microwave Frequencies (8-12GHz) [1, 2]. Given the critical need for data security in the current era of information explosion, it would be a huge advantage if one could develop a technology which can offer such security in the physical level itself, without converting the data to higher software-based levels, as such software levels tend to have latency based delays, extremely slowing down the communication throughput [3, 4, 5, 6].

The present work purports to address this problem in a radically different way, and this is achieved in two steps.

1. First, it is observed that if the output of an X Band Source (Gunn Oscillator) is connected to one of the three terminals of a Microwave Transistor, a low frequency sinusoidal source connected to a second is able to tune the nature of the output, taken at the third terminal [7, 8]. It is observed that this tunability heavily depends on the controlling low frequency. Also, the envelope of the output is seen to show a chaotic nature, when characterized using standard measures such as Lyapunov Exponent and Kolmogorov Entropy [9, 10, 11].
2. It is well established that most communication systems involving OFDM rely on a domain transform to modulate the subcarriers. In the present work, it is proposed to use a wavelet based transform, rather than the conventional Fourier based Transform [12, 13]. This is because, in the present work, the a hyperbolic secant based solitary function modulated with the chaotic envelope obtained in the previous step is proposed as the wavelet function, and it is seen that the wavelet thus formed possesses extreme smoothness and compactness with vanishing higher moments, with the additional advantage of a frequency controlled security layer obtained by using the chaotic envelope. For this reason, these wavelets are called 'Secure Solitary Wavelets' [14, 15, 16, 17].

The extreme simplicity of design in generating a chaotic envelope in X-Band, coupled with the smoothness, compactness and security aspects of the Secure Solitary Wavelets obtained using such chaotic envelopes forms the novelty of the present work.

## 2. Frequency Controlled Tunability in the X Band

### 2.1. Experimental Setup

In the present work, a Gunn Diode Oscillator connected to a PIN modulator forms the source ('Master') of Microwave X-Band [7]. The generated output is connected to the base terminal of the BFU725F/N1, a Microwave Bipolar Junction

Transistor (BJT) having a noise figure of 0.7db at 5.8GHz and a maximum stable gain of 27dB at 1.8 GHz, through an Isolator and a and Test Gig based Adapter [18, 19]. A low frequency function generator (‘Slave’) has been connected to the emitter terminal of the transistor. Output is measured at the collector terminal of the transistor by using oscilloscope. The experimental setup is shown in Fig. (1).

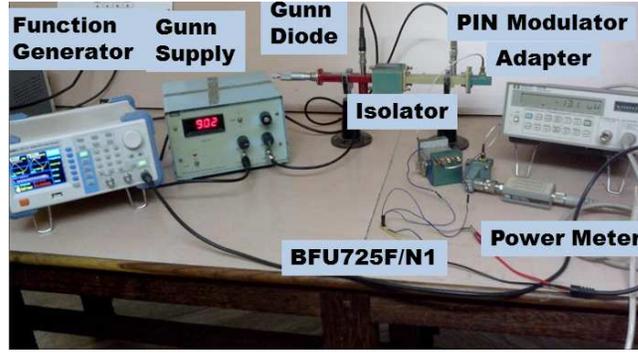


Figure 1: Experimental Setup for the Generation of Frequency Controlled Tunability

The frequency of the slave signal generated by function generator is initially kept constant at 1MHz and the Gunn bias voltage is varied from 5.22V to 9.03V. It is seen that the range of operation from 6.82V to 9.03V shows Negative Resistance Region which results in output signal in microwave region(X-Band). Based on this result, the Gunn Bias is set to 8V, in the negative resistance region [7]. In order to study the effects of transistor nonlinearity and slave signal on the microwave signal, the power spectra of obtained output microwave X-Band signal overlaid on the output power spectra of the Gunn based Source is shown in Fig. (2).

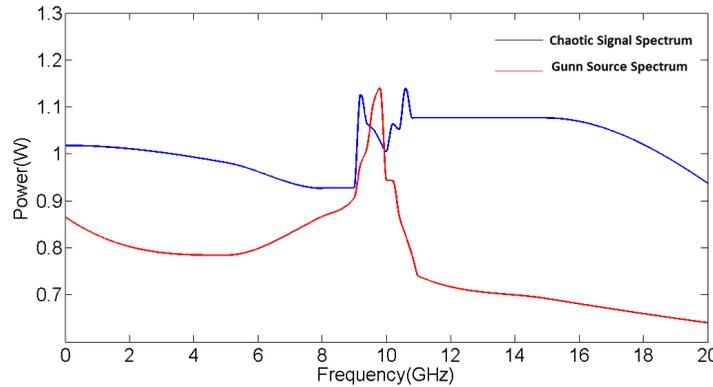


Figure 2: Plot showing the power spectrum of obtained output (chaotic) signal (blue) overlaid on Gunn Source power spectrum (red)

## 2.2. Characterization of Chaos in the Envelope

The envelope of the output microwave signal is obtained and the corresponding phase portrait is plotted in Fig. (3). The phase portrait plots the time derivative of a signal ( $dV/dt$ ) as a function of the signal ( $V$ ) illustrating the dynamics of the signal in the phase space and describing the stability aspects of the chaotic system behavior [20]. From the figure, it is qualitatively established that the envelope indeed shows chaotic behaviour.

The most suitable measure to ascertain the presence of and to characterize chaos in the generated output is the Largest Lyapunov Exponent (LLE), a measure of a system’s sensitive dependence on initial conditions [9]. In the present work, Rosenstein’s algorithm is used to compute the Lyapunov Exponents  $\lambda_i$  from the voltage waveform, where the sensitive dependence is characterized by the divergence samples  $d_j(i)$  between nearest trajectories represented by  $j$  given as  $d_j(i) = C_j e^{\lambda_i(i\delta t)}$ ,  $C_j$  being a normalization constant [10].

The second parameter used to characterize chaos is the Kolmogorov Entropy, which is essentially a statistical measure of the uncertainty of the signal. By assigning each of the  $N$  quantifiable states of the output amplitude as an event  $i$ , the Kolmogorov Entropy  $K_2$  obtained depends on their probabilities  $p_i$  according to the relation  $K_2 = - \sum_{i=1}^N p_i \log p_i$  [11].

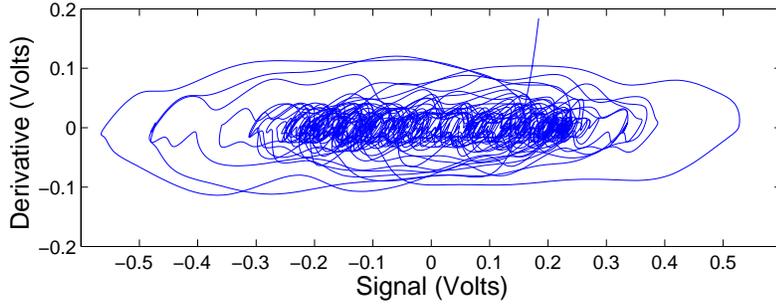


Figure 3: Phase Portrait of the Output Envelope Signal

Table 1: Effect of Slave Frequency on the Nature of Chaos in Output Envelope

Amplitude(V)	Frequency (MHz)	LLE	K2 (Nats/s)
0.015	0.001	22.1526	7.3596
0.015	0.05	20.3935	7.3474
0.015	0.1	21.2456	7.3676
0.015	0.5	19.6538	7.3518
0.015	1	15.3481	7.3746
0.015	5	13.8087	7.3602
0.015	7.5	11.939	7.3477
0.015	10	10.7907	7.3446

The gunn bias voltage is maintained constant at 8V, as mentioned earlier, and the frequency of the slave signal applied at the emitter terminal is varied in the range of 10KHz to 10MHz and the nature of chaos of the output envelope is tabulated in Table 1.

From the table, it is indeed observed that the frequency of the slave is able to significantly alter the nature of chaos in the output envelope and thus pertains to a case of ‘Frequency Controlled Chaos’.

### 3. Formation of the Secure Solitary Wavelet (SSW)

The first step to form an SSW is to define the Scaling Function, also called the ‘Father Wavelet’  $\phi$  in continuous time.  $\phi$  is defined as the mathematical product of a hyperbolic secant  $sech(t)$  and the generated chaotic envelope  $C$ . i.e.  $\phi(t) = C \times sech(t)$ . The choice of hyperbolic secant is dictated by the fact that hyperbolic secant solitons are known to be very smooth, compact functions, making them ideal choices for a wavelet [14, 15].

The Secure Solitary Father Wavelet  $\phi$  thus defined is used as the basis to form the Secure Solitary ‘Mother Wavelet’  $\psi$ , such that the following criteria are satisfied [15, 16]

1.  $\psi(t)$  belongs to a subspace of the space  $L^1(\mathfrak{R}) \cap L^2(\mathfrak{R})$ , the space of absolutely and square integrable measurable functions.
2.  $\phi(t)$  and  $\psi(t)$  are orthogonal to each other.
3.  $\psi(t)$  has zero mean, i.e.  $\int_{-\infty}^{\infty} \psi(t) dt = 0$ .
4.  $\psi(t)$  has unity square norm, as per  $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$ .
5. It is preferable, but not a mandatory criterion to ensure that  $\psi(t)$  possesses a higher number  $M$  vanishing moments. In other words, for all  $m < M$ ,  $\int_{-\infty}^{\infty} t^m \psi(t) dt = 0$ .

Based on the above procedure, the Secure Solitary father and mother wavelets have been formed using the MATLAB Wavelet Toolbox [14, 15, 16]. The Father and Mother Wavelet Signals are plotted in Fig. (4).

Physically, the existence of vanishing higher moments signifies that the wavelet has a compact, continuous, smooth structure, and that the analysis of bursts in signals with such wavelets can be carried out with minimal filtering [14]. In order to investigate and characterize the performance of the Secure Solitary wavelet, the moments upto the tenth order of the Secure Solitary mother wavelet (SSW) are computed and compared with the corresponding moments of three established wavelets, namely Daubechies 4 (DB4), Biorthogonal 4.4 (BIOR4.4) and the Discrete Meyer Wavelet (DMEY) [14, 15, 16].

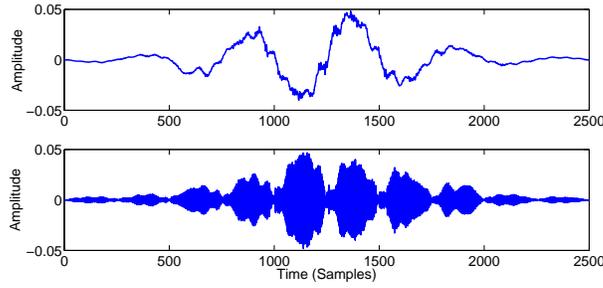


Figure 4: Father (above) and Mother (below) Signals of the Secure Solitary Wavelet

It is observed that even in the Meyer wavelet, known to exhibit vanishing moments up to very large orders, moments increase after a certain order (sixth). In order to capture the trends of the higher moments, the moments of the above mentioned wavelets as well as few others, from the third order onwards are plotted on a logarithmic scale in Fig. (5). It

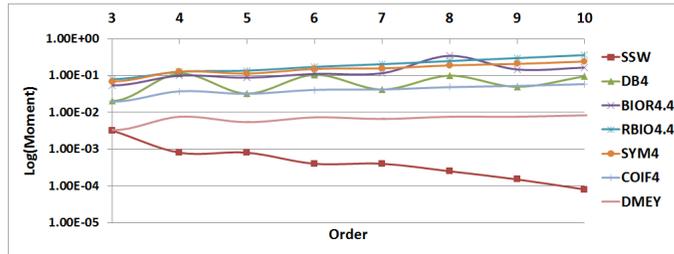


Figure 5: The Moments (upto Tenth Order) of Secure Solitary Wavelets compared with contemporary wavelets on a logarithmic scale. The negative slope vanishing fast towards zero clearly sets apart the Secure Solitary wavelet

is clearly seen that while all the other wavelet moments including those of the Daubechies and Meyer show an increasing trend, the Secure Solitary wavelet moments show a decreasing trend with a negative logarithmic slope. This indicates that the moments of the solitary wavelet rapidly decay and vanish toward zero. This gives the Secure Solitary wavelet the exclusive advantages of smoothness, compactness along with the inherent security of the frequency dependent chaos with the control parameter  $r$  acting as the secure key.

In order to assess the performance of the Secure Solitary Wavelet in state-of-the-art Wireless Communication Systems, a Quadrature Phase Shift Keying based Orthogonal Frequency Division Multiplexing (QPSK-OFDM) system with the Fourier Transform block replaced with the Secure Solitary Wavelet Transform Block is proposed and numerically implemented using MATLAB. Additive White Gaussian Noise (AWGN) is used to simulate the channel with the Signal to Noise Ratio (SNR) is set to 10dB. The Power Spectral Density of the transmitted wideband signal in Fig. (??).

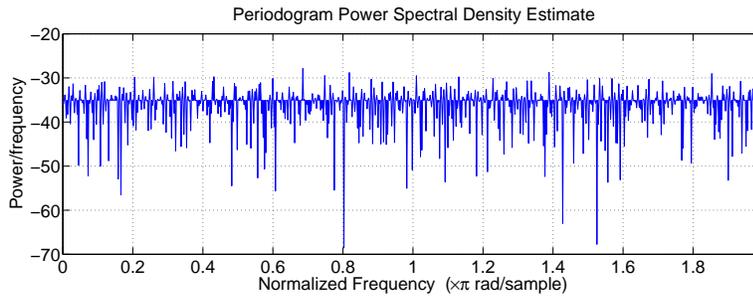


Figure 6: Power Spectral Density of the Wide Band Signal using Secure Solitary Wavelet based QPSK-OFDM

Finally, the Bit Error Rates are for the Secure Solitary Wavelet based OFDM (SSW) plotted in Fig. (7) as a function of SNR in comparison with conventional Fourier-OFDM systems, as well as Wavelet-based OFDM systems using other wavelets, such as Daubechies and Discrete Meyer Wavelets.

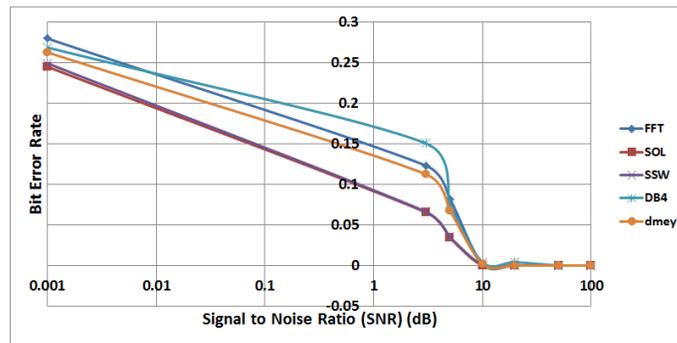


Figure 7: Bit Error Rates of the SSW-QPSK-OFDM as a function of SNR, compared with conventional Fourier OFDM as well as Wavelet-OFDM using Db4 and DMeyer Wavelets

#### 4. Conclusion

A Gunn diode based X-Based is connected to a microwave transistor and the effect of frequency based tuning of a low frequency ‘slave’ signal on the nature of output is examined. Specifically, characterization using Lyapunov Exponents and Kolmogorov Entropy indicates chaotic nature of the output envelope signal. This is used as the basis for modulating a hyperbolic secant function to form a ‘secure solitary wavelet’ (SSW), which is found to have rapidly vanishing higher moments with a distinct negative logarithmic slope. This enables the SSW to have extreme smoothness and compactness, rendering it very useful as a wavelet, while the chaotic envelope is controlled by the slave frequency, thus acting as a ‘key’ providing an additional layer of security. The performance of the SSW is assessed using an OFDM system replacing Fourier transform blocks by the SSW Transform. It is opined that this SSW would be the curtain riser for the new era of fully hardware, microwave communication with high throughput and security, operating fully on the physical level without having to resort to higher software based levels.

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